EXERCISES 1

INTRO TO QUANTUM COMPUTING
MAY 2019
TOKYO TECH

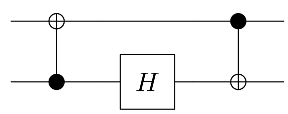
NAME

INSTRUCTIONS: Please ask any questions about the exercises in class. The point values indicate the relative difficulty of the problem.

QUOTE: Facts are meaningless. You can use facts to prove anything that's even remotely true.
Homer Simpson

1 20 points

Find the output of the following quantum circuit for $\frac{3}{5}|01\rangle + \frac{4}{5}|00\rangle$ (the left bit is represented by the top 'wire'). Show your work!



2 10 points

Given the vectors $a = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$, $b = \frac{1}{\sqrt{2}}(i|0\rangle + |1\rangle)$, and $c = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$, which ones represent the same quantum state?

a) a and b; b) b and c; c) c and a; d) all are the same e) all are different;

3 10 points

Given the vectors $a = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$, $b = (\frac{3i}{5}|0\rangle + \frac{4}{5}|1\rangle)$, and $c = \frac{1}{2}(|0\rangle + i|1\rangle)$, which ones represent a quantum state?

a) a and b; b) b and c; c) c and a; d) all do; e) none;

4 20 points

Write the truth table and (describe the) design (of) a (classical) boolean circuit that computes $f : \mathbb{F}_2^3 \to \mathbb{F}_2$ given by $f(x, y, z) = x \oplus (y \otimes z)$ (here $\oplus$ and $\otimes$ are the addition and the multiplication in $\mathbb{F}_2$, respectively) using only $\vee$, $\wedge$, and $\neg$ gates.

5 20 points

Let U be a quantum transformation on the space of quantum 2-states such that $U|0\rangle = \frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle$. Is U uniquely determined by this data? (10 points). Find (one of possibly many, if the answer to the previous question is no) U (10 points).

6 20 points

Given the quantum state $\frac{1}{\sqrt{14}}|001\rangle + \frac{2}{\sqrt{14}}|111\rangle + \frac{3}{\sqrt{14}}|110\rangle$ what is the probability that the third bit is measured as 1 (10 points)? What is the quantum state after such measurement (10 points)?