# ON *VEDĀŃGA* ASTRONOMY: THE EARLIEST SYSTEMATIC INDIAN ASTRONOMY

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**Abstract:** In the history of astronomy in India, the *Vedāriga* astronomy is the first systematic mathematical astronomy. In this paper, I would like to describe this system, and compare it with ancient Mesopotamian astronomy. In the case of the *Vedāriga* astronomy, some people suspected that it was influenced by ancient Mesopotamian astronomy, but I shall show that they are independent, and they were created at their own places which are located at different latitude.

# 1 INTRODUCTION

The chronological development of Indian astronomy is overviewed by Ôhashi (1998; 2009) but can roughly be summarised as follows:

- (1) Indus Valley civilisation period.
- (2) Vedic period (ca.1500 BC ca.500 BC).
  - (2a) Rg-vedic period (ca.1500 BC ca.1000 BC).
  - (2b) Later Vedic period (ca.1000 BC ca.500 BC).
- (3) Vedānga astronomy period.
  - (3a) Period of the formation of the *Vedāriga* astronomy (sometime between the 6<sup>th</sup> and 4<sup>th</sup> centuries BC?).
  - (3b) Period of the continuous use of the *Vedā nga* astronomy (up to sometime between the  $2^{nd}$  and  $5^{th}$  centuries AD?).
- (4) Period of the introduction of Greek astrology and astronomy.
  - (4a) Period of the introduction of Greek horoscopy (the second (?) or third century AD).
  - (4b) Period of the introduction of Greek mathematical astronomy (sometime around the fourth century AD?).
- (5) Classical Siddhānta period (Classical Hindu astronomy period) (the end of the fifth century to the twelfth century AD).
- (6) Coexistent period of the Hindu astronomy and Islamic astronomy (the thirteenth/fourteenth century to the eighteenth/nineteenth century AD).
- (7) Modern period (Co-existent period of the modern astronomy and traditional astronomy) (the eighteenth/nineteenth century onwards).

Among them, the *Vedāniga* astronomy is the first systematic mathematical astronomy. In this paper, I would like to discuss this system from astronomical point of view, and compare it with Mesopotamian astronomy. This paper is based on the first half of my oral paper "Two Systems of Indian Astronomy" presented at ICOA-7 (2010, Tokyo), whose second half was on Classical Hindu astronomy. The second half is omitted here due to the page limitation imposed.<sup>1</sup>

# 2 VEDĀŅGA ASTRONOMY

### 2.1 The Beginnings of Indian Astronomy

The earliest decipherable literature in India is a class of texts called *Veda*, which was produced by Indo-Aryans who are said to have migrated to Northwest India in ca. 1600 BC. They first produced the *Rg-veda* between 1500 and 1000 BC or so, and then produced Later Vedic literature between 1000 and 500 BC or so.

The Indo-Aryans were originally pastoral people. They were mainly in the Panjab in the Rg-vedic period, and had some astronomical and calendrical knowledge.

They migrated to the basin of the Ganga River in the Later Vedic period, and became essentially agriculturalists. Accordingly, a more accurate calendar was required. It should be noted that some rituals which symbolize the divisions of time were developed in this period, including the new and full moon offerings and four monthly offerings.

Towards the end of the Later Vedic period, the *Vedāriga*, which is a kind of supplemental learning for the *Veda* and consists of six divisions, was created. One of its divisions is on calendrical astronomy and is called *jyotisa*. Let us call the type of Indian astronomy present at this stage '*Vedāriga* astronomy'. Its fundamental text, entitled *Jyotisa-vedāriga* (or *Vedāriga-jyotisa*), is extant (see Sastry, and Sarma, 1984; and also Kauņḍinnyāyana, 2005; Mishra, 2005). I think that it was created sometime between the sixth and fourth centuries BC. There are two recensions (Rg-vedic and Yajur-vedic) of this text, but they are not so different.

The *Vedāriga* calendar was a luni-solar calendar with 366-day year, and had two intercalary months in five years. The reason why calendrical astronomy was included in the *Vedāriga* was that a calendar was

necessary for the determination of the proper time of rituals. We should not forget that the development of the rituals was connected with the development of agriculture and the calendar. The origin of the Indian astronomy looks ritualistic religion at first sight, but the astronomy actually originated in productive activities such as agriculture, and the rituals were the symbols of these activities.

In the modern Indian languages (such as Hindi), the word *jyotişa* means astrology rather than astronomy, but the original *jyotişa* of the *Vedāriga* was calendrical astronomy and was not astrology. It is wrong to consider that ancient astronomy was mixed with astrology. As an accurate calendar is indispensable for productive activities such as agriculture, calendrical astronomy was forced to be rational from the beginning. For further details of Vedic astronomy see Ôhashi (1993).

