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Mathematics and astronomy in traditional India

The history of Indian astronomy can roughly be summarised as follows.

(i) Indus civilization period (ca.2600/2500 BCE – ca.1900 BCE).
(ii) Vedic period (ca.1500 BCE – ca.500 BCE).
(iii) *Vedāṅga* astronomy period (From sometime between the 6th and 4th centuries BCE up to sometime between the 3rd and 5th centuries CE?).
(iv) Period of the introduction of Greek astrology and astronomy (Sometime around the 3rd and 4th century CE?).
(v) Classical *Siddhānta* period (Classical Hindu astronomy period). (From the end of the 5th century up to the 12th century).
(vi) Coexistent period of the Hindu astronomy and Islamic astronomy (From the 13/14th century up to the 18/19th century).
(vii) Modern period (Coexistent period of the modern astronomy and traditional astronomy). (From the 18/19th century onwards).

[For an overview of Indian astronomy, see Ôhashi (1998) in Japanese or more detailed Ôhashi (2009) in English. For an overview of Indian mathematics, see Datta and Singh (1935 – 38), Sarasvati Amma (1979), and Plofker (2009) in English, and Hayashi (1993) in Japanese.]

Indian numerals:

Decimal place value system.

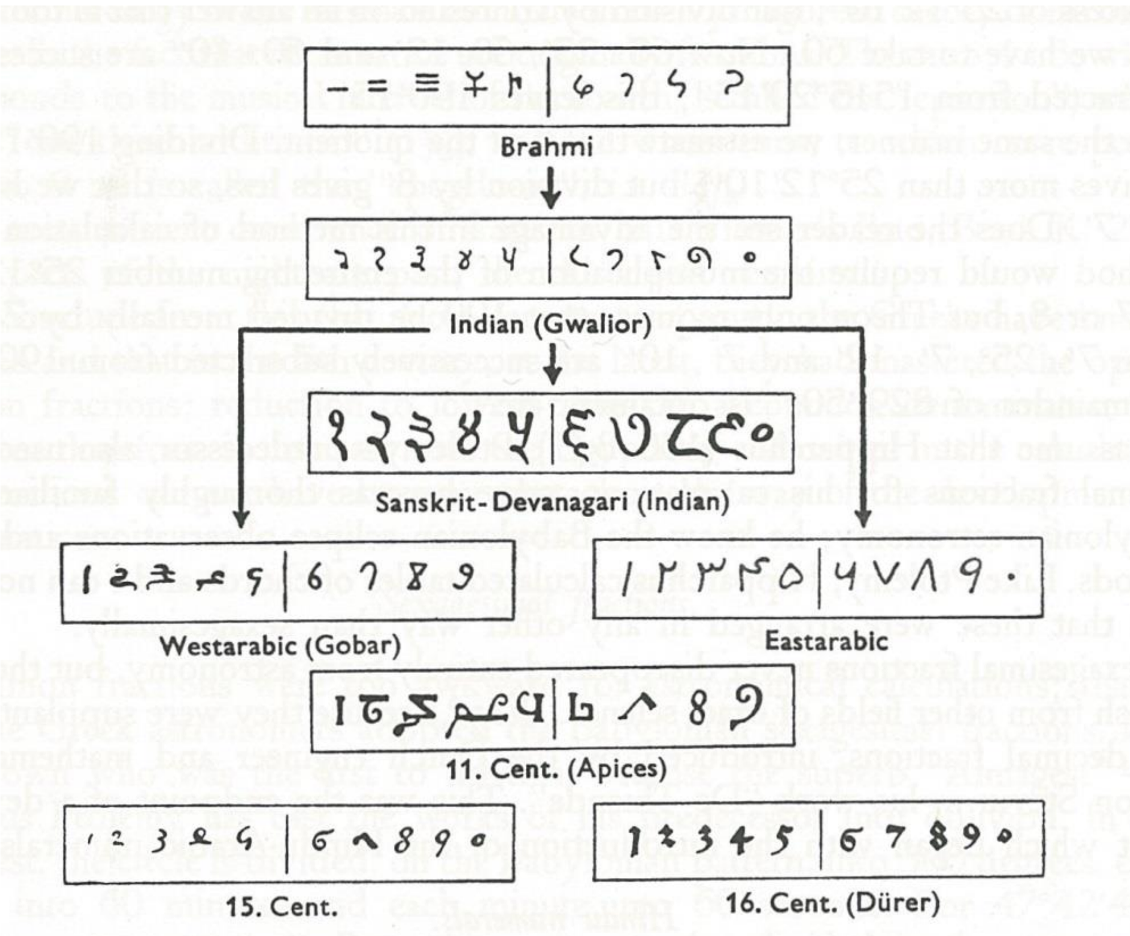


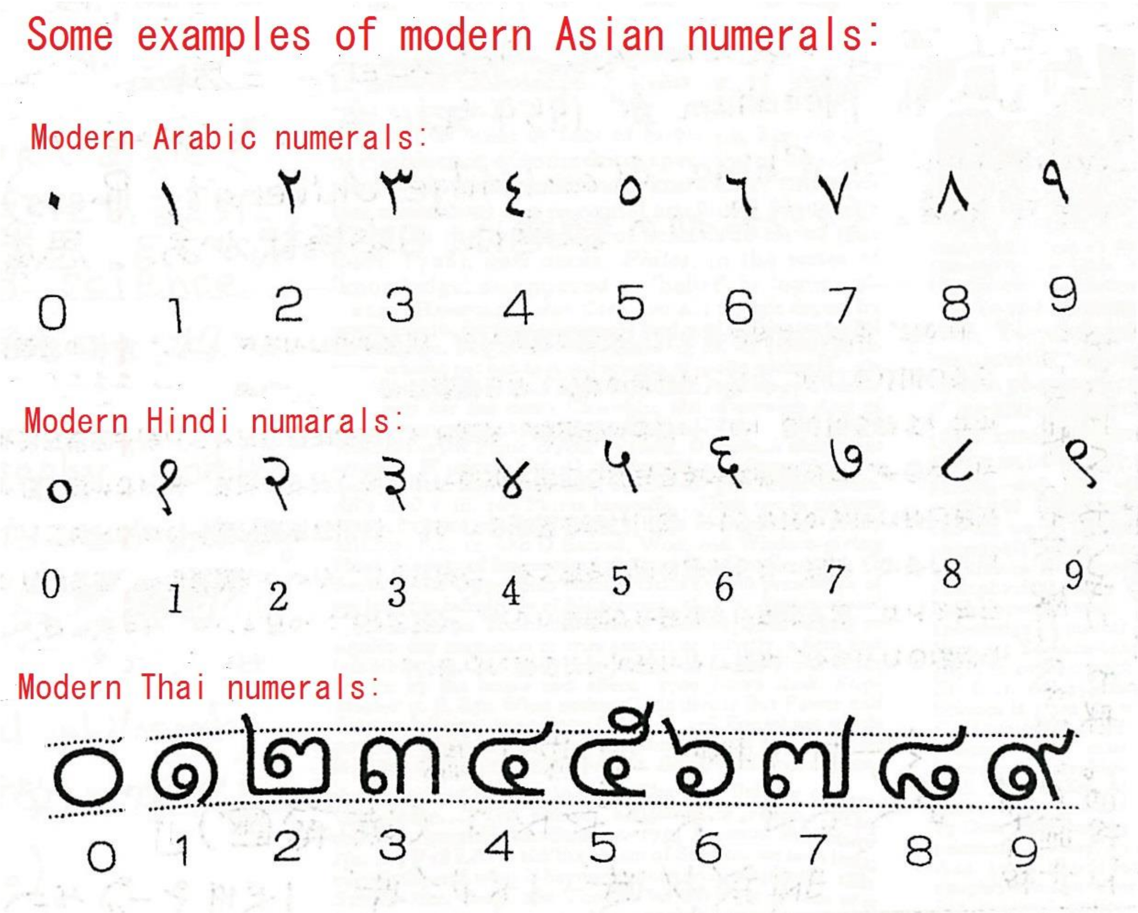
Fig. 10. Genealogy of our digits. From Menninger, Zahlwort und Ziffer. p. 329

(From van der Waerden: *Science Awakening I*, 1961, p.52.)

The Indian system was transmitted to the Islamic World, and then transmitted to Europe.

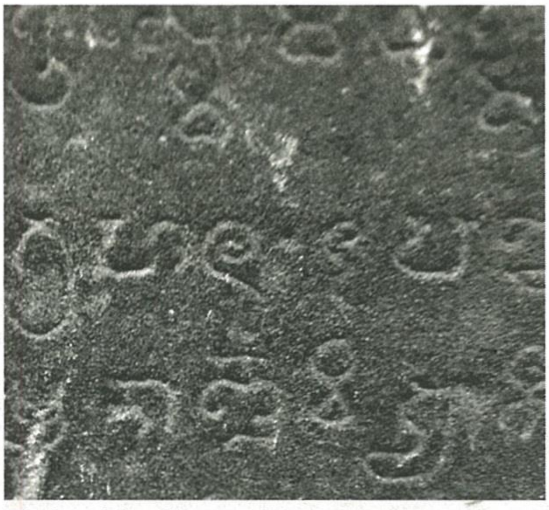
It should be noted that the shape of the figure which is now usually used is the modern European style, and the shapes of traditional figures in different areas are different.

Let us see some examples of modern Asian numerals.



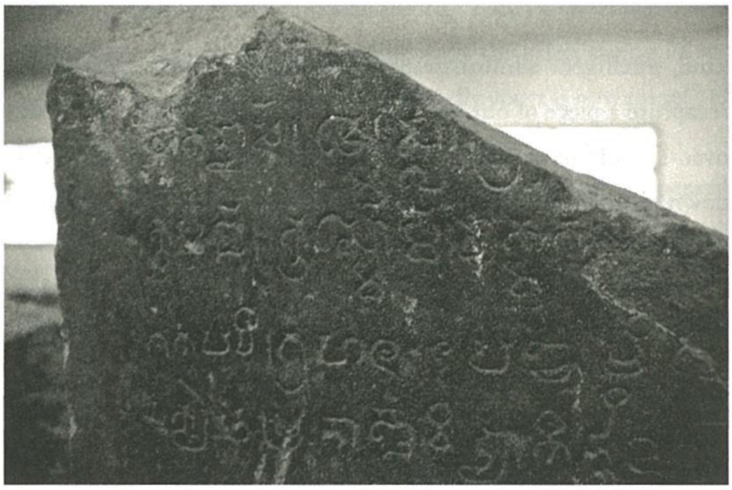
Now, let us see the numerals in an early inscription and a manuscript:

One of the earliest inscriptions which has the symbol of "zero":



The face of the stele K-127, showing the ancient writings, including the number 605.

The numerals 605, the zero being the dot in the middle, from the photograph above.



The upper part of artifact K-127, showing its broken top. The numerals 605 are on the second line from the bottom.

An inscription found in Cambodia. It has the date "Śaka 605" which corresponds to 683 CE. (Śaka is an Indian Era.)

(From Aczel: *Finding Zero*, New York, St. Martin's Press, 2015, p.177.)

The Indian numerals in the Bakhshālī Manuscript:

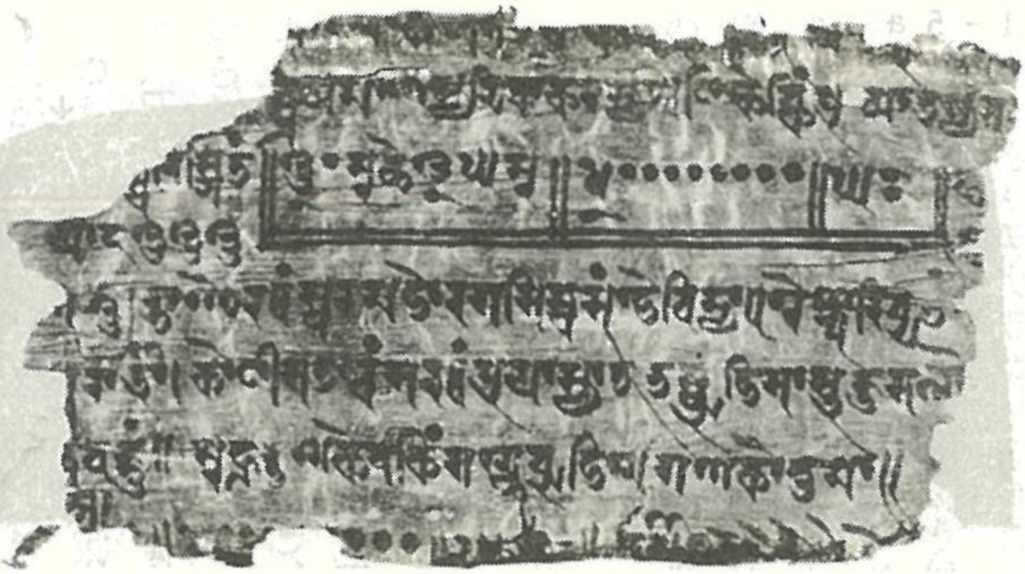


図1－6 『バクシャーリー写本』のゼロ記号

3行目の枠囲いは、

30 mu che 2 gha mu	500000000	gha 1
1 1	1	1

これは、「1日（＝30ムフールタ）で50コーティ（＝50×10⁷）ヨージャナ進む太陽は、1ガティカー（＝½ムフールタ）ではどれだけ進むか？」という問題に対する三量法の表示（G.R. Kaye, *The Bakhshālī Manuscript : A Study in Medieval Mathematics*, Archaeological Survey of India, 1927/33, fol. 37R）

(From Hayashi: 林隆夫『インドの数学』、中公新書、1993, p.30.)

