

集積回路設計

14. トピック：算術演算回路と セルライブラリ合成

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集積回路設計期末試験

■ 日時・場所

- 6月7日(金) 13:20～14:50、S321講義室
- 出題範囲: 第1回～第13回(本日講義分は除く)
- ノート・講義資料・その他: 持ち込み不可

VLSIの設計方式 (1)

■ フルカスタム方式

- 論理設計、回路設計、レイアウト設計を全て手動設計
- 回路面積・動作速度・消費電力を最適化できる
- 開発コスト(設計コスト)は高いが、量産効果による製品単価の大幅削減が可能(汎用CPU、メモリ)
- アナログ回路は通常フルカスタム方式で設計される

■ スタンドダードセル方式

- 論理設計、回路設計、レイアウト設計を全て自動化
- 論理セル: 数十～数百種のレイアウト済み設計物
- 集積度・性能はフルカスタムに劣るが、開発コストを大幅削減可能

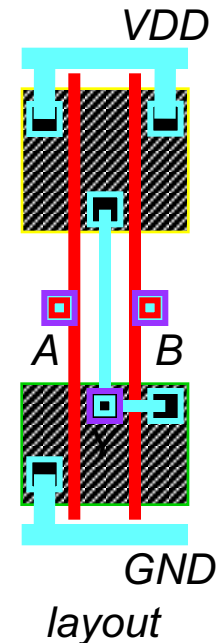
Technology Library (1)

A) Standard-cell library

- Cell types :
 - Primitive/complex gates : NAND, NOR, INV, AOI, MUX, tristate-buffer, half-adder, full-adder, carry-generator, ...
 - Storage elements : DFF, LATCH
- Models :
 - Functional (technology-independent)
 - Delay, layout (technology-dependent)

Load :
A : 0.025pF
B : 0.024pF
Internal delay :
A=>Y(rise) : 0.055 ns
A=>Y(fall) : 0.051 ns
B=>Y(rise) : 0.073 ns
B=>Y(fall) : 0.060 ns
Output transition delay :
Y(rise) : 1.532 ns/pF
Y(fall) : 1.153 ns/pF

delay



Technology Library (2)

B) Memory module library/generator

- Input parameters
 - capacity, word size
 - access timing (synchronous / asynchronous)
 - # of ports (ex. 1 R/W, 1R + 1W, 2R + 1W)
- Layout data : predesigned or auto-generated

C) Arithmetic module library/generator

- Input parameters
 - Functionality (add, subtract, multiply, divide, ...)
 - Word size
 - Speed
- “Hard” module : layout data
- “Soft” module : synthesizable RTL or gate-level netlist

Technology Library (3)

D) Complex function modules

(IP : intellectual property)

- Complex functions take long time to design → design reuse
- Types of IPs : CPU-core, interface, customized datapath
- “Hard” IP and “Soft” IP
- IP designers and IP users are different people (different organization in most cases)
 - Need very good documentation of IP. (Difficulty of understanding how the IP works and how to use the IP is the major problem in IP-based designs)

Hard Module vs. Soft Module

- Hard module (layout data)
 - Optimize at transistor-level and layout
 - Smaller, faster, power efficient
 - Technology-dependent
 - need to redesign for each different technology
 - expensive
- Soft module (gate-level / RTL netlist)
 - Optimize at architecture level
 - Technology-independent
 - can take advantage of the latest process technologies
 - easier to incorporate into user's designs
- *Soft module is becoming more popular than hard modules*
- *Need good cell library !*

Cell Synthesis

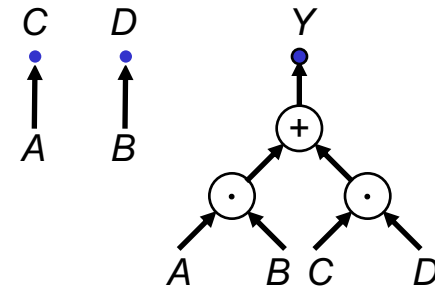
- Purpose :
 - Standard-cell library compilation
 - Transister-level custom module synthesis
 - Because of the fast change in process technology, libraries have to be designed quickly → more tasks are being automated
- Input : logic function, layout design rule (technology)
- Output : cell layout
- Process flow :
 - Logic function
 - ↓
 - CMOS circuit topology
 - ↓
 - CMOS placement & routing (abstract / stick diagram)
 - ↓
 - CMOS layout

CMOS Circuit Topology Enumeration (1)

