QIP Course 14: Refuting the local realism view of the universe

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Implicit assumption in the classical physics

1. The real nature of the universe can be completely described by mathematics, and the complete description of an object at a time t allows deterministic prediction of its future trajectory.
2. Effects of an event cannot propagate instantaneously to another place.

The latter is called the locality. Without it everything could interact with everything in the universe, and it would be hopeless to have some theories to predict the future.

The former is called the realism. The purpose of this unit is to explain how to verify whether or not both claims hold simultaneously. It is experimentally confirmed that at least one of them is false.

CHSH inequality

by John Clauser, Michael Horne, Abner Shimony, and Richard Holt in 1969, based on the paper by John Stewart Bell who died when he was nominated a Nobel prize for the work explained today.

Suppose there are two observers Alice and Bob. They measure the photon polarizations by slits, and map pass/absorption to $+1$ or -1. Alice uses two kinds of directions of slits, and outcomes (± 1) are denoted by the random variable A for one direction and A' for the other.
Similarly, Bob uses two kinds of directions of slits, and outcomes (± 1) are denoted by the random variable B for one direction and B' for the other.

Assuming the realism

If there is a complete description of the physical reality, independent of which measurements are made, then there must exist a definite value of B' even when the slit direction for B is used and a value of B is recorded. This realism assumption implies

- Either $B - B'$ or $B + B'$ is zero, and
- the other is ± 2.

From the above, we have

$$S = AB + A'B + AB' - A'B' = A(B + B') + A'(B - B') \leq 2. \tag{1}$$

How to perform an experiment

- Each photon pair must arrive at slits of Alice and Bob almost simultaneously so that they can exclude the possibility of a photon being affected by the measurement outcome of the other.
- Alice and Bob choose their slit directions
  - after each photon pair is generated, otherwise we cannot exclude the possibility that the photon generator makes $AB = A'B = AB' = -A'B' = 1$ and $S > 2$,
  - before the effect of the measurement outcome of the opposite side could reach, otherwise we cannot exclude the possibility that the random slit selection is somehow affected by the measurement outcome of the opposite side.

Checking CHSH value

Performing the experiment and compute the sample average of the random variable S. If both realism and locality (effects cannot propagate faster than light) are true, then the sample average of S must be ≤ 2.
On the other hand, when the quantum state of the pair of photons is

$$\frac{1}{\sqrt{2}}(|+\rangle|-\rangle - |-\rangle|+\rangle),$$

where $|+\rangle$ ($|-\rangle$) is the eigenvector of X belonging to eigenvalue $+1$ (-1), and A corresponds to the observable Z, A' does to X, B does to $-(Z+X)/\sqrt{2}$, and B' does to $(Z-X)/\sqrt{2}$, the sample average of S becomes

$$2\sqrt{2}. \tag{2}$$

Most of experimental results support the quantum theory so far.

- Bell thought that experiments would support the local realism until the first experimental report was provided.
- Bell thought the locality instead of the reality should be discarded.

Source: `http://en.wikipedia.org/wiki/John_Stewart_Bell`

Homework for getting the course credit

Submit your answers by using Tokyo Tech OCW by 16 September. When your report has an incorrect answer, your will be notified of your error and have an oppotunity to revise your report. **Please write the detail of your computations**.

1. Verify (2). (80 points)
2. Solve Q6 of the 10th handout with $x = 4$. Other values are the same. (20 points)
3. (Optional) Your comments on the lecture and/or quantum information in general. (0 point)

1. Suppose that the XZ error occured at the 5th qubit of the Shor code. Describe the changes of the state in the error correction process step by step.
Let

$$
\begin{aligned}
|\varphi\rangle &= \alpha\frac{(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)}{2\sqrt{2}} \\
&\quad +\beta\frac{(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)}{2\sqrt{2}},
\end{aligned}
$$

whose oritinal information is $\alpha|0\rangle + \beta|1\rangle$.