The Bakhshālī Manuscript is the earliest Sanskrit mathematical manuscript in India (probably composed in the 7th century CE or so).

Now, let us see the development of mathematics and astronomy in India from ancient time.

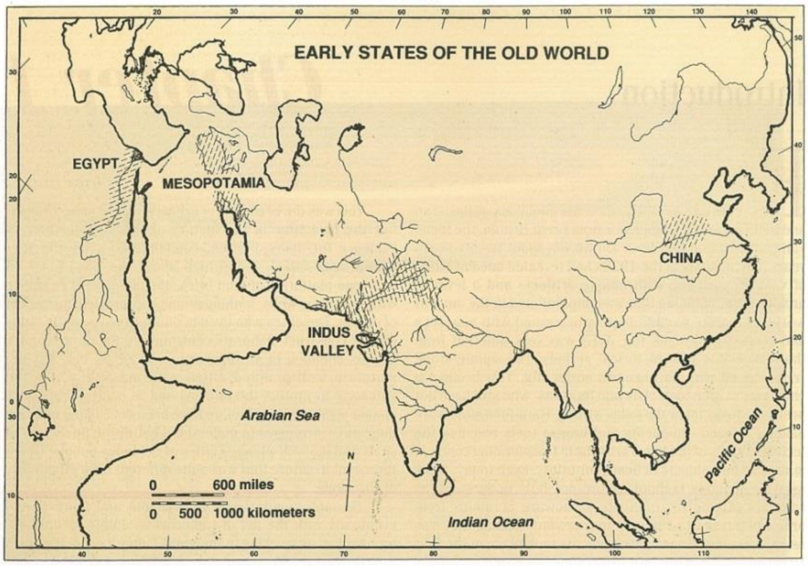
(I) Indus civilization period

(ca.2600/2500 BCE – ca.1900 BCE).

It is usually inferred that Munda people (one branch of the Austro-Asiatic people) (mainly in the eastern part of India) and Dravidian people (mainly in the western part of India) were already living in Indian subcontinent thousands of years ago, and later (around 1500 BCE or so), Indo-Aryan people migrated to Indian subcontinent from the northwest. Dravidian people are now living mainly in South India.

The Indus civilization (ca.2600/2500 BCE ~ ca.1900 BCE) is the earliest urban civilization in Indian subcontinent (before the immigration of Indo-Aryan people). It developed in the Indus Valley and nearby area of the western part of the Indian subcontinent. Its cities Harappa and Mohenjo-daro were excavated since 1920s, and some other cities were excavated later. It had Indus scripts, and some people are trying to decipher them as Dravidian language, but the results are still at the stage of hypothesis.

The cities are well planned, and we can suppose that they had certain knowledge to determine cardinal directions. And also, they had standardized system of weights and measures. As their agriculture was well developed, we can suppose that they had certain knowledge of calendar and related astronomy, which are necessary for agriculture..



(From Kenoyer: *Ancient Cities of the Indus Valley Civilization*, Oxford University Press, 1998, p.16)

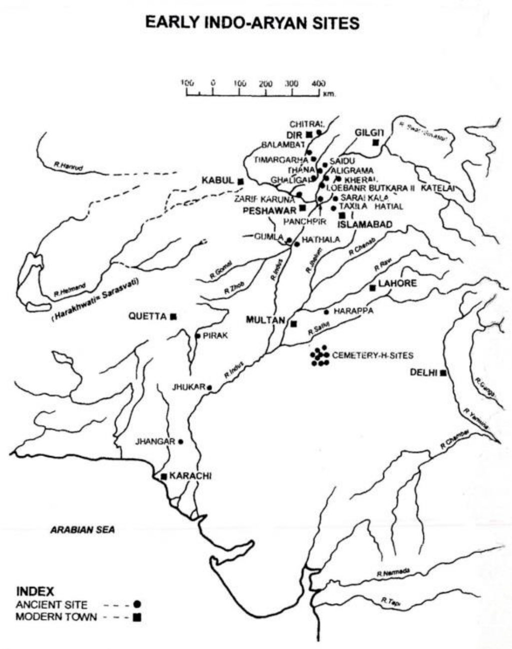
(II) Vedic period (ca.1500 BCE – ca.500 BCE).

After the period of the Indus civilization (ca.2600 BCE ~ ca.1900 BCE), Aryans appeared in Northwest India in ca.1500 BCE or so. The Aryans were originally pastoral people. The Aryans produced a set of Brahmanic literature called *Veda* in India

. There are four *Veda*s, namely, the *Ṛg-veda*, the *Sāma-veda*, the *Yajur-veda*, and the *Atharva-veda*. Each of the four *Veda*s consists of the *Saṁhitā*, the *Brāhmaṇa*, the *Āraṇyaka*, and the *Upaniṣad*.

Firstly, the *Ṛg-veda-saṁhitā* was produced in Northwest India (present Punjab) during ca.1500 BCE and ca.1000BCE. Let us call this period "Ṛg-vedic period".

Then, the Aryans advanced towards east, and produced Later Vedic literature (Vedic literature except for the *Ṛg-veda-saṁhitā*) in North India (roughly the western part of the plain of the Ganga) during ca.1000 BCE and ca.500 BCE. Let us call this period "Later Vedic period".



MAP 5 Early Indo-Aryan Sites. *Courtesy ASI*

Ṛg-vedic period
(c. 1500 ~ 1000 BCE)
(From Sharma (2005), facing p.95)

MAP 6 Distribution of Painted Grey Ware. *Courtesy ASI*

Period of Painted Grey Ware
corresponds to the
Later Vedic Period
(c. 1000 ~ 500 BCE)
(From Sharma (2005), facing p.118)

(From Sharma, R. S.: *India's Ancient Past*, New Delhi, Oxford University Press, 2005)

Ṛg-vedic period (ca.1500 BCE ~ ca.1000 BCE):

Certain calendrical knowledge (connected with annual monsoon) is recorded.

In a late portion of the *Ṛg-veda*, the intercalary month seems to have been mentioned.

Later Vedic period (ca. 1000 BCE ~ ca. 500 BCE):

The Aryans advanced towards east.

The society had become essentially agricultural in this stage.

The intercalary month is explicitly mentioned.

The complete set of *nakṣatra*s (lunar mansions) is given in later Vedic literature.

One year was divided into six seasons, namely, *vasanta* (spring), *grīṣma* (summer), *varṣā* (rainy), *śarad* (autumn), *hemanta* (winter), and *śiśira* (cool).

In the Vedic period, the regular calendar was symbolized in rituals.

(III) *Vedāṅga* period.

Towards the end of the Later Vedic period, a class of works regarded as auxiliary to the *Veda* was produced, which is called *Vedāṅga* (limbs of the *Veda*).

The *Vedāṅga* consists of six divisions, namely, phonetics, metrics, grammar, etymology, astronomy and ceremonial.

The ceremonial texts called "*śulba-sūtra*s" contain certain geometrical knowledge.

It is this period when astronomy, which was called "*jyotiṣa*" in Sanskrit, was established as an independent learning.

(III.1) *Śulba-sūtra*s (probably composed sometime between the 6th century BCE and the 2nd centurie CE).

The *Śulba-sūtra*s (literally mean "rules of the measuring-cords") are the texts to construct the altar for Brahmanic ceremonies.

There are four main texts, namely, *Āpastamba-śulba-sūtra, Baudāyana-śulba-sūtra, Kātyāyana-śulba-sūtra, Mānava-śulba-sūtra*.

Certain geometrical knowledge is used to construct altars.

Let us see some examples.

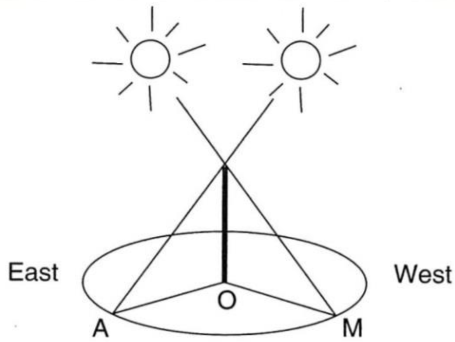


Figure 2.1 Determining the east-west line with shadows cast by a stake.

The preliminary step is the drawing of a baseline running east and west. We do not know for sure how this was accomplished in the time of the early *Śulba-sūtra* authors, but the later *Kātyāyana-śulba-sūtra* prescribes using the shadows of a gnomon or vertical rod set up on a flat surface, as follows:

> Fixing a stake on level [ground and] drawing around [it] a circle with a cord fixed to the stake, one sets two stakes where the [morning and afternoon] shadow of the stake tip falls [on the circle]. That [line between the two] is the east-west line. Making two loops [at the ends] of a doubled cord, fixing the two loops on the [east and west] stakes, [and] stretching [the cord] southward in the middle, [fix another] stake there; likewise [stretching it] northward; that is the north-south line. (*KāSS* 1.2)

The first part of the procedure is illustrated in figure 2.1, where the base of the gnomon is at the point O in the center of a circle drawn on the ground.[14] At some time in the morning the gnomon will cast a shadow OM whose tip falls on the circle at point M, and at some time in the afternoon the gnomon will cast a shadow OA that likewise touches the circle. The line between points A and M will run approximately east-west.

[14]Note that the text itself is purely verbal and contains no diagrams. This figure and all the remaining figures and tables in this chapter are just modern constructs to help explain the mathematical rules.

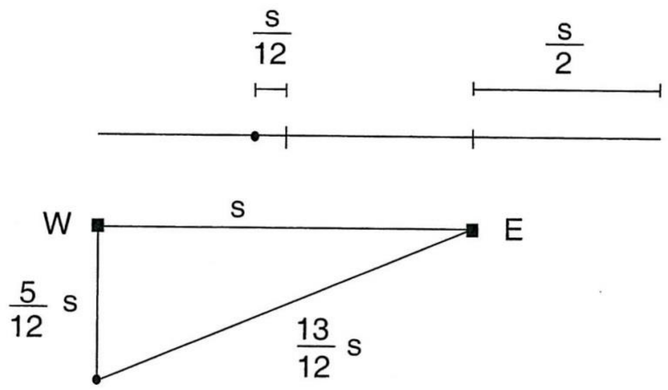


Figure 2.2 Determining the perpendicular sides of a square with a marked cord.

Then a cord is attached to stakes at the east and west points, and its midpoint is pulled southward, creating an isosceles triangle whose base is the east-west line. Another triangle is made in the same way by stretching the cord northward. The line connecting the tips of the two triangles is a perpendicular bisector running north and south. Similar ways of stretching a cord into a triangle are also used for basic determinations of right-angled figures, as in the following construction of a square:

> The length is as much as the [desired] measure; in the western third of [that length] increased by its half, at the [place] less by a sixth part [of the third], one makes a mark. Fastening [the ends of the cord] at the two ends of the east-west line, stretching [the cord] southward by [holding] the mark, one should make a marker [at the point that it reaches]. In the same way [one should stretch the cord] northward; and in the other two directions after reversing [the ends of the cord]. That is the determination. [There is] shortening or lengthening [of the side to produce the desired half-side of the square with respect to] that marker. (*ĀpSS* 1.2)

Here a cord with length equal to the desired side of a square, say s, is increased to a total length of $\frac{3}{2}s$, and a mark is made at a distance of $\frac{5}{12}s$ from one end, as shown in figure 2.2. So when the endpoints are fixed a distance s apart along the east-west line, pulling the mark downwards creates a 5-12-13 right triangle to make the sides perpendicular. The same technique is also used with 3-4-5 right triangles (e.g., in *BauSS* 1.5, *KāSS* 1.4).

(From Plofker: *Mathematics in India*, Princeton, 2009, pp.19 – 20.)

Mathematical constants in the *Śulba-sūtra*s:

Table 2.1 *Śulba-sūtra* constants

Sūtra	Rule and modern equivalent	Remarks; value
BauSS 2.9, *MāSS* 1.8, *ĀpSS* 3.2, *KāSS* 3.11	Half diagonal of square, minus difference of half diagonal and half side, plus one-third that difference, is radius of circle: $r = \frac{s}{2} + \frac{s\sqrt{2}/2 - s/2}{3}$	$\pi \approx 3.08831$
BauSS 2.10	Seven-eighths diameter of circle, plus one twenty-ninth of remaining eighth, minus one sixth of that twenty-ninth diminished by its eighth, is side of square: $s = \frac{2r}{8}\left(7 + \frac{1}{29} - \left(\frac{1}{29 \cdot 6} - \frac{1}{29 \cdot 6 \cdot 8}\right)\right)$	$\pi \approx 3.08833$
BauSS 2.11, *ĀpSS* 3.3, *KāSS* 3.12	Thirteen-fifteenths of diameter of circle is side of square: $s = 2r \cdot \frac{13}{15}$	Called "approximate" $\pi \approx 3.004$
BauSS 2.12, *ĀpSS* 1.6, *KāSS* 2.9	Side of square plus its third plus a fourth of the third minus one thirty-fourth of the fourth is the diagonal $s\sqrt{2} = s \cdot \left(1 + \frac{1}{3} + \frac{1}{3 \cdot 4} - \frac{1}{3 \cdot 4 \cdot 34}\right)$	*KāSS* says "approximate" $\sqrt{2} \approx 1.4142$
MāSS 11.9–10	$s^2 = \frac{3(2r)^2}{4}$	So interpreted in [Hay1990] $\pi \approx 3$
MāSS 11.15	$r = \frac{4}{5} \cdot \frac{\sqrt{2}}{2} s$	So interpreted in, e.g., [Gup2004b] $\pi \approx 3.125$

(From Plofker: *Mathematics in India*, Princeton, 2009, p.27.)