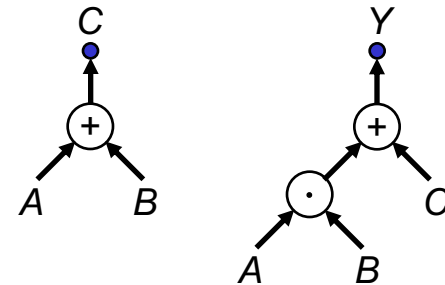
Ex. 2-input XOR : $Y = A \oplus B$

A) Decompose to negative-unate functions

$$\begin{aligned} \text{i. } Y &= A \cdot (\sim B) + (\sim A) \cdot B \\ &= \sim((\sim A) \cdot (\sim B) + A \cdot B) \\ C &= \sim A \\ D &= \sim B \\ Y &= \sim(C \cdot D + A \cdot B) \end{aligned}$$



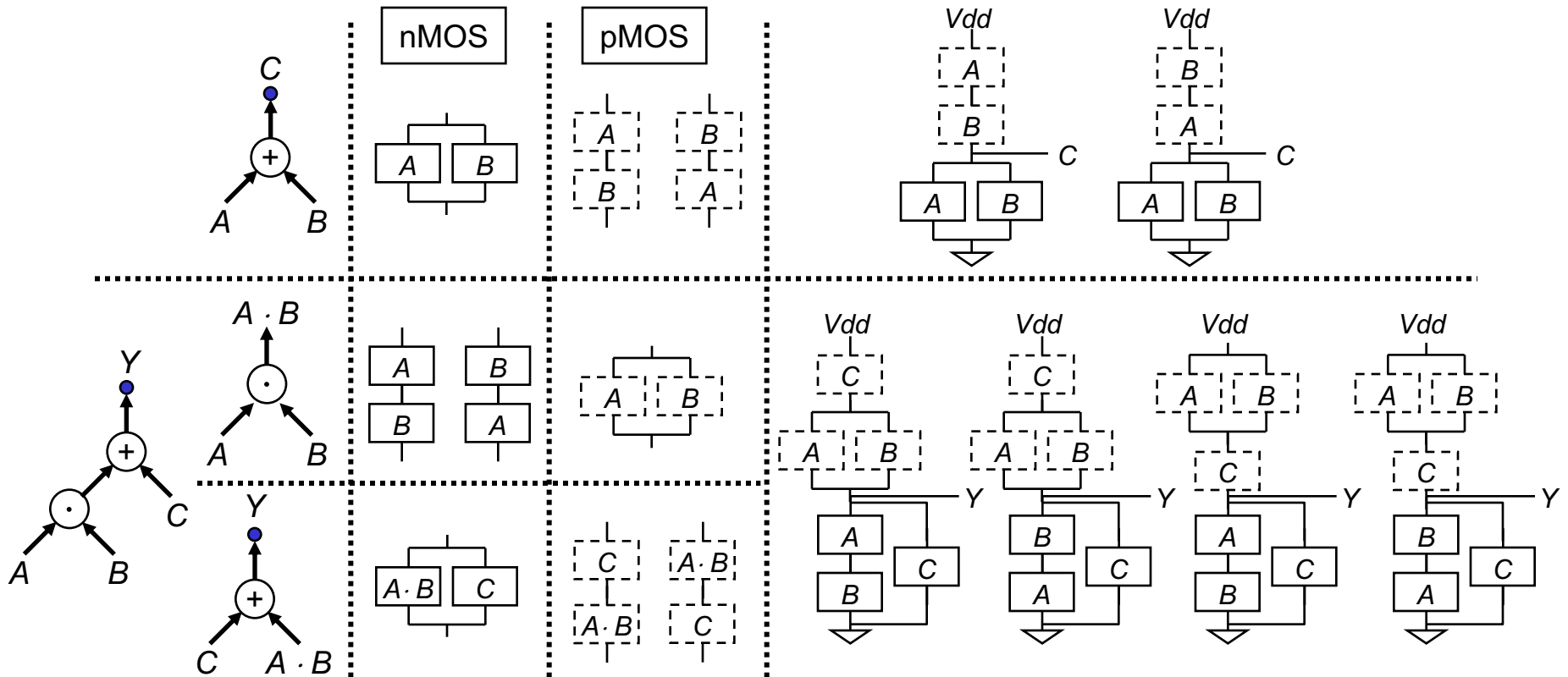
$$\begin{aligned} \text{ii. } Y &= \sim((\sim A) \cdot (\sim B) + A \cdot B) \\ &= \sim(\sim(A + B) + A \cdot B) \\ C &= \sim(A + B) \\ Y &= \sim(C + A \cdot B) \end{aligned}$$



CMOS Circuit Topology Enumeration (2)