#### 2.2 The Structure of *Vedān*ga Astronomy

The contents of the *Jyotiṣa-vedāriga* are almost exclusively calendrical. The calendar described there is a kind of luni-solar calendar, where two intercalary months are inserted in a five-year cycle called *yuga*. Two intercalary months were inserted at the middle and the end of the five-year *yuga*. The calendrical system of the *Jyotiṣa-vedāriga* can be summarized as follows.

1 sāvana day (civil day) is from sunrise to sunrise.

- 1 sāvana month is 30 sāvana days.
- 1 *tithi* is 1/30 of a synodic month.
- 1 synodic month is from new moon to new moon.
- 1 solar month is 1/12 of a solar year.
- 1 *rtu* (season) is 1/6 of a solar year.
- 1 solar year is from the winter solstice to the winter solstice.
- 1 solar year = 2 ayanas (half years);
  - = 6 *rtu*s (seasons);
  - = 12 solar months;
  - = 366 sāvana days (civil days);
  - = 372 *tithi*s.
- 1 *yuga* = 5 years;
  - = 60 solar months;
  - = 61 sāvana months = 1830 sāvana days;
  - = 62 synodic months = 1860 *tithi*s;
  - = 67 sidereal months;
  - = 1835 sidereal days.

In the *Jyotişa-vedāriga*, celestial longitude was expressed using a *nakṣatra* (lunar mansion). One *nakṣatra* used there is a segment which is equivalent to 1/27 of the ecliptic. The system of 28 or 27 *nakṣatra*s already appeared in some of the later Vedic literature, but the *nakṣatra*s described in the later Vedic literature must have consisted of the actual visible stars. The *Jyotiṣa-vedāriga* started to use it as an artificial system of coordinates. It may be mentioned here that the systems of 28 and 27 *nakṣatra*s are used for different purposes in the later Hindu astronomy since the Classical Siddhānta period, the system of 28 *nakṣatra*s as actual stars, and the system of 27 *nakṣatra*s as artificial coordinates. The *Vedāriga* astronomy may be considered to be the beginning of this division.

#### 2.3 Length of Day Time

2.3.1 The Indian Linear Zig-zag Function with the Ratio 3:2

A kind of zig-zag function was used in order to calculate the length of day time in *Vedāriga* astronomy as well as in Ancient Mesopotamia. Some people suspected that the Mesopotamian function was transmitted to India (e.g. see Pingree, 1973). However, I would like to show that both Indian and Mesopotamian functions were created independently, and that each function was based upon actual observations made in each region, but at quite different latitudes.

The Jyotișa-vedāriga (Rg -vedic recension vs.7, and Yajur-vedic recension vs.8) reads:

The increase of day time and decrease of night time is [the time equivalent of] one *prastha* of water [in the clepsydra per day] during the northward course [of the Sun]. They are in reverse during the southward course. [The total difference is] 6 *muhūrtas* during a half year.

Another verse (Rg -vedic recension vs.22, and Yajur-vedic recension vs.40) reads:

[The number of days] elapsed in the northward course or remaining in the southward course is doubled, divided by 61, and added to 12. The result is the length of day time ... [in terms of *muhūtras*].

The above rule can be expressed as follows:

$$T = \left(12 + \frac{2}{61}n\right)$$

(1)

Here, *T* is the length of day time in terms of *muhūrta*s, and *n* is the number of days that have elapsed from or remain until the winter solstice. One *muhūrta* is 1/30 of a day. This is a kind of linear zigzag function,

where the length of the day time changes by one *muhūrta* during one solar month.

According to Equation (1), the proportion of day time and night time is 2:3 at the time of the winter solstice. This proportion is observed at the latitude of around 35° N. So, some people have conjectured that this function was produced at about this latitude (e.g. see Pingree, 1973). This conjecture is based on the assumption that this function is the result of the interpolation of data at the solstices (see Figure 1). However, there is also a possibility that this function is the result of the extrapolation of data around the equinoxes. Let us discuss about this possibility.

A source belonging to Later Vedic literature tells that Vedic people observed the Sun (most probably in the direction of sunrise) which moved constantly during its northward and southward courses, and considered that it was stationary around the solstices. The *Kauṣītaki-brāhmaṇa* (XIX.3), one of the Later Vedic texts, reads:

On the new moon of Māgha he rests, being about to turn northwards ... He goes north for six month ... Having gone north for six months he stands still, being about to turn southwards ... He goes south for six months ... Having gone south for six months he stands still, being about to turn north ... (Keith, 1920: 452).