For original text and English translation of the four *Śulba-sūtra*s, see Sen and Bag (1983). A Japanese translation of the *Āpastamba-śulba-sūtra* by Ikari is included in Yano (1980). Datta (1932) is an early monumental study of geometry in the *Śulba-sūtra*s.

(III.2) *Jyotiṣa-vedāṅga* (probably composed sometime between the 6th and 4th centuries BCE).

The *Jyotiṣa-vedāṅga* of Lagadha is a small monograph of astronomy written in Sanskrit. It has two recensions, namely, the Ṛg-vedic recension entitled *Ārca-jyotiṣa*, and the Yajur-vedic recension entitled *Yājuṣa-jyotiṣa*.

The calendrical system of the *Jyotiṣa-vedāṅga* can be summarized as follows.

1 *sāvana* day (civil day) is from sunrise to sunrise.
1 *sāvana* month (civil month) is 30 *sāvana* days.
1 *tithi* is 1/30 of a synodic month.
1 synodic month is from new moon to new moon.
1 solar month is 1/12 of a solar year.
1 *ṛtu* (season) is 1/6 of a solar year.
1 solar year is from winter solstice to winter solstice.
1 solar year = 2 *ayana*s (half years),
= 6 *ṛtu*s (seasons),
= 12 solar months,
= 366 *sāvana* days (civil days),
= 372 *tithi*s.
1 *yuga* = 5 years,
= 60 solar months,
= 61 *sāvana* months = 1830 *sāvana* days,
= 62 synodic months = 1860 *tithi*s,
= 67 sidereal months,
= 1835 sidereal days.

In the *Jyotiṣa-vedāṅga*, celestial longitude was expressed using *nakṣatra* (lunar mansion). One *nakṣatra* used there is a segment which is equivalent to 1/27 of the ecliptic. The system of 28 or 27 *nakṣatra*s already appeared in some of the later Vedic literature. The *nakṣatra*s described in the later Vedic literature must have been consisted of the actual visible stars. The *Jyotiṣa-vedāṅga* started to use it as an artificial system of coordinates. It may be mentioned here that the systems of 28 and 27 *nakṣatra*s are used for different purposes in the later Hindu astronomy since the Classical Siddhānta period, the system of 28 *nakṣatra*s as actual stars, and the system of 27 *nakṣatra*s as artificial coordinates. The *Vedāṅga* astronomy may be considered to be the beginning of this division.

Accuracy of the *Vedāṅga* astronomy:

1 solar year = 366 civil days,

1 *yuga* = 5 years,

= 1830 civil days,

= 62 synodic months,

= 67 sidereal months,

Modern accurate value:

1 solar year = 365.24219 days.

1 synodic month = 29.530589 days.

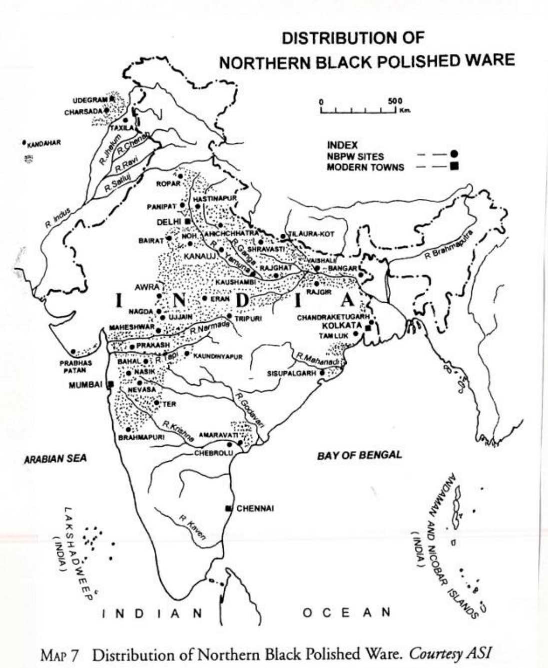
--- 62 synodic months = 1830.8965 days.

1 sidereal month = 27.321662 days.

--- 67 sidereal months = 1830.5485 days.

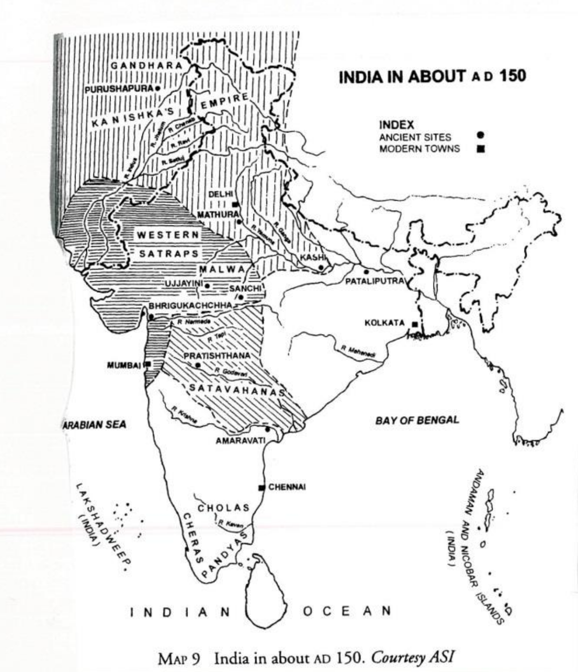
--- Enough exact regarding lunar months!

(For more detail, see Ôhashi, Yukio: "On *Vedāṅga* astronomy: The Earliest Systematic Indian Astronomy", in Nakamura, Orchiston, Sôma and Strom (eds.): *Mapping the Oriental Sky. Proceedings of the Seventh International Conference on Oriental Astronomy*, Tokyo, National Astronomical Observatory of Japan, 2011, pp.164 – 170. For other references, see this paper.)



MAP 7 Distribution of Northern Black Polished Ware. *Courtesy ASI*

Period of Northern Black Polished Ware
(c. 500 BCE ~)
(From Sharma (2005), facing p.158)



MAP 9 India in about AD 150. *Courtesy ASI*

India in about 150 CE
(From Sharma (2005), facing p.222)

(From Sharma, R. S.: *India's Ancient Past*, New Delhi, Oxford University Press, 2005)

(IV) Period of the introduction of Greek astrology and astronomy

(Sometime around the 3rd and 4th century CE?).

Greek horoscopy is mentioned in the *Yavana-jātaka* (ca. 3rd century CE?) of Sphujidhvaja. The most of the contents of the *Yavana-jātaka* is Greek horoscopic astrology.

With Greek astrology, zodiacal signs, seven-day week, etc. were introduced into India.

Greek mathematical astronomy seems to have been introduced into India sometime around the 4th century CE. There is little source material of the development of astronomy during this period. The *Pañca-siddhāntikā* of Varāhamihira (6th century CE) (see below) gives the most important information, although its information is fragmental.

It seems that the Indian traditional *Vedāṅga* astronomy was continually used until the 3rd ~ 5th centuries CE. After the end of the 5th century, India did not receive foreign influence for sometimes, and created its own classical mathematics and astronomy.

(V) Classical *Siddhānta* period

(Classical Hindu astronomy period).

(The end of the 5^{th} century ---- the 12^{th} century).

In the Classical Hindu Astronomy period (Classical *Siddhānta* period) (from the end of the 5^{th} century to the 12^{th} century), Indian astronomy did not receive apparent foreign influence, and developed individually. Some of the Sanskrit works in this period are still considered to be authoritative by modern traditional Hindu calendar makers etc. This period can be called Classical *Siddhānta* period or Classical Hindu Astronomy period. The "*Siddhānta*" is the fundamental treatise of mathematical astronomy in Sanskrit.

Famous astronomers and mathematicians in this period:

Āryabhaṭa (b.476 CE) --- astronomy and mathematics,
Varāhamihira (6^{th} century) --- astronomy and astrology,
Bhāskara I (fl.629) --- astronomy and mathematics,
Brahmagupta (b.598) --- astronomy and mathematics,
Śrīdhara (8^{th} century) --- mathematics,
Lalla (ca.8^{th} or 9^{th} century) --- astronomy and astrology,
Mahāvīra (9^{th} century) --- mathematics,
Vaṭeśvara (b.880) --- astronomy,
Mañjula (fl.932) --- astronomy,
Śrīpati (fl.1039/1056) --- astronomy, mathematics and astrology,
Bhāskara II (b.1114) --- astronomy and mathematics.

And also the anonymous *Sūrya-siddhānta* (ca.10^{th} or 11^{th} century) is a very popular Sanskrit astronomical text of this period.

The Bakhshālī Manuscript (probably composed in the 7^{th} century CE or so) is also an important Sanskrit mathematical manuscript in India.

Main contents of Hindu classical astronomy:

“Graha-gaṇita” (calculation of planetary position):

----- Calculation of the sun, moon, and planets for calendar making using eccentric and epicyclic models..

----- (mean motion, true motion, “three problems” (direction, place and time), lunar and solar eclipses, conjunction of planets and stars, heliacal rising and setting, lunar phase etc.)

“Gola” (spherics).

----- Topics concerning celestial sphere, astronomical instruments etc.

Main contents of Hindu classical mathematics:

“Pāṭī-gaṇita” (arithmetical mathematics),

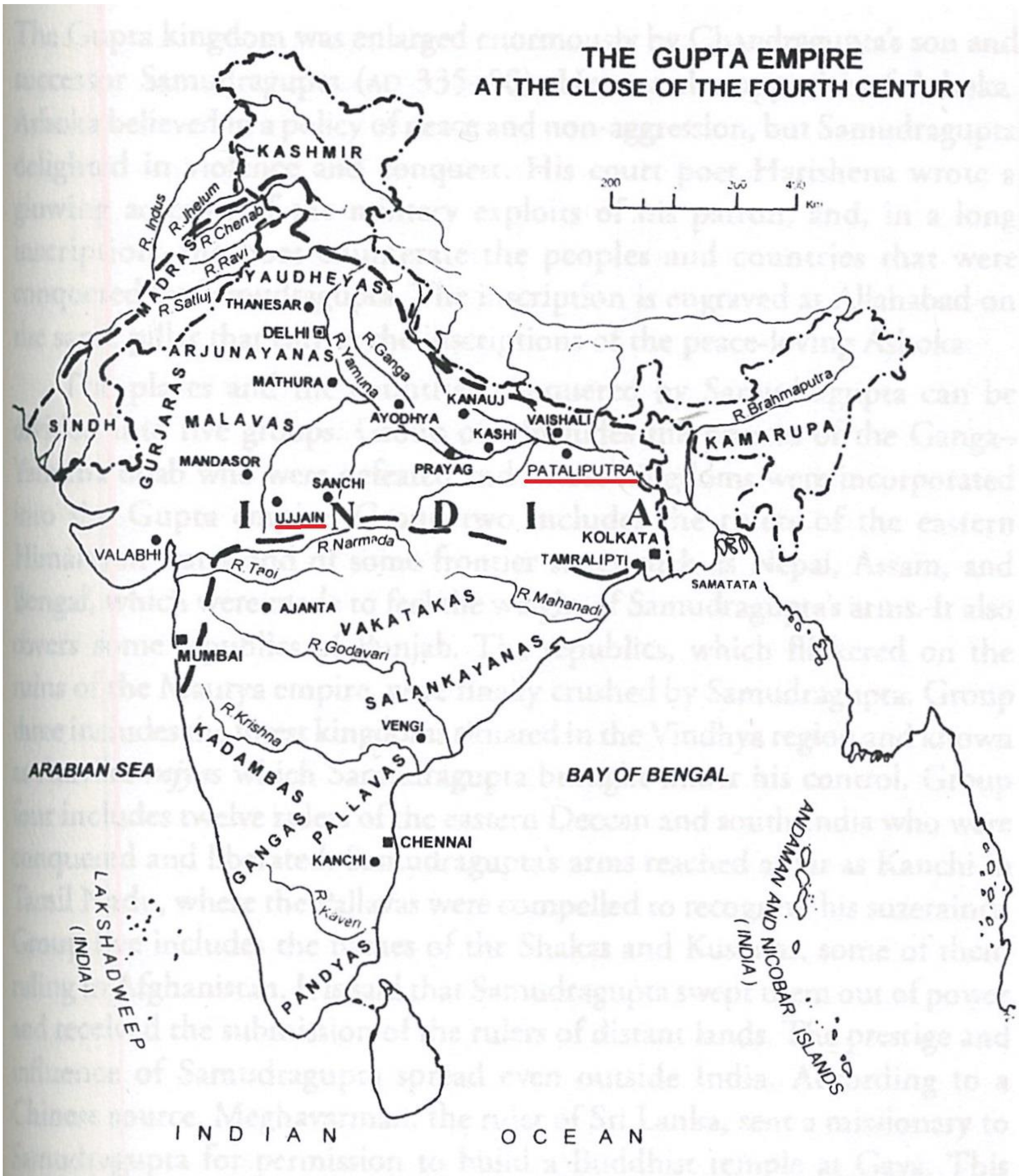
----- Calculation of known quantities using certain algorithms.