Answers to problems of handout 13 II

After the error, the state becomes

$$
\begin{aligned}
& (I^{\otimes 4} \otimes XZ \otimes I^{\otimes 4})|\varphi\rangle \\
= \quad & \alpha \frac{(|000\rangle + |111\rangle)(|010\rangle - |101\rangle)(|000\rangle + |111\rangle)}{2\sqrt{2}} \\
& + \beta \frac{(|000\rangle - |111\rangle)(|010\rangle + |101\rangle)(|000\rangle - |111\rangle)}{2\sqrt{2}},
\end{aligned}
$$

The both measurements of $Z_1 \otimes Z_2$ and $Z_2 \otimes Z_3$ gives outcome $+1$ and we do nothing to correct errors.
The both measurements of $Z_4 \otimes Z_5$ and $Z_5 \otimes Z_6$ gives outcome -1, we conclude that the X error occured at the 5th qubit, and we apply X_5^{-1}. The state becomes

Answers to problems of handout 13 III

$$
\begin{aligned}
& (I^{\otimes 4} \otimes Z \otimes I^{\otimes 4})|\varphi\rangle \\
= \quad & \alpha\frac{(|000\rangle + |111\rangle)(|000\rangle - |111\rangle)(|000\rangle + |111\rangle)}{2\sqrt{2}} \\
& +\beta\frac{(|000\rangle - |111\rangle)(|000\rangle + |111\rangle)(|000\rangle - |111\rangle)}{2\sqrt{2}},
\end{aligned}
$$

The both measurements of $Z_7 \otimes Z_8$ and $Z_8 \otimes Z_9$ gives outcome $+1$ and we do nothing to correct errors.
The both measurements of $X_1 \otimes \cdots \otimes X_6$ and $X_4 \otimes \cdots \otimes X_9$ gives outcome -1, we conclude that the Z error occured at the 4th, 5th, or 6th qubit, and we apply Z_4^{-1}. The state becomes

Answers to problems of handout 13 IV

$$
\begin{aligned}
& |\varphi\rangle \\
= \;& \alpha\frac{(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)}{2\sqrt{2}} \\
& +\beta\frac{(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)}{2\sqrt{2}},
\end{aligned}
$$

2. Verify that I, X, Z, and XZ form a basis for the linear space of 2×2 matrices.
Answer:

$$
\begin{aligned}
& \begin{pmatrix} a & b \\ c & d \end{pmatrix} && (3) \\
= \;& \frac{a+d}{2}I + \frac{a-d}{2}Z + \frac{c+b}{2}X + \frac{c-b}{2}XZ && (4)
\end{aligned}
$$

Answers to problems of handout 13 V

Any matrix can be written as a linear combination of I, X, Z, and XZ. On the other hand, if Eq. (3) is zero, then all coefficnents in Eq. (4) are zero. These two facts imply that they are a basis.
3. Suppose that the H error occured at the 5th qubit of the Shor code, where

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \quad H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}.$$

Describe the changes of the state in the error correction process step by step.
Answer: Observe that $H = (X + Z)/\sqrt{2}$. The state after error is

$$I^{\otimes 4} \otimes (X + Z)/\sqrt{2} \otimes I^{\otimes 4}|\varphi\rangle.$$

The both measurements of $Z_1 \otimes Z_2$ and $Z_2 \otimes Z_3$ gives outcome $+1$ and we do nothing to correct errors.

The measurements of $Z_4 \otimes Z_5$ gives outcome -1 with probability 0.5, or, outcome $+1$ with probability 0.5.
For a while assume that the outcome is -1. Then the state after the measurement is

$$
\begin{aligned}
& (I^{\otimes 4} \otimes X \otimes I^{\otimes 4})|\varphi\rangle \\
= \quad & \alpha \frac{(|000\rangle + |111\rangle)(|010\rangle + |101\rangle)(|000\rangle + |111\rangle)}{2\sqrt{2}} \\
& +\beta \frac{(|000\rangle - |111\rangle)(|010\rangle - |101\rangle)(|000\rangle - |111\rangle)}{2\sqrt{2}}.
\end{aligned}
$$

The above state after the measurement can be easily computed. Because $X_5|\varphi\rangle$ belongs to the eigenvalue -1 of $Z_4 \otimes Z_5$, and $Z_5|\varphi\rangle$ belongs to the eigenvalue $+1$. The state is already a linear combination of eigenstates.