In this text, "he" refers to the Sun. It is seen from this text that Vedic people noticed that the Sun (probably the direction of sunrise) moved constantly, except for a certain period around the solstices. It was also considered that the Sun "stands still" around the solstices. From this fact, we are obliged to think that Vedic people thought that a linear function should be based on the data excluding those around the solstices, and that Equation (1) in the *Jyotiṣa-vedānga* was not obtained by interpolation from the observations at the solstices, but was obtained by extrapolation from observations of the length of day time around the equinoxes. Practically, there were two possibilities: (I) If the formula was obtained from

one *muhūrta*'s difference of the length of day time during one solar month after the equinox, the most suitable latitude for this observation was around 27°N. (2) If it was obtained from two *muhūrtas*' difference during two solar months, the most suitable latitude was around 29°N. I graphed Equation (1) together with the actual seasonal change of the length of day time at the latitudes 35°N, 29°N, and 27°N in Figure 1.

From the above consideration, I conclude that Equation (1) is the result of extrapolation of data around the equinoxes observed at a latitude of 27°~29°N, and that the *Jyotişa-vedāriga* was produced in North India (most probably in the western part of the basin of the Ganga River where Later Vedic people resided) without apparent foreign influence (see, also, Ôhashi, 1993 and 2002).



#### 2.3.2 Mesopotamian Functions

The history of ancient Mesopotamia can roughly be divided as follows:

- (1) Sumerian and Old Akkadian Periods.
- (2) Old Babylonian and Old Assyrian Periods (from about the twentieth century BC to about the sixteenth century BC).
- (3) Middle Babylonian and Middle Assyrian Periods (from about the sixteenth century BC to ca.1000 BC).
- (4) Neo-Babylonian and Neo-Assyrian Periods (from ca.1000 BC to the end of the seventh century BC).<sup>2</sup>
- (5) Late Babylonian Period (from the end of the seventh century BC to the first century AD).
  - (5.1) Neo-Babylonian (Chaldean) Dynasty (625 BC ~ 539 BC).
  - (5.2) Achaemenid Dynasty (559 BC ~ 330 BC).
  - (5.3) Seleucid Dynasty (312 BC ~ 64 BC).

#### 2.3.2.1 The Mesopotamian Linear Zigzag Function with the Ratio 2:1

Already in the Old Babylonian tablet BM 17175+17284<sup>3</sup> the lengths of day time and night time at the equinoxes and solstices are given (see Hunger and Pingree, 1989: 163-164 and Plate XIIa; cf. Brown, 2000: 128-129, 249; Hunger and Pingree, 1999: 50). The length is given in terms of *mina*, which is used to measure the weight of water poured into a water clock. One *mina* corresponds to 4 hours, and one day corresponds to 6 *mina*. It has the following values.

Summer solstice: day time = 4 *mina*, night time = 2 *mina*. Autumnal equinox: day time = 3 *mina*, night time = 3 *mina*. Winter solstice: day time = 2 *mina*, night time = 4 *mina*, Vernal equinox: day time = 3 *mina*, night time = 3 *mina*.

It is not clear whether the above value was meant to be a step function or a linear zig-zag function.

The linear zigzag function of the length of day time with the ratio 2:1 is given in *Enūma Anu Enlil* XIV, Table C (Al-Rawi and George, 1991/1992: 57-58; cf. Brown, 2000: 128-129, 254-256; Hunger and Pingree, 1999: 44-50; Rochberg, 2004: 73-75). It tells us that the length of day time (or night time) changes from

14.4

13.6

12.8

12

Length of daytime (hours)



35°N

30

2 mina to 4 mina, and that the length changes linearly by one-sixth *mina* per quarter month. I graphed this function of the length of the day time in Figure 2 together with the actual of seasonal change in the length of the day time at the latitude 35°N. The Mesopotamian function looks different from the actual change at first sight, and was considered to be "very inaccurate", "incorrect for Mesopotamia" etc. by previous researchers. However, I suspect that the Mesopotamian linear function is the result of the extrapolation of data collected around the equinoxes. We should note that the Mesopotamian linear function gives a more or less acceptable value around the equinoxes. This point will become clearer when we discuss the later development of the polygonal function.