“Bīja-gaṇita” (algebraic mathematics).

----- Methods to solve equations using letters to express unknown quantities.

The above mentioned “Graha-gaṇita”, “Gola”, “Pāṭī-gaṇita”, and “Bīja-gaṇita” are usually used Sanskrit term.

Let us see some examples.



MAP 10 The Gupta Empire at the Close of the Fourth Century.

Courtesy ASI

(From Sharma, R. S.: *India's Ancient Past*, New Delhi, Oxford University Press, 2005, facing p.232.)

Gupta Empire (From the late 3rd century to the mid-6th century)

Āryabhaṭa (b.476 CE) and Varāhamihira (6th century) lived around this period.

Āryabhaṭa (b.476 CE)

Āryabhaṭa is an astronomer who was born in 476 CE. He probably lived in Kusumapura (=Pāṭaliputra) in ancient Magadha, i.e. modern Patna in Bihar. He is the earliest astronomer in the Classical Siddhānta period whose name and date are definitely known

Āryabhaṭa composed two astronomical works, the *Āryabhaṭa-siddhānta* (now lost) and the *Āryabhaṭīya* (499 CE).

The *Āryabhaṭa-siddhānta* is a lost text belonging to the Ārdharātrika school. Only its fragments are found to be quoted in later works.

The *Āryabhaṭīya* (499 CE) is a celebrated work of Hindu astronomy. It consists of four sections, namely, the *Gītikā* section on astronomical constants, the *Gaṇita* section on mathematics, the *Kālakriyā* section on the reckoning of time, and the *Gola* section on the celestial sphere.

Significance of this work is that the rotation of the earth is mentioned there. In the *Āryabhaṭīya* (I. 3, and I. 6), Āryabhaṭa mentions the eastward rotation of the earth. This theory was, however, not accepted by other Hindu astronomers. What Āryabhaṭa told is the rotation of the earth, and not the revolution of the earth. The earth was considered to be at the centre of the universe. Therefore, Āryabhaṭa's theory is different from Copernican heliocentric theory. This difference should not be confused.

Āryabhaṭa explained several topics in the section of mathematics, and it is the beginning of the systematic mathematics in India.

For its text and English translation, see Shukla and Sarma (1976).
There is also a Japanese translation by Yano in Yano (1980).

The contents of the *Āryabhaṭīya*

(from Shukla and Sarma (1976), pp.vii-xii):

I. THE GITIKĀ SECTION

II. GAṆITA OR MATHEMATICS

III. KĀLAKRIYĀ OR THE RECKONING OF TIME

IV. GOLA OR THE CELESTIAL SPHERE

Rotation of the Earth in the *Āryabhaṭīya*:

(The quotations from the *Āryabhaṭīya* are from Shukla and Sarma (1976).)

Āryabhaṭīya (I. 3-4):

REVOLUTION-NUMBERS AND ZERO POINT

युगरविभगणाः ख्युघृ, शशि
चयगियिङुशुछ्लृ, कु ङिशिबुण्लृष्खृ[1] प्राक् ।
शनि ढुङ्विघ्व, गुरु ख्रि-
च्युभ, कुज भद्लिझ्नुखृ, भृगुबुधसौराः ॥ ३ ॥

चन्द्रोच्च [2]र्जुष्खिध, बुध
सुगुशिथृन, भृगु जषबिखुछृ, शेषार्काः ।
बुफिनच पातविलोमा,
बुधाह्न्यजार्कोदयाच्च लङ्कायाम् ॥ ४ ॥

3-4. In a *yuga*, the eastward revolutions of the Sun are 43,20,000; of the Moon, 5,77,53,336; of the Earth,[3] 1,58,22,37,500; of Saturn, 1,46,564; of Jupiter, 3,64,224; of Mars, 22,96,824; of Mercury and Venus, the same as those of the Sun; of the Moon's apogee, 4,88,219; of (the *śīghrocca* of) Mercury, 1,79,37,020; of (the *śīghrocca*) of Venus, 70,22,388; of (the *śīghroccas* of) the other planets, the same as those of the Sun; of the moon's ascending node in the opposite direction (*i.e.*, westward), 2,32,226.[4] These revolutions commenced at the beginning of the sign Aries on Wednesday at sunrise at Laṅkā (when it was the commencement of the current *yuga*).

The 'Moon's apogee' is that point of the Moon's orbit which is at the remotest distance from the Earth, and the 'Moon's ascending node' is that point of the ecliptic where the Moon crosses it in its northward motion.

The *śīghroccas* of Mercury and Venus are the imaginary bodies which are supposed to revolve around the Earth with the heliocentric mean angular velocities of Mercury and Venus, respectively, their directions from the Earth being always the same as those of the mean

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1. Go. ख्षृ.
2. C.D. Kṛ. Pa. Sū. ज्रुष्खिघ; Bh. Nī. Pa. (alt.), Ra. So. र्जुष्खिध.
3. These are the rotations of the Earth, eastward.
4. These very revolutions, excepting those of the Earth, are stated in *MBh*, vii. 1-5; *LBh*, i. 9-14; and *ŚiDVṛ*, *Grahagaṇita*, i. 3-6.

positions of Mercury and Venus from the Sun. It will thus mean that the revolutions of Mars, the *śīghrocca* of Mercury, Jupiter, the *śighrocca* of Venus, and Saturn, given above, are equal to the revolutions of Mars, Mercury, Jupiter, Venus and Saturn, respectively, round the Sun.

The following table gives the revolutions of the Sun, the Moon and the planets along with their periods of one sidereal revolution. The sidereal periods according to the Greek astronomer Ptolemy (A.D. c. 100-c. 178) and the modern astronomers are also given for the sake of comparison.

Table 2. Mean motion of the planets

Planet	Revolutions in 43,20,000 years	Sidereal period in terms of days		
		Āryabhaṭa I	Ptolemy[1]	Moderns[2]
Sun	43,20,000	365·25868	365·24666	365·25636
Moon	5,77,53,336	27·32167	27·32167	27·32166
Moon's apogee	4,88,219	3231.98708	3231·61655	3232·37543
Moon's asc. node	2,32,226	6794·74951	6796·45587	6793·39108
Mars	22,96,824	686·99974	686·94462	686·9797
Śīghrocca of Mercury	1,79,37,020	87·96988	87·96935	87·9693
Jupiter	3,64,224	4332·27217	4330·96064	4332·5887
Śīghrocca of Venus	70,22,388	224·69814	224·69890	224·7008
Saturn	1,46,564	10766·06465	10749·94640	10759·201

The epoch of the planetary motion mentioned in the text marks the beginning of the current *yuga* and not the beginning

1. Taken from Bina Chatterjee, "*The Khaṇḍa-khādyaka of Brahmagupta*", World Press, Calcutta, 1970, vol. I, Appendix VII, p. 281.

2. Taken from H.N. Russell, Dugan and J.Q. Stewart, *Astronomy*, Part I : *The Solar system*, Revised edition, Ginn and Company, Boston, Appendix. Also, see *ibid.*, pp. 150, 159. The sidereal periods of Moon's apogee and ascending node are taken from P.C. Sengupta and N.C. Lahiri's introduction (p. xiv) to Babuāji Miśra's edition of Śrīpati's *Siddhānta-śekhara*.

of the current *Kalpa* as was supposed by P.C. Sengupta. The current *Kalpa*, according to Āryabhaṭa I, started on Thursday 1,98,28,80,000 years or 7,24,26,41,32,500 days before the beginning of the current *yuga*; and 1,98,61,20,000 years or 7,25,44,75,70,625 days before the beginning of the current Kaliyuga.[1] The current Kaliyuga began on Friday, February 18, 3102 B.C., at sunrise at Laṅkā (a hypothetical place on the equator where the meridian of Ujjain intersects it), which synchronized with the beginning of the light half of the lunar (synodic) month of Caitra.

One thing that deserves special notice is the statement of the Earth's rotations. Āryabhaṭa I is, perhaps, the earliest astronomer in India who advanced the theory of the Earth's rotation and gave the number of rotations that the Earth performs in a period of 43,20,000 years. The period of one sidereal rotation of the Earth according of Āryabhaṭa I is $23^h\ 56^m\ 4^s{\cdot}1$. The corresponding modern value is $23^h\ 56^m\ 4^s{\cdot}091$.[2] The accuracy of Āryabhaṭa I's value is remarkable.

Of the other Indian astronomers who upheld the theory of the Earth's rotation, mention may be made of Pṛthūdaka (A.D. 860) and Makkibhaṭṭa (A.D. 1377). In the *Skanda-purāṇa* (1. 1. 31. 71), too, the Earth is described as revolving like a *bhramarikā* (spinning top, potter's wheel or whirlpool).

The commentators of the *Āryabhaṭīya*, who hold the opinion that the Earth is stationary, think that Āryabhaṭa I states the rotations of the Earth because the asterisms, which revolve westward around the earth by the force of the provector wind, see that the Earth rotates eastward.

These commentators were indeed helpless because Āryabhaṭa I's theory of the Earth's rotation received a severe blow at the hands of Varāhamihira (d. A.D. 587) and Brahmagupta (A.D. 628) whose arguments against this theory could not be refuted by any Indian astronomer.

It is noteworthy that the Greek astronomer Ptolemy, following Aristotle (B.C. 384-322), believed that the Earth was stationary and adduced arguments in support of his view.

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1. *Vide infra* notes on verse 5.
2. See W. M. Smart, *Text-Book on Spherical Astronomy*, Cambridge, 1940, p. 420.

(From Shukla and Sarma (1976), p.6 – 8.)

Astronomical constants concerning the movement of heavenly bodies are given by their revolution (or rotation) numbers (with respect to fixed stars) in a "*yuga*" (4320000 years). The rotation number of the Earth corresponds to the number of sidereal days in a *yuga*.

Āryabhaṭīya (I. 6):

PLANETARY ORBITS, EARTH'S ROTATION

शशिराशयष्ठ चक्रं
तेंऽशकलायोजनानि य-व-ञ-गुणाः ।
प्राणेनैति कलां भूः,[1]
खयुगांशे ग्रहजवो, भवांशेऽर्कः ॥ ६ ॥

6. **Reduce the Moon's revolutions (in a *yuga*) to signs, multiplying them by 12 (lit. using the fact that there are 12 signs in a circle or revolution). Those signs mutiplied successively by 30, 60 and 10 yield degrees, minutes and *yojanas*, respectively. (These *yojanas* give the length of the circumference of the sky). The Earth rotates through (an angle of) one minute of arc in one respiration (=4 sidereal seconds). The circumference of the sky divided by the revolutions of a planet in a *yuga* gives (the length of) the orbit on which the planet moves.[2] The orbit of the asterisms divided by 60 gives the orbit of the Sun.[3]**

Thus we have

$$\text{Orbit of the sky} = 57753336 \times 12 \times 30 \times 60 \times 10 \textit{ yojanas}$$
$$= 12474720576000 \textit{ yojanas}$$

$$\text{Orbit of the asterisms} = 173260008 \textit{ yojanas}$$

$$\text{Orbit of the Sun} = 2887666\tfrac{4}{5} \textit{ yojanas}$$

$$\text{Orbit of the Moon} = 216000 \textit{ yojanas}$$

$$\text{Orbit of Mars} = 5431291\frac{132027}{287103} \textit{ yojanas}$$

$$\text{Orbit of (}\textit{Śīghrocca of}\text{) Mercury} = 695473\frac{373277}{896851} \textit{ yojanas}$$

$$\text{Orbit of Jupiter} = 34250133\frac{699}{1897} \textit{ yojanas}$$

$$\text{Orbit of (}\textit{Śīghrocca}\text{ of) Venus} = 1776421\frac{255221}{585199} \textit{ yojanas}$$

$$\text{Orbit of Saturn} = 85114493\frac{5987}{36641} \textit{ yojanas.}$$

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1. Br. Pṛ. Ud. भूः ; all others भं.
2. *Cf.* Someśvara : ग्रहजवो ग्रहपरिधिः ग्रहकक्ष्येत्यर्थः ।
3. The same rule, excepting the rate of the Earth's motion, occurs in *MBh*, vii. 20, also.

These orbits are hypothetical and are based on the following two assumptions :

1. That all the planets have equal linear motion in their respective orbits.[1]
2. That one minute of arc (1′) of the Moon's orbit is equal to 10 *yojanas* in length.[2]

From the second assumption, the length of the Moon's orbit comes out to be 216000 *yojanas*. Multiplying this by the Moon's revolution-number (*viz*. 57753336), we get 12474720576000 *yojanas*. This is the distance described by the Moon in a *yuga*. From the first assumption, this is also the distance described by any other planet in a *yuga*. Hence

$$\text{Orbit of a planet} = \frac{\text{distance described by a planet in a } yuga}{\text{revolution-number of that planet}}.$$

This is how the lengths of the orbits of the various planets stated above have been obtained.

In the case of the asterisms, it is assumed that their orbit is 60 times the orbit of the Sun. By saying that "the orbit of the asterisms divided by 60 gives the orbit of the Sun", Āryabhaṭa I really means to say that "the orbit of the asterisms is 60 times the orbit of the Sun."