Answers to problems of handout 13 VII

The both measurements of $Z_5 \otimes Z_6$ gives outcome -1 with probability 1. We conclude that the X error occured at the 5th qubit and apply X_5^{-1}. The state becomes $|\varphi\rangle$.
Since we get the original state $|\varphi\rangle$, all the measurement outcomes of $Z_7 \otimes Z_8$, $Z_8 \otimes Z_9$, $X_1 \otimes \cdots \otimes X_6$, and $X_4 \otimes \cdots \otimes X_9$ is $+1$, and we do nothing to correct errors.
On the other hand, suppose that the measurements of $Z_4 \otimes Z_5$ gives outcome $+1$. Then the state after the measurement is

$$\begin{aligned} & (I^{\otimes 4} \otimes Z \otimes I^{\otimes 4})|\varphi\rangle \\ = \quad & \alpha \frac{(|000\rangle + |111\rangle)(|000\rangle - |111\rangle)(|000\rangle + |111\rangle)}{2\sqrt{2}} \\ & + \beta \frac{(|000\rangle - |111\rangle)(|000\rangle + |111\rangle)(|000\rangle - |111\rangle)}{2\sqrt{2}}. \end{aligned}$$

Answers to problems of handout 13 VIII

The both measurements of $Z_5 \otimes Z_6$ gives outcome $+1$ with probability 1. We conclude that the no X error occured at the 4th, 5th , or 6th qubit.
The both measurements of $Z_7 \otimes Z_8$ and $Z_8 \otimes Z_9$ gives outcome $+1$ with probability 1 and we do nothing to correct errors.
The both measurements of $X_1 \otimes \cdots \otimes X_6$ and $X_4 \otimes \cdots \otimes X_9$ gives outcome -1 with probability 1, we conclude that the Z error occured at the 4th, 5th, or 6th qubit, and we apply Z_4^{-1}. Then we get $|\varphi\rangle$.
4. Show that the fidelity between $\alpha|0\rangle + \beta|1\rangle$ and ρ' is at least $(1-p)^3 + 3p(1-p)^2$.
Answer: omitted.
5. Show that $0 \leq \langle\varphi|\rho|\varphi\rangle \leq 1$.
Answer: Let

$$\rho = \lambda_1|\psi_1\rangle\langle\psi_1| + \cdots + \lambda_n|\psi_n\rangle\langle\psi_n|$$

Answers to problems of handout 13 IX

such that $|\psi_i\rangle$ is an eigenvector belonging to the eigenvalue λ_i of ρ. Then we have

$$\begin{aligned} & \langle\varphi|\rho|\varphi\rangle \\ = & \sum_{i=1}^{n} \lambda_i \langle\varphi|\psi_i\rangle\langle\psi_i|\varphi\rangle \\ = & \sum_{i=1}^{n} \lambda_i |\langle\varphi|\psi_i\rangle|^2. \end{aligned}$$

Since all the eigenvalues are nonnegative by the definition of density matrices in page 7-4, the above value is nonegative.

Answers to problems of handout 13 X

Let $\{|\varphi\rangle, |\varphi_2\rangle, ..., |\varphi_n\rangle\}$ be an ONB. Then we have

$$\begin{aligned} & \langle\varphi|\rho|\varphi\rangle \\ \leq & \langle\varphi|\rho|\varphi\rangle + \sum_{i=2}^{n}\langle\varphi_i|\rho|\varphi_i\rangle \\ = & \mathrm{Tr}[\rho], \end{aligned}$$

which is ≤ 1 by the definition of density matrices in page 7-4.