# 2.3.2.2 The Mesopotamian Linear Zigzag Function with the Ratio 3:2

According to Brown (2000: 120, 261), the earliest attestation of the ratio 3:2 is in the late Neo-Assyrian period in BM 36731 (see Neugebauer and Sachs, 1967: 183-190). This text implies the ratio 3:2 of the length of day time and night time at the solstices, but it is not clear whether the linear zigzag function was implied or not. A table which is based on the linear zigzag function with the ratio 3:2 is given in the Late Babylonian tablet BM 29371 (see Brown, Fermor and Walker, 1999/2000: 144-148). In this tablet, the value is given for every 5 days.

I graphed this function of the length of day time in Figure 3 together with the actual seasonal change of the length of the day time at the latitude 35°N. The Mesopotamian function is evidently the result of the interpolation of data obtained at the solstices.

#### 2.3.2.3 The Mesopotamian Polygonal Function with the Ratio 3:2

900

Fig.3

1200

Mesopotanian zig-zag function (ratio 3:2)

The mathematical astronomical texts in the Seleucid Dynasty, which is the last phase of the Late Babylonian Period, have been collected together in the *Astronomical Cuneiform Texts* (≡ ACT) of Otto Neugebauer (1955).

Table 1: Changes in the length of day time with solar longitude.

600

Solar longitude (°)	Length of day time (large-hours)
0	3;00
30	3;20
60	3;32
90	3;36
120	3;32
150	3;20
180	3;00
210	2;40
240	2;28
270	2;24
300	2;28
330	2;40

The polygonal function of the length of day time with the ratio 3:2 is given in Section 2 of ACT 200 (= BM 32651) (see Neugebauer, 1955,1: 187; 3: Plates 223 and 234) and ACT 200b (= BM 33631) (see Neugebauer, 1955,1: 214; 3: Plate 236), which belong to the 'Procedure texts' of "System A" (cf. Neugebauer, 1955,1:47, and 1975, 1:369-371). The value is given for every 30° of the solar longitude starting from the vernal equinox as listed in Table 1. For other longitudes, the value is obtained by linear interpolation. Therefore, this is a kind of polygonal function. The length of day time is given in terms of 'large-hours', where one 'large-hour' is 4 hours, and one day is 6 'large-hours'. Fractions are sexagesimally expressed. A sexagesimal point is

expressed by a semicolon instead of a dot for the decimal point.

I graphed this polygonal function of the length of day time in Figure 4, together with the linear zigzag

function with the ratio 2:1 and that with the ratio 3:2, and the actual change at the latitude 35°N.

#### 2.3.2.4 The Polygonal Function as a Synthesis of Extrapolation and Interpolation

From Figure 4 it is seen clearly that the polygonal function from the solar latitude 0° to 30° is exactly the same as the linear zigzag function with the ratio 2:1. From this fact, I suppose that the astronomers during the Seleucid Dynasty considered that the linear zigzag function with the ratio 2:1 was the result of the extrapolation of data collected around the equinoxes, as I suspected. So, it will be justified to say that the polygonal function is the result of the synthesis of extrapolation (linear function with the ratio 2:1) and interpolation (linear function with the ratio 3:2).

#### 2.3.3 The Indian Function and the Mesopotamian Function

We have seen that the Indian function is based on the data observed at the latitude of 27°~29° N, and that their preference of extrapolation can be understood from a description in the Vedic literature. And also, we have seen that Mesopotamian functions are

based on the data collected at the latitude of about 35° N. From these facts, I conclude that the Indian function and the Mesopotamian function are independent. This point will become clearer following a comparison of the functions of the length of the gnomon shadow.

### 2.4 The Gnomon Shadow

#### 2.4.1 The Gnomon Shadow in India

The Artha-śāstra, of which the actual date is controversial, is a political work and is traditionally attributed to Kautilya, a minister of Candragupta Maurya (throned in 321 BC). The Artha-sastra also records knowledge of Vedāriga astronomy (and for an edited text with an English translation see Kangle, 1965-1972).

The Artha-śāstra (II.20.41-42) gives the annual variation of the length of the gnomon shadow, and I have graphed it in Figure 5, together with the actual variation at 21° N, 27° N, and 35° N. From this figure, it is clear that the variation of the Artha-śāstra is based on the observation in North India (around 27° N).

The Artha-śāstra (II.20.39–40) also lists the diurnal variation in the length of the gnomon shadow which is given by the following equation (after Abraham, 1981), which gives almost the correct value at the summer solstice at the Tropic of Cancer.

$$\frac{d}{2t} = \frac{s}{g} + 1$$

Here, t/d is the fraction of day time (d stands for the duration of day time, and t the time elapsed since sunrise or remaining until sunset measured by the same unit as d), s is the length of the shadow of the gnomon of length g.