Indian astronomers, particularly the followers of Āryabhaṭa I, believe that the distance described by a planet in a *yuga* denotes the circumference of the space, supposed to be spherical, which is illumined by the Sun's rays. This space, they call 'the sky' and its circumference 'the orbit of the sky'. Bhāskara I says :

> "(The outer boundary of) that much of the sky as the Sun's rays illumine on all sides is called the circumference or orbit of the sky. Otherwise, the sky is beyond limit; it is impossible to state its measure."[3]

> "For us the sky extends to as far as it is illumined by the rays of the Sun. Beyond that, the sky is immeasurable."[4]

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1. See *Ā*, iii, 12.
2. This is implied in the text under discussion.
3. See Bhāskara I's commentary on *Ā*, i. 6, in Vol. II.
4. See Bhāskara I's commentary on *Ā*, iii. 12, in Vol. II.

According to the Indian astronomers, therefore,

$$\text{Orbit of a planet} = \frac{\text{Orbit of the sky}}{\text{Planet's revolution-number}}$$

The statement of the Earth's rotation through 1′ in one respiration,[1] stated in the text, has been criticised by Brahmagupta, who says :

> "If the Earth moves (revolves) through one minute of arc in one respiration, from where does it start its motion and where does it go ? And, if it rotates (at the same place), why do tall lofty objects not fall down ?"[2]

The reading *bhaṁ* (in place of *bhūḥ*) adopted by the commentators is evidently incorrect. The correct reading is *bhūḥ*, which has been mentioned by Brahmagupta (A.D. 628), Pṛthūdaka (A. D. 860) and Udayadivākara (A.D. 1073).[3]

LINEAR DIAMETERS

नृ-षि योजनं, ञिला भू-
व्यासो, ऽर्केन्द्वोर्घ्रिञा गिण, क मेरोः ।
भृगु-गुरु-बुध-शनि-भौमाः
शशि-ङ-ञ-ण-न-मांशकाः, समार्कसमाः ॥ ७ ॥

7. **8000 *nṛ* make a *yojana*. The diameter of the Earth is 1050 *yojanas*; of the Sun and the Moon, 4410 and 315 *yojanas*, (respectively) ;[4] of Meru, 1 *yojana* ; of Venus, Jupiter, Mercury, Saturn and Mars (at the Moon's mean distance), one-fifth, one-tenth, one-fifteenth, one-twentieth, and one-twentyfifth, (respectively), of the Moon's diameter.[5] The years (used in this work) are solar years.**

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1. 1 respiration = 4 seconds of time.
2. प्राणेनैति कलां भूर्यदि तर्हि कुतो व्रजेत् कमध्वानम् ।
   आवर्तनमुर्व्याश्चेन्न पतन्ति समुच्छ्रयाः कस्मात् ॥
   BrSpSi, xi. 17.
3. See his commentary on *LBh*, i. 32-33.
4. The same values are given in *MBh*, v. 4 ; *LBh*, iv. 4.
5. *Cf. MBh*, vi. 56.

(From Shukla and Sarma (1976), pp.13-14.) (From Shukla and Sarma (1976), pp.13-15.)

The rotating earth theory is explained in the following verses. (*Āryabhaṭīya* (IV. 9-10)).

APPARENT MOTION OF THE STARS DUE TO THE EARTH'S ROTATION

अनुलोमगतिर्नौस्थः पश्यत्यचलं विलोमगं यद्वत् ।
अचलानि भानि तद्वत् समपश्चिमगानि लङ्कायाम् ॥ ९ ॥

उदयास्तमयनिमित्तं नित्यं प्रवहेण वायुना क्षिप्तः ।
लङ्कासमपश्चिमगो भपञ्जरः सग्रहो भ्रमति ॥ १० ॥

9. Just as a man in a boat moving forward sees the stationary objects (on either side of the river) as moving backward, just so are the stationary stars seen by people at Laṅkā (on the equator), as moving exactly towards the west.

10. (It so appears as if) the entire structure of the asterisms together with the planets were moving exactly towards the west of Laṅkā, being constantly driven by the provector wind, to cause their rising and setting.

The theory of the Earth's rotation underlying the above passage was against the view generally held by the people and was severely criticised by Varāhamihira (*d.* A.D. 587) and Brahmagupta (628 A.D.) The followers of Āryabhaṭa I, who were unable to refute the criticism against the theory, fell in line with Varāhamihira and others of his ilk and have misinterpreted the above verses as conveying the contrary sense. See how the commentator Someśvara interprets the above verses :

"Just as one seated on a boat sees the stationary objects such as trees etc. standing on the two sides of the river or sea moving in the contrary direction, in the same way those situated on the Earth rotating eastwards see the stationary stars located in the sky as moving in the opposite direction towards the west. Likewise, those living in Laṅkā see the stars as moving towards the west. Laṅkā is only a token, others also see in the same way. So, it is the Earth that moves towards the east; the stars are fixed. And that part of the circle of the asterisms which lies (at the moment) towards the east appears to rise, that which lies in the middle of the sky appears to culminate, and that which lies towards the west appears to set. Otherwise, the rising and setting of the stars is impossible." After saying all this he adds :

"This is the false view. For, if the Earth had a motion, the world would have been inundated by the oceans, the tops of the trees and castles would have disappeared, having been blown away by the storm caused by the velocity of the Earth, and the birds etc. flying in the sky would never have returned to their nests. So, there exists not a single trace of the Earth's motion. Hence this stanza must be interpreted in another way (as follows) :

"Just as a man seated on a boat moving forward sees the stationary objects moving in the contrary direction, in the same way the asterisms driven by the provector wind, due to their own motion, see the objects at Laṅkā as moving in the opposite direction, *i e.*, they see the stationary Earth lying below as if it were rotating. Apparently also the asterisms rise in the east and move towards the west."

Pṛthūdaka (860 A.D.) in his commentary on the *Brahma-sphuṭa-siddhānta*, supports Āryabhaṭa I's theory of the Earth's rotation. The followers of Āryabhaṭa I, who misinterpreted Āryabhaṭa I, were, according to him, afraid of the public opinion which was against the motion of the Earth.

It is noteworthy that the Greek astronomor Ptolemy (*c.* A. D. 100-178) holds that the Earth is stationary and does not move in any way locally.[1]

1. See The *Almagest*, translated by R.C. Taliaferro, pp. 10-12.

(From Shukla and Sarma (1976), pp.119-120.)

Sine table in the *Āryabhaṭīya* (II.12):

Verse 12] SINE-DIFFERENCES 29

RSINE-DIFFERENCES

मखि भखि फखि धखि णखि ञखि
ङखि हस्झ स्ककि किष्ग श्घकि किघ्व[1] ।
घ्लकि किग्र हक्य धकि किच[2]
स्ग श्झ[3] ङ्व क्ल प्त फ छ कलार्धज्याः ॥ १२ ॥

12. 225, 224, 222, 219, 215, 210, 205, 199, 191, 183, 174, 164, 154, 143, 131, 119, 106, 93, 79, 65, 51, 37, 22, and 7—these are the Rsine-differences (at intervals of 225 minutes of arc) in terms of minutes of arc.

The following table gives the Rsines and the Rsine-differences at intervals of 225′ (or 3° 45′) according to Āryabhaṭa I and the corresponding modern values correct to three decimal places.

Table 10. Rsines and Rsine-differences at the intervals of 225′ or 3°45′

	Āryabhaṭa I's values		Modern Values	
Arc	Rsine	Rsine-differences	Rsine	Rsine-differences
225′	225′	225′	224′.856	224′.856
450′	449′	224′	448′.749	223′.893
675′	671′	222′	670′.720	221′.971
900′	890′	219′	889′.820	219′.100
1125′	1105′	215′	1105′.109	215′.289
1350′	1315′	210′	1315′.666	210′.557
1575′	1520′	205′	1520′.589	204′.923
1800′	1719′	199′	1719′.000	198′.411
2025′	1910′	191′	1910′.050	191′.050
2250′	2093′	183′	2092′.922	182′.872
2475′	2267′	174′	2266′.831	173′.909
2700′	2431′	164′	2431′.033	164′.202
2925′	2585′	154′	2584′.825	153′.792
3150′	2728′	143′	2727′.549	142′.724
3375′	2859′	131′	2858′.592	131′.043
3600′	2978′	119′	2977′.395	118′.803
3825′	3084′	106′	3083′.448	106′.053
4050′	3177′	93′	3176′.298	92′.850
4275′	3256′	79′	3255′.546	79′.248
4500′	3321′	65′	3320′.853	65′.307
4725′	3372′	51′	3371′.940	51′.087
4950′	3409′	37′	3408′.588	36′.648
5175′	3431′	22′	3430′.639	22′.051
5400′	3438′	7′	3438′.000	7′.361

1. D.G. Sū. किघ्वा ; others किघ्व.
2. A. हक्य धा कि कि च; Bh. Śa. हक्य धकि किच ; E. हक्य धाह्रा किच. Pa. Ra. Sū. हक्य धाह्रा स्त ; So. किग्र क्यकि किघ किच.
3. Bh. भ्रश ; others श्झ.

30 GITIKĀ SECTION [Gitikā Sn.

The twenty-four Rsines given in the *Sūrya-siddhānta*[1] are exactly the same as those in column 2 above. P.C. Sengupta is of the opinion that the author of the *Sūrya-siddhānta* has based his Rsines on the Rsine-differences given by Āryabhaṭa I.[2]

The 16th Rsine, *viz.*, 2978, was modified by Āryabhaṭa II[3] (c. A.D. 950) who replaced it by the better value 2977. The table of Rsines given by Bhāskara II[4] (A.D. 1150) is the same as that of Āryabhaṭa II (c. A.D. 950).

Astronomer Sumati of Nepal, who lived anterior to Āryabhaṭa II (c. A.D. 950), gives[5] the values of the 4th and 16th Rsines as 889′ and 2977′ respectively instead of 890′ and 2778′ given by Āryabhata I. Sumati's table contains ninety Rsines at the intervals of one degree.

1. ii. 17-22.
2. See P. C. Sengupta's introduction (p. xix) to E. Burgess' *Translation of the Sūrya-siddhānta.*
3. See *MSi,* iii. 4-6.
4. See *SiŚi, Grahagaṇita,* ii. 3-6.
5. Both in *Sumati-mahātantra* and *Sumati-karaṇa.*

(From Shukla and Sarma (1976), pp.29-30.)

The relationship between arc and chord was used in ancient Greece for astronomical calculation, but the relationship between arc and half-chord (which corresponds to sine) was first used in India. This is the direct origin of the modern trigonometry.

Varāhamihira (6th century, probably expired in 587)

Vrāhamihira is an astronomer and astrologer of the 6th century CE. He resided in Avanti, i.e. Ujjain in modern Madhya Pradesh.

He composed an astronomical work *Pañca-siddhāntikā*, and several astrological works. Among his astrological works, the *Bṛhaj-jātaka* (on the *horā* branch of astrology, or horoscopic astrology) and the *Bṛhat-saṁhitā* (on the *saṁhitā* branch of astrology, or the astrology concerning several natural phenomena) are famous and important. Especially, the *Bṛhat-saṁhitā* is an encyclopaedic work, and is an important source material of Indian science, technology, and culture at his time.

The *Pañca-siddhāntikā* is a compilation of five earlier astronomical works, namely, the *Paitāmaha-siddhānta*, the *Vāsiṣṭha-siddhānta*, the *Pauliśa-siddhānta*, the *Romaka-siddhānta*, and the *Saura-siddhānta*. The *Paitāmaha-siddhānta* is a remnant of Vedāṅga astronomy after the introduction of Greek horoscopy. The *Pauliśa-*, *Romaka-*, and *Saura-siddhānta* are the texts after the introduction of Greek astronomy, and the *Saura-siddhānta* (also called *Sūrya-siddhānta*) is, according to Varāhamihira's own word, more accurate than other works. (This *Sūrya-siddhānta* should not be confused with the famous modern *Sūrya-siddhānta*, which was composed in ca. 10th – 11th century CE.) This *Pañca-siddhāntikā* of Varāhamihira gives very important information of the early history of Hindu astronomy.

Brahmagupta (b.598 CE)

Brahmagupta is an astronomer who was born in 598 CE. He probably lived at Bhillamāla (modern Bhinmal in the southwest of Rajasthan)

Brahmagupta composed two works, namely, the *Brāhma-sphuṭa-siddhānta* (628 CE) (Precise treatise of the Brāhma school), which is a basic text of the Brāhma school which was also followed by Bhāskara II, and the *Khaṇḍa-khādyaka* (665 CE) ("Candied sugar"). Brahmagupta criticized Āryabhaṭa (b.476 CE) in his *Brāhma-sphuṭa-siddhānta*, and Brahmagupta himself was criticized by Vaṭeśvara (b.880 CE) who followed the Ārya school which was founded by Āryabhaṭa. Brahmagupta later accepted the system of Ārdharātrika school, another school founded by Āryabhaṭa, in his *Khaṇḍa-khādyaka*. Brahmagupta was contemporary with another Indian astronomer Bhāskara I, but it is not known whether they knew each other.

The *Brāhma-sphuṭa-siddhānta* has some chapters of mathematics.

Contents of two mathematical chapters of the *Brāhma-sphuṭa-siddhānta*:

BRAHMEGUPTA.

CHAPTER XII. *ARITHMETIC* (*Gańita.*)

CHAPTER XVIII. *ALGEBRA* (*Cuttaca.*)

(From Colebrooke (1817).)

Brahmagupta mentioned the basic calculations including "zero" as follows. Besides the Indisn numerals including symbol of "zero", the calculations including "zero" are also great significance of Indian mathematics.

SECTION II.

ALGORITHM.[1]

31. Rule for addition of affirmative and negative quantities and cipher: § 19. The sum of two affirmative quantities is affirmative; of two negative is negative; of an affirmative and a negative is their difference; or, if they be equal, nought. The sum of cipher and negative is negative; of affirmative and nought is positive; of two ciphers is cipher.

32—33. Rule for subtraction: § 20—21. The less is to be taken from the greater, positive from positive; negative from negative. When the greater, however, is subtracted from the less, the difference is reversed. Negative, taken from cipher, becomes positive; and affirmative, becomes negative. Negative, less cipher, is negative; positive, is positive; cipher, nought. When affirmative is to be subtracted from negative, and negative from affirmative, they must be thrown together.

34. Rule for multiplication: § 22. The product of a negative quantity and an affirmative is negative; of two negative, is positive; of two affirmative, is affirmative. The product of cipher and negative, or of cipher and affirmative, is nought; of two ciphers, is cipher.

35—36. Rule for division: § 23—24. Positive, divided by positive, or negative by negative, is affirmative. Cipher, divided by cipher, is nought. Positive, divided by negative, is negative. Negative, divided by affirmative,

[1] *Shaṭ-trinśat-paricarman.* Thirty-six operations or modes of process. See Arithm. § 1. *Vij.-gan.* § 3.

is negative. Positive, or negative, divided by cipher, is a fraction with that for denominator:[1] or cipher divided by negative or affirmative.[2]

[36 Concluded.] Rule for involution and evolution: § 24. The square of negative or affirmative is positive; of cipher, is cipher. The root of a square is such as was that from which it was [raised].[3]

[1] *Tach-ch'héda,* having that for denominator: having, in this instance, cipher for denominator, to a finite quantity for numerator. See *Vij.-gan.* § 16.

[2] Is in like manner expressed by a fraction having a finite denominator to a cipher for numerator.

[3] The root is to be taken either negative or affirmative, as best answers for the further operations. Com.

(From Colebrooke (1817), pp.339-140.)

Bhāskara II (b.1114 CE)

Bhāskara II is an astronomer who was born in 1114 CE. The number "II" is added by modern historians only for convenience' sake in order to differentiate from his namesake (Bhāskara I) of the 7th century. Bhāskara II probably lived in Vijjaḍaviḍa (possibly present Bijapur in the north of Karnataka). His father was Maheśvara who was also an astronomer.

Bhāskara II composed the *Siddhānta-śiromaṇi* (1150 CE) with his auto-commentary, the *Karaṇa-kutūhala* (1183 CE), and the *Śiṣyadhī-vṛddhida-vivaraṇa* (Commentary on the *Śiṣyadhī-vṛddhida-tantra* of Lalla (ca. 8th century CE)).

The *Siddhānta-śiromaṇi* (1150 CE), which was written at the age of 36 with his own commentary, is a comprehensive treatise of mathematics and astronomy. It consists of 4 parts as follows.

(1) *Līlāvatī* on arithmetical operations [Its English translation is included in Colebrooke, H.T. (1817). Its Japanese translation by Hayashi and Yano is included in Yano (1980).],
(2) *Bījagaṇita* on algebraic operations [Its English translation is included in Colebrooke, H.T. (1817). For its Japanese translation, see Hayashi (2016).],
(3) *Graha-gaṇita-adhyāya* (= *Grahagaṇitādhyāya*) on the calculation of the position of planets [For its English translation, see Arkasomayaji, D. (1980/2000).], and
(4) *Gola-adhyāya* (= *Golādhyāya*) on spherics [For its English translation, see Sastri and Wilkinson (1861).].

The first two parts on mathematics are sometimes treated as independent works. In the last two parts, the word "adhyāya" stands for "chapter".

Contents of the *Līlāvatī* and *Bījagaṇita*:

BHÁSCARA.

ARITHMETIC (Lílávatí.)

ALGEBRA (Vija-gańita.)

(From Colebrooke (1817).)

In the *Bījagaṇita*:, indeterminate equations, multiple variable equations, equations of higher degree etc. are systematically explained. For example:

Examples of quadratic indeterminate equation given by Bhāskara II

表 7.10. バースカラの与える平方始原数の例題

(原書に述べられる順序. BG=『ビージャガニタ』)

1)	$8x^2+1=y^2$	(BG74)	9)	$6x^2+75=y^2$	(BG80)
2)	$11x^2+1=y^2$	(BG74)	10)	$6x^2+300=y^2$	(BG80)
3)	$67x^2+1=y^2$	(BG76)	11)	$32x^2+1=y^2$	(BG83)
4)	$61x^2+1=y^2$	(BG76)	12)	$9x^2+52=y^2$	(BG85)
5)	$13x^2-1=y^2$	(BG79)	13)	$4x^2+33=y^2$	(BG85)
6)	$8x^2-1=y^2$	(BG79)	14)	$13x^2-13=y^2$	(BG86)
7)	$6x^2+3=y^2$	(BG80)	15)	$13x^2+13=y^2$	(BG86)
8)	$6x^2+12=y^2$	(BG80)	16)	$-5x^2+21=y^2$	(BG87)

Multiple variable equations of degree two and more explained by Bhāskara II

表 7.11. バースカラが規則を与える二次以上の多色方程式のタイプ

(原書に述べられる順序. BG=『ビージャガニタ』)

1)	$ax^2+bx+c=u^2$	(BG151)
2)	$ax^4+bx^2=u^2$	(BG154)
3)	$ax^6+bx^4=u^2$	(BG154)
4)	$ax^2+bx+c=a'y^2+b'y+c'$	(BG157)
5)	$ax^2+by^2+c=u^2$	(BG159)
6)	$a^2x^2+bxy+cy^2=u^2$	(BG162)
7)	$x+y+a=s^2,\ x-y+a=t^2,\ x^2+y^2+b=u^2,\ x^2-y^2+c=v^2$	(BG166)
8)	$ax+b=u^2$	(BG169)
9)	$ax+b=u^3$	(BG169)
10)	$y=(ax^2+c)/b$	(BG174, 176, 177, 178)
11)	$y=(x^3+b)/a$	(BG174)
12)	$axy=bx+cy+d$	(BG185)

Examples of equation of higher degree given by Bhāskara II

表 7.12. バースカラが例題にあげる高次方程式

(原書に述べられる順序. BG=『ビージャガニタ』)

1)	$x^2+y^2+z^2+u^2=x^3+y^3+z^3+u^3$	(BG106)
2)	$x^2+y^2=u^3,\quad x^3+y^3=v^2$	(BG109)
3)	$x^3+12x=6x^2+35$	(BG122)
4)	$x^4-2(x^2+200x)=10^4-1$	(BG123)
5)	$(x+y)^2+(x+y)^3=2(x^3+y^3)$	(BG153)
6)	$5x^4-100x^2=u^2$	(BG155)
7)	$x-y=u^2,\quad x^2+y^2=v^3$	(BG156)
8)	$x^3+y^2=u^2,\quad x+y=v^2$	(BG161)
9)	$3x+1=u^3,\quad 3u^2+1=v^2$	(BG171)
10)	$y=(x^3-6)/5$	(BG179)
11)	$20(x+y+z+u)=xyzu$	(BG183)

(From Hayashi (1993), p.253.)

An example from the *Bījagaṇita*:

132. Example: The square-root of half the number of a swarm of bees is gone to a shrub of jasmin; and so are eight-ninths of the whole swarm: a female is buzzing to one remaining male, that is humming within a lotus, in which he is confined, having been allured to it by its fragrance at night. Say, lovely woman, the number of bees.[1]

Put the number of the swarm of bees *ya v* 2. The square-root of half this is *ya* 1. Eight-ninths of the whole swarm are *ya v* $\frac{16}{9}$. The sum of the square-root and fraction, added to the pair of bees specified, is equal to the amount of the swarm, namely *ya v* 2. Reducing the two sides of the equation to a common denomination, and dropping the denominator, the equation is

$$\begin{array}{lll} ya\ v\ 18 & ya\ 0 & ru\ \ 0 \\ ya\ v\ 16 & ya\ 9 & ru\ 18 \end{array}$$

and, subtraction being made, the two sides are

$$\begin{array}{lll} ya\ v\ 2 & ya\ \dot{9} & ru\ \ 0 \\ ya\ v\ 0 & ya\ 0 & ru\ 18 \end{array}$$

Multiplying both these by eight, and adding the number eighty-one, and extracting both roots, the statement of them for an equation is

$$\begin{array}{ll} ya\ 4 & ru\ \ \dot{9} \\ ya\ 0 & ru\ 15 \end{array}$$

Whence the value *yávat-távat* comes out 6. By substituting the square of this, the number of the swarm of bees is found 72.

[1] This example is repeated from the *Lílávatí*, § 68.

(From Colebrooke (1817), pp.211-212.)

The following is an exposition by Hayashi (2009).

E61=L 71: A flock of bees.

x = number of the bees.

Statement of problem. $\sqrt{x/2} + 8 \cdot (x/9) + 2 = x$.

Solution. Let $x = 2s^2$ (= yāva 2). Then, $\sqrt{x/2} = s$, $8 \cdot (x/9) = (16/9)s^2$, and $s + (16/9)s^2 + 2 = 2s^2$. Reduce all terms to a common denominator and eliminate the denominators: $18s^2 = 16s^2 + 9s + 18$. By the equal subtraction, $2s^2 - 9s = 18$. Multiply both sides by 8 and add 9^2 to them: $16s^2 - 72s + 81 = 225$. Take the square-roots of both sides: $4s - 9 = 15$. Hence follows $s = 6$. Raised by this, $x = 72$.

In the L, this problem is given as an example for the algorithm of the 'multiplier operation' (L 65–66) and, in order to apply that algorithm, the statement is rewritten as $(1/2)\sqrt{x/2} + (8/9)(x/2) + 1 = x/2$ in the prose part of L 71.

(From Hayashi (2009), p.139.)

Contents of the *Sūrya-siddhānta*:

The anonymous *Sūrya-siddhānta* (ca.10th or 11th century) is a very popular Sanskrit astronomical text belonging to the Saura school of Hindu astronomy. There are two English translations of this text (Sastri and Wilkinson (1861), and Burgess (1860).).

The contents of the *Sūrya-siddhānta*:

Chapter I.—Called Madhya-gati which treats of the Rules for finding the mean places of the planets,

Chapter II.—Called Sphuṭa-gati which treats of the Rules for finding the true places of the planets,

Chapter III.—Called the Tripras'na, which treats of the Rules for resolving the questions on time, the position of places, and directions,

Chapter IV.—On the Eclipses of the Moon,

Chapter V.—On the Eclipses of the Sun,

Chapter VI.—On the projection of Solar and Lunar Eclipses,

Chapter VII.—On the conjunction of the planets,

Chapter VIII.—On the conjunction of the planets with the stars,

Chapter IX.—On the heliacal rising and setting of the planets and stars,

Chapter X.—On the phases of the Moon and the position of the Moon's cusps,

Chapter XI.—Called Pátádhikára, which treats of the Rules for finding the time at which the declination of the Sun and Moon become equal,

Chapter XII.—On Cosmographical matters,

Chapter XIII.—On the construction of the armillary sphere and other astronomical instruments,

Chapter XIV.—On kinds of time,

(From Sastri and Wilkinson (tr.): *The Súrya Siddhánta*, Calcutta, 1861)

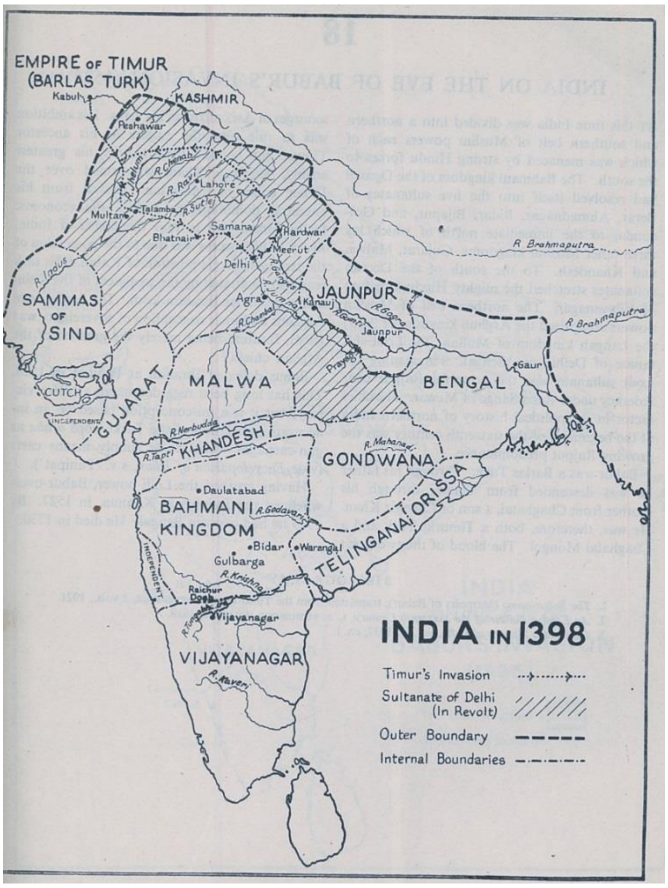
(VI) Coexistent period of the Hindu astronomy and Islamic astronomy

(The 13/14th century ----- the 18/19th century).