The Yavana-jātaka (AD 269/270) of Sphujidhvaja is the earliest extant Sanskrit work on Greek horoscopy (and for an edited text with

an English translation see Pingree, 1978). Most of its contents is Greek horoscopic astrology,<sup>4</sup> and only its last chapter (Chapter 79) is devoted to mathematical astronomy. The text states that "the instruction of the The text Greeks" (Yavana-upadeśa) is explained there, but developed Greek geometrical astronomy (the epicyclic theory, etc.) is not found there. On the contrary, according to my analysis, a certain theory which is part of Indian Vedānga astronomy is found there. Let us examine the variation in the length of the gnomon shadow.



Fig.5



(2)



The Yavana-jātaka (Chapter 79, verse no.32) tells us that the diurnal variation of the gnomon shadow is given by the following formula (after Abraham, 1981):

$$\frac{d}{2t} = \frac{s-s'}{\sigma} + 1 \tag{3}$$

Here, t/d is the fraction of day time which has elapsed since sunrise or remains until sunset, *s* is the length of the shadow of the gnomon of length *g*, and *s'* is its midday shadow.

I have graphed the diurnal variation according to Equation (3)<sup>5</sup> at the equinoxes and solstices in Figure 6, together with the actual variation at latitude 23.7° N (latitude of the Tropic of Cancer at the time of Kauțilya).

As regards the value of the midday shadow, I used the correct value.

#### 2.4.2 The Gnomon-shadow in Mesopotamia

According to Otto Neugebauer (1975, 1: 544-545) the Mesopotamian shadow table in Mul.Apin (II.ii.21-42; see Hunger and Pingree, 1989: 96-101, 153-154) can be obtained from the following formula.

$$t = \frac{c}{s}$$

(4)

Here, *t* is the time after sunrise which is counted in time degrees (1 day =  $360^{\circ}$ ), *s* is the length of the shadow in terms of cubits, and *c* is a constant. The amount of the constant, *c*, is 60 at the winter solstice, 75 at the equinoxes, and 90 at the summer solstice. Strangely, the midday shadow is always 5/6 cubit, and the reason of this unreality is not known.

I have graphed the diurnal variation according to the Equation (4) at the equinoxes and solstices in Figure 7 together with the actual variation at the latitude of 35° N. As regards the value of the midday shadow, I used the correct value.

#### 2.4.3 The Gnomon Shadow in India and Mesopotamia

From Figures 6 and 7 we can suppose that the Indian Equation (3) originated from observation made in India at the time of the summer solstice, while the Mesopotamian Equation (4) seems to have originated from observations made in Mesopotamia at the time of equinoxes. They are mathematically different, and must have been derived independently.



3 CONCLUSION

No researcher is free from preconception. It is also a fact that previous authorities' wrong preconceptions, sometimes even against their own wishes, produced chances for newcomers to make breakthroughs.

The assumption that the ancient astronomers only used interpolation is also a preconception. Some previous researchers considered that the Indian linear zigzag function with the ratio of 3:2 and the Mesopotamian linear zigzag function with the ratio of 2:1 were quite inaccurate, because they assumed that the ancient astronomers used interpolation. Some of the

previous researchers even suspected that the Indian linear zigzag function with the ratio 3:2 was borrowed from Mesopotamia, although there is no historical evidence to prove this.

I have shown here that ancient astronomers in India and Mesopotamia must have used extrapolation, and that they must have produced their astronomy independently, which could have been developed at a particular latitude. The Indian preference for extrapolation can be understood given the historical background provided in the Vedic literature. And also, the dialectical development of the Mesopotamian function can be traced, which must have taken place at their proper latitude.

Some previous researchers laid too much stress on the transmission of ancient astronomical ideas, but what is more important is to understand ancient astronomy in its own cultural context.

## 4 NOTES

1. Some parts of the first half of this paper, which is presented in this volume, were presented as an unpublished oral paper at the 22<sup>nd</sup> International Congress of History of Science (Beijing, 2005), and its Japanese summary has been published as Ôhashi (2006). The second half, which is omitted here, was further discussed at the 4<sup>th</sup> Symposium on "History of Astronomy" held at the National Astronomical Observatory of Japan, Tokyo, in January 2011, and will be published in its proceedings.

2. This 'Neo-Babylonian period' should not be confused with the 'Neo-Babylonian Dynasty' which appears later in this listing.

3. Here 'BM' is used to denominate tablets in the British Museum, London.

4. Along with Greek astrology, zodiacal signs and the seven-day week were introduced into India.

5. Equation (3) becomes the same as the Equation (2) at the summer solstice.

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