After the establishment of Islamic dynasties in North India, the coexistent period of Hindu astronomy and Islamic astronomy (the 13/14th century ~ the 18/19th century) began.

(Actually, the earliest Sanskrit work which mentions a kind of the information of Islamic calendar is the *Kālacakra-tantra* (an esoteric Buddhist work, probably composed in the 11th century), where the year of Hijra is mentioned with two years' error, which I shall mention in the section of Tibetan astronomy.)

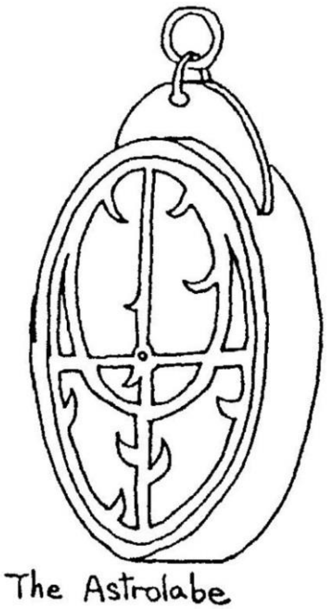
(a) Delhi Sultanate period (1206 ~ 1526 AD)

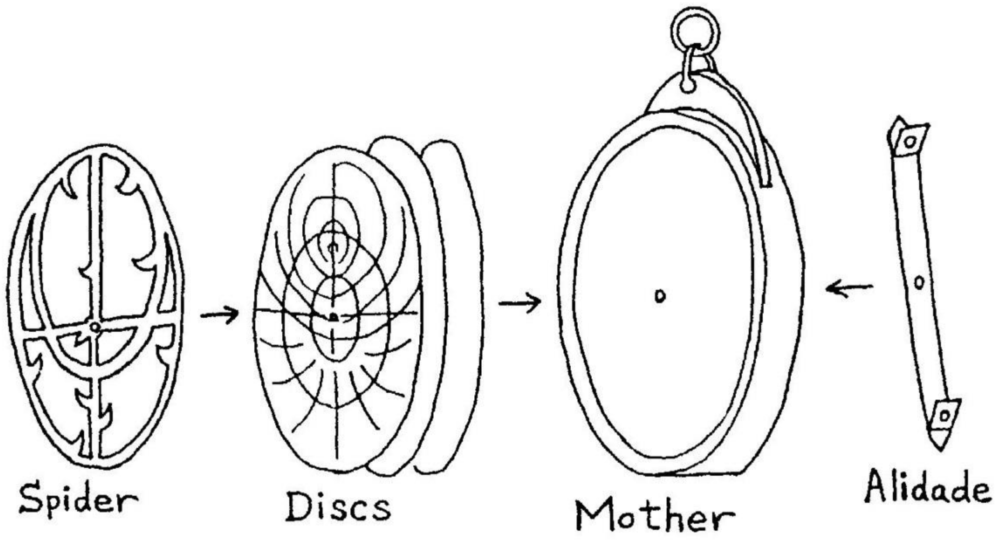


(From Davis: *An Historical Atlas of the Indian Peninsula*, 1959, p.37.)

The first Sanskrit work in which Islamic astronomy is explained in detail is the *Yantra-rāja* (1370) of Mahendra Sūri, which is the first Sanskrit work on the astrolabe. At this time, some Sanskrit works on Hindu astronomical sciences were also translated into Persian by the order of Fīrūz Shāh (reign 1351 ~ 1388), a Sulṭān of the Tughluq dynasty. These events mark the real beginning of the coexistent period of Hindu and Islamic astronomy. For convenience' sake, let us divide this period into two subdivisions, namely the Delhi Sultanate period and the Mughal Empire period.

The astrolabe is a very convenient astronomical instrument.





Components of the astrolabe

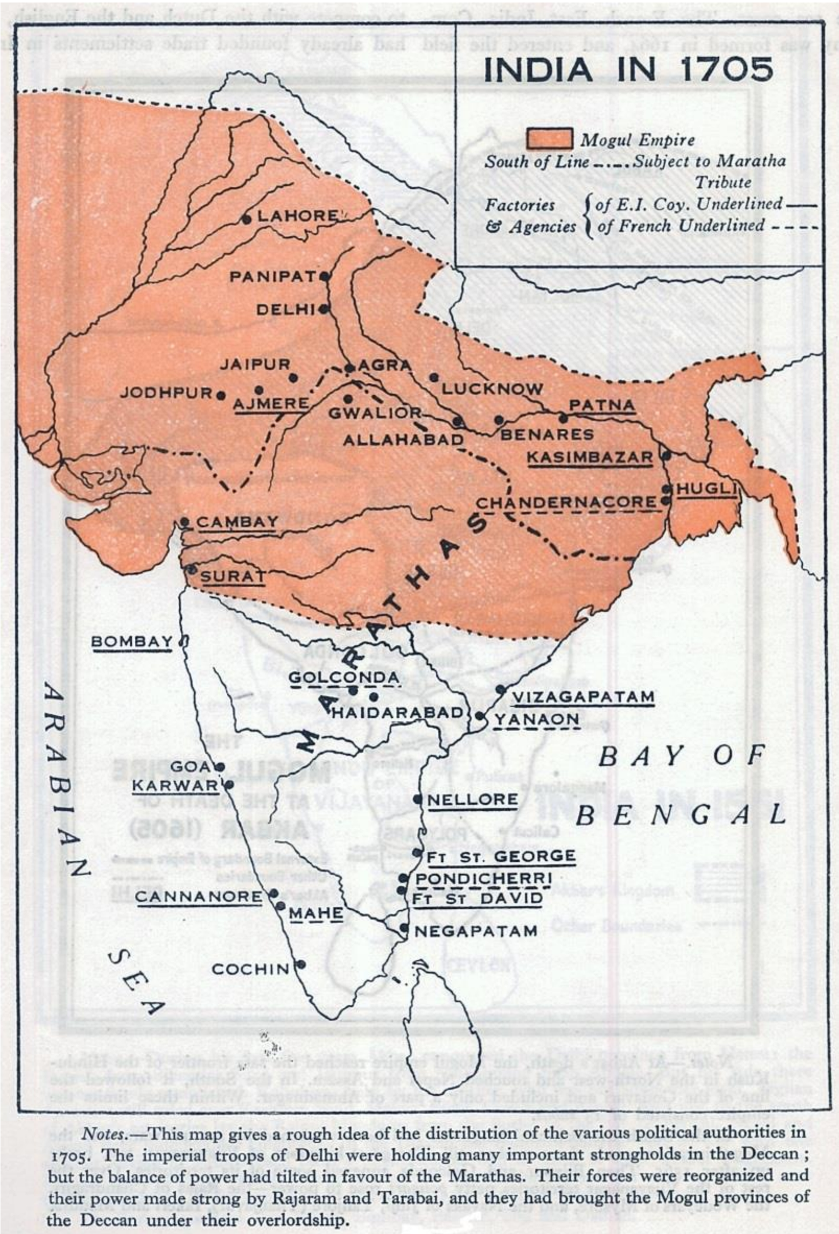
(For detail about introduction of the astrolabe into India, see Ôhashi, Yukio: "Early History of the Astrolabe in India", *Indian Journal of History of Science*, **32**(3), 1997, 199-295.)

During the Delhi Sultanate period, only one *siddhānta* (fundamental treatise of astronomy) was produced. It is the *Sundara-siddhānta* (also called *Siddhānta-sundara*) (1503 CE) of Jñānarāja.

Some interesting *karaṇa*s (handy practical works of astronomy) were produced in this period. One is the *Karaṇa-kautuka* (1496 CE) of Keśava. Keśava's son Gaṇeśa (b. 1507 CE) was also a great astronomer, and his *Graha-lāghava* (1520 CE) is a quite popular *karaṇa*.

There is also a popular Sanskrit astronomical table *Makaranda-sāraṇī* (1478 CE) of Makaranda, which is based on the *Sūrya-siddhānta*.

(b) Mughal Empire period (1526 ~ 1858 AD)



(From Kini and Rao: *Oxford Pictorial Atlas of Indian History*, 1967, p.32.)

Some new *siddhānta*s were composed during the Mughal Empire period. Nityānanda wrote the *Siddhānta-sindhu* (1628 CE) (I have not seen this text.), and the *Siddhānta-rāja* (1639 AD) under the reign of Emperor Shāh Jahān (reign 1628~1658). At the same time, Farīd ad-Dīn Mas‘ūd ibn Ibrāhīm Dihlawī, a court astronomer of Shāh Jahān, composed the *Zīj-i Shāh Jahānī* (Astronomical table dedicated to Emperor Shāh Jahān)(1629 CE) in Persian.

Munīśvara (b.1603) wrote the *Siddhānta-sārva-bhauma* in 1646 CE.

Kamalākara wrote the *Siddhānta-tattva-viveka* in 1658 CE, which basically follows the *Sūrya-siddhānta*.

Obervatories of Sawai Jai Singh (18th century)

In the first half of the 18th century, five traditional astronomical observatories, among which four still exist, were built by Sawai Jai Singh (or Savāī Jaya Siṁha in literal transcription of Nāgarī script) (reign 1699 ~ 1743), a mahārāja who constructed the city of Jaipur.

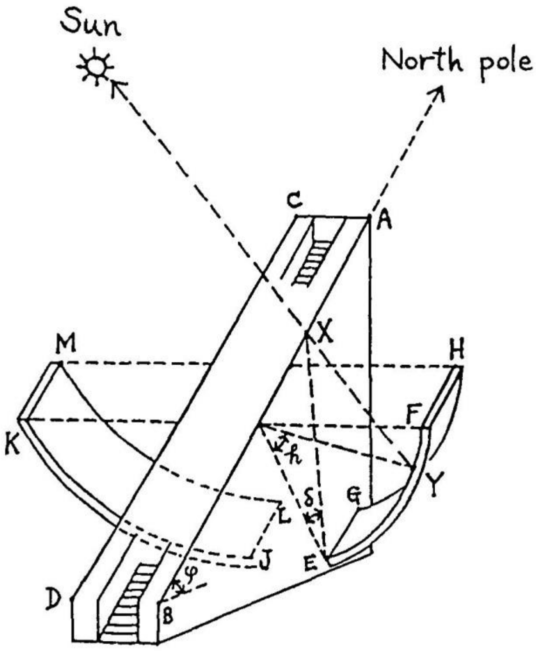
At his court, some astronomical works in Sanskrit and Persian were composed, for example, the *Zīj-i jadīd-i Muḥammad Shāhī* (New astronomical table dedicated to Emperor Muḥammad Shāh) (1728) in Persian. And also, at his court, Jagannātha translated aṭ-Ṭūsī's Arabic version of Ptolemy's *Almagest* into Sanskrit as the *Samrāṭ-siddhānta*, and aṭ-Ṭūsī's Arabic version of Euclid's *Elements* into Sanskrit as the *Rekhā-gaṇita*.

Jai Singh's observatory is extant at Jaipur, Delhi, Banaras, and Ujjain. His observatory at Mathura is not extant. Among them, the Jaipur observatory is the largest.

An overview of the Jaipur observatory.

Samrāṭ-yantra:

Among the several instruments in Jai Singh's observatories, the most famous instrument is probably the *Samrāṭ-yantra* ("emperor instrument"). It is a kind of equatorial sundial. In the figure, φ is the latitude of the observer, δ is the sun's declination, and *h* is the sun's hour angle. In the afternoon, the shadow of the gnomon (AB) is cast on the quadrant (EFGH), and its position (Y) indicates time. The position (X) indicates the sun's declination. In the forenoon, the shadow of the gnomon (CD) is cast on the quadrant (JKLM).

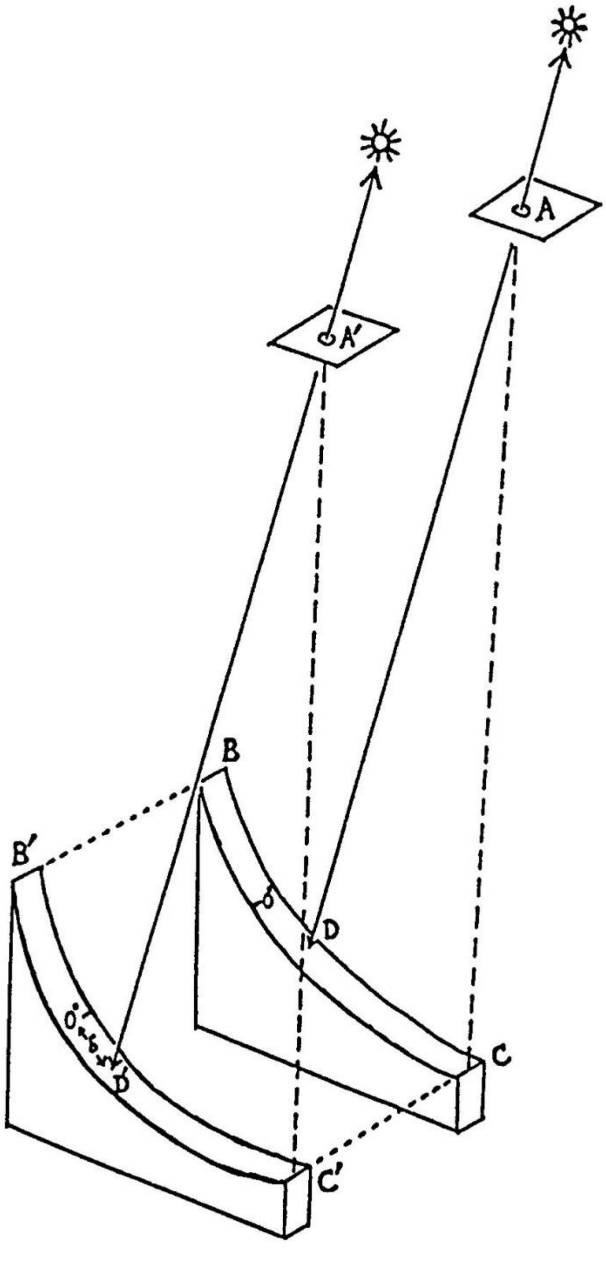


(*Samrāṭ-yantra*)

(*Samrāṭ-yantra* in Jaipur)

Ṣaṣṭhāṁśa-yantra

Under the each quadrant of the larger Samrāṭ-yantra in the Jaipur observatory is constructed a chamber in which the *Ṣaṣṭhāṁśa-yantra* ("sextant instrument") is kept. The image of the sun (D and D') through a pair of pinholes (A and A') at its ceiling is cast on a pair of mural sextants (BC and B'C') at midday. The sun's declination and zenith distance are obtained by this instrument. This is probably the most precise instrument in Jai Singh's observatories.



(*Ṣaṣṭhāṁśa-yantra*)

(*Ṣaṣṭhāṁśa-yantra* in Jaipur)

(*Ṣaṣṭhāṁśa-yantra* in Jaipur)

Miśra-yantra in Delhi

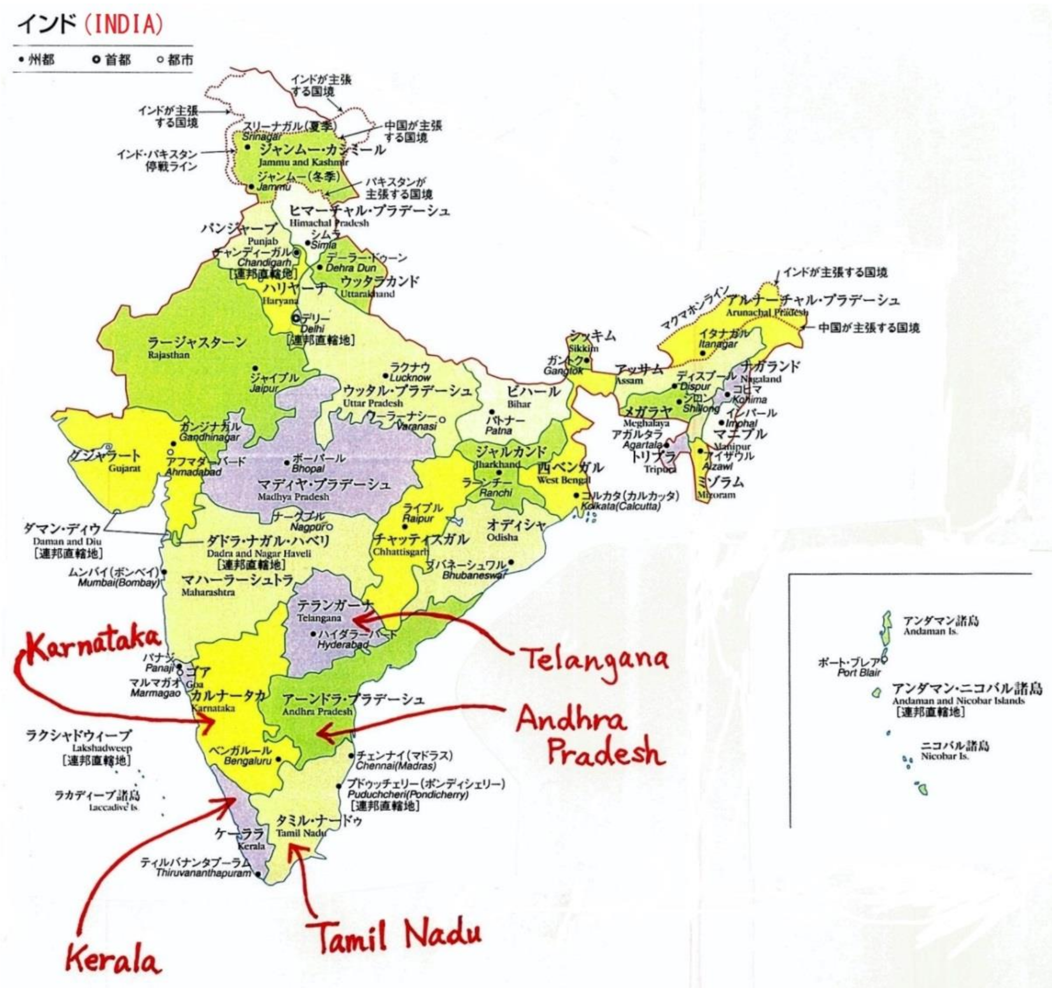
The *Miśra-yantra* is a unique instrument in the Delhi observatory, which is a combination of some instruments. Its front side is used to observe a heavenly body's declination four times a day. It may not be so useful astronomically, but is certainly a beautiful art object. There are several other interesting instruments in Jai Singh's observatories.

(The *Miśra-yantra* in Delhi, looking from south**)**

(The *Miśra-yantra* in Delhi, looking from northheast**)**

Development of mathematics and astronomy in several regions:

Hindu traditional mathematics and astronomy further developed in several regions of India, and the development in South India (Telangana, Andhra Pradesh, Karnataka, Kerala and Tamil Nadu) is significant.



(From 『プレミアム アトラス 世界地図帳』、新訂第 3 版、平凡社、2017.)

The *Āryabhaṭīya*, was popular in Kerala and Tamil Nadu, while the *Sūrya-siddhānta* was popular in the Telugu speaking area (Telangana and Andhra Pradesh).

Telugu speaking area (Telangana and Andhra Pradesh):

Already in 1825, John Warren pointed out in his *Kala Sankalita* that the solar calendar of Tamil region is based on the *Ārya-siddhānta* (=*Āryabhaṭīya*), and the luni-solar calendar of "Telingana" region is based on the *Sūrya-siddhānta*, while Muslims are using lunar calendar. It is known that Mallikārjuna Sūri, the author of the earliest extant commentary of the *Sūrya-siddhānta*, wrote the commentary both in Sanskrit and Telugu. Therefore, the tradition of the *Sūrya-siddhānta* in Telugu speaking area is very important.

Kerala:

There were two major systems of Kerala astronomy.

One is the *Parahita* system, which is based on the *Āryabhaṭīya* (499 CE) of Āryabhaṭa, and was started by Haridatta (ca.650 – 700) who composed the *Graha-cāra-nibandhana*. It is said that this *Parahita* system was started in AD 683.

The other system is the *Dṛk* system started by Parameśvara (ca.1360 – 1455) who composed the *Dṛg-gaṇita* (1431) etc.

Parameśvara's teacher Mādhava (ca.1340 – 1425) was a great mathematician and astronomer. Parameśvara's son's disciples Nīlakaṇṭha Somayājin (1443 – ca.1543) and Jyeṣṭhadeva (ca.1500 – 1610), and Jyeṣṭhadeva's disciple Acyuta Piṣāraṭi (ca.AD 1550 – 1621) were also great astronomers.

Putumana Somayājin (ca.AD 1700 – 1760) and Śaṁkaravarman (AD 1800 – 38) were also great astronomers.

Tamil Nadu:

Astronomy in Tamil Nadu is also closely connected with the astronomy of Kerala. Before the introduction of modern astronomy, Tamil calendars were solely based on the *Vākya-karaṇa* (ca.1300 AD) and its auxiliary tables. This is a practical work for calendar making, and is basically based on the *Mahā-bhāskarīya* of Bhāskara I, who was a follower of the school of Āryabhaṭa, and the *Parahita* system of Haridatta.

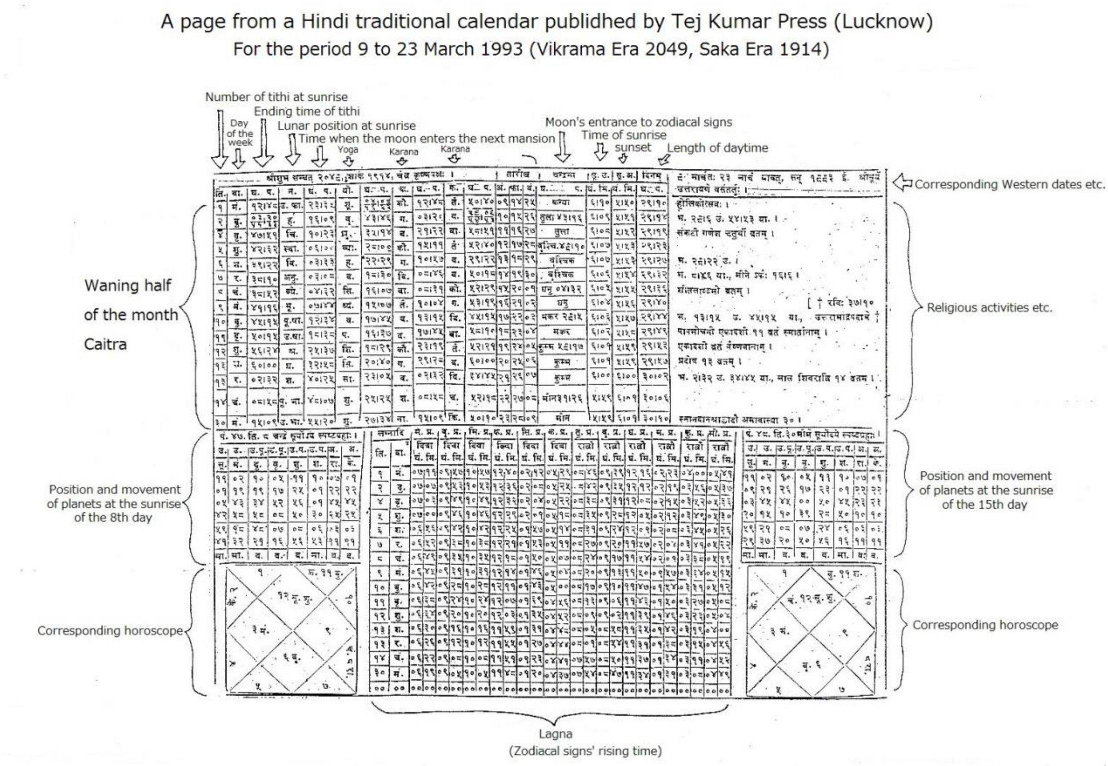
(VII) Modern period

(Coexistent period of the modern astronomy and traditional astronomy).

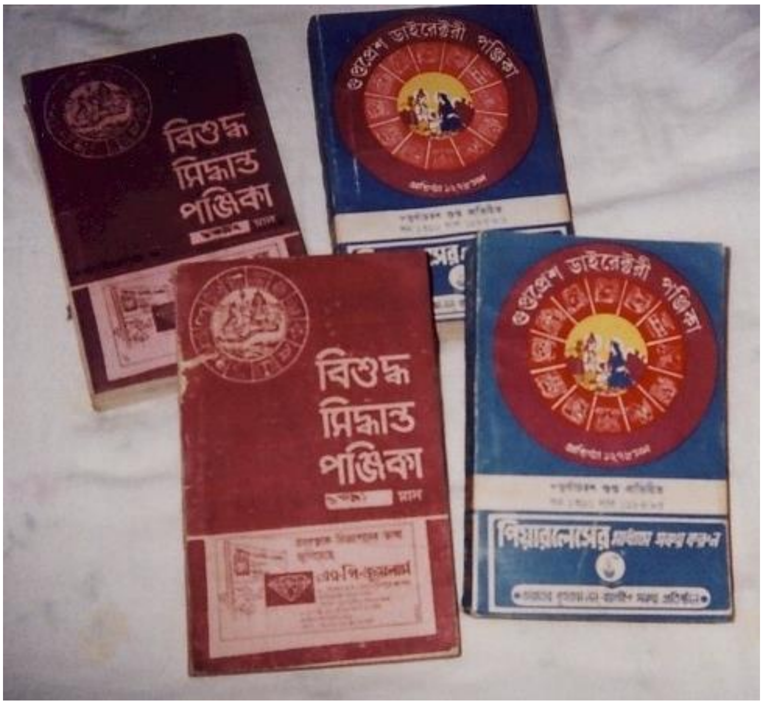
(From the 18/19th century onwards).

Hindu Classical Astronomy is still used in order to make traditional regional calendars etc. There are several traditional regional calendars in several different places. Let us see some example of traditional calendars.

(Some examples of Hindi traditional calendars in Hindi language)



(A page from a Hindi traditional calendar with my notes)



(Some examples of Bengali traditional calendars)

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----- And many more.

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