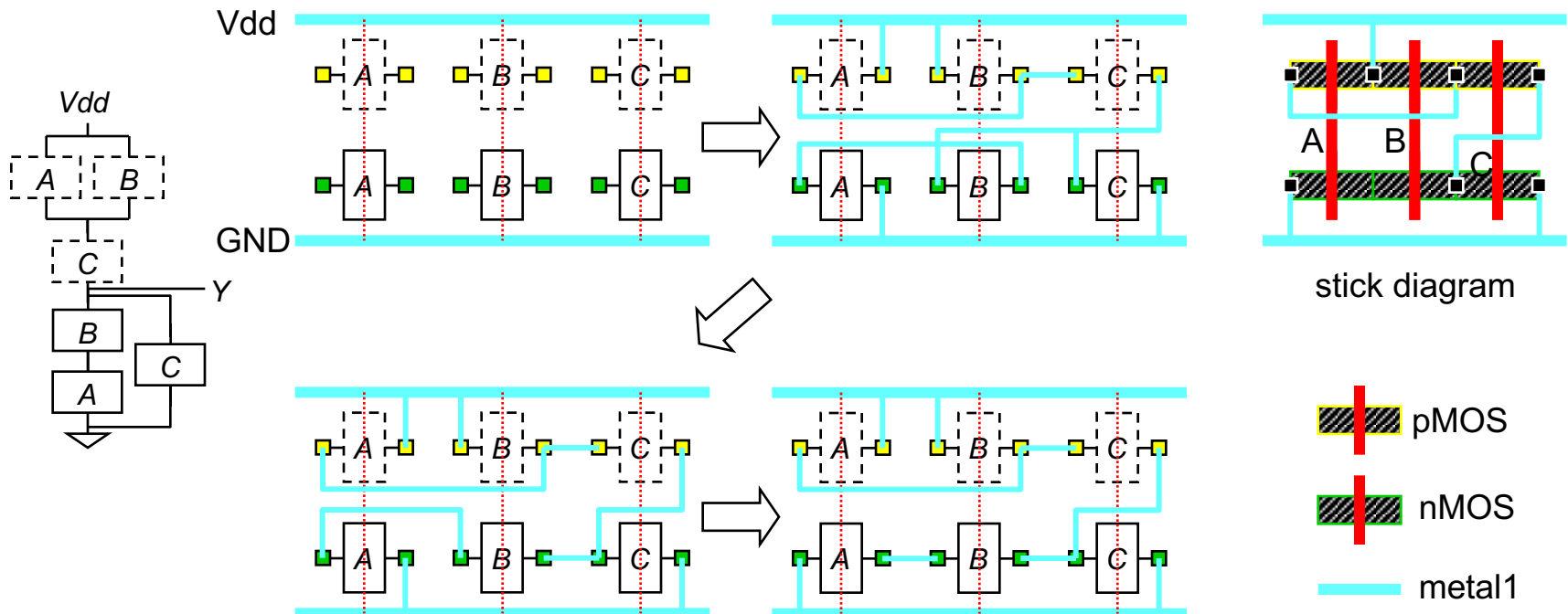
B) Construct circuit topology

- OR : nMOS = parallel, pMOS = serial
- AND : nMOS = serial, pMOS = parallel



CMOS Placement and Routing

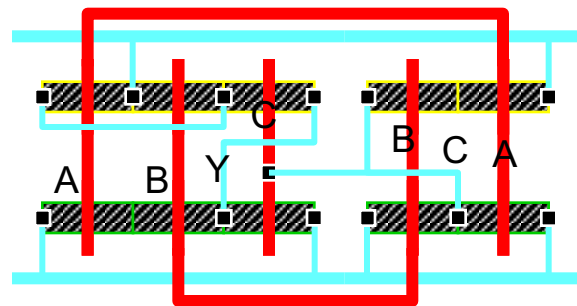
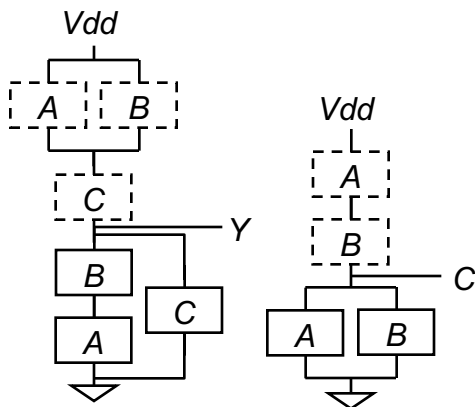
- A) Transistor placement (gate ordering)
 - place the pMOS and nMOS with the same gate input at the same vertical line
 - Vdd : top-horizontal, GND : horizontal-bottom
- B) Permutate transistor pin (drain/source) direction
- C) Cost function : area (drain/source sharing), # of routing tracks , # of routing layers (*routing cost is not easy to evaluate before actual layout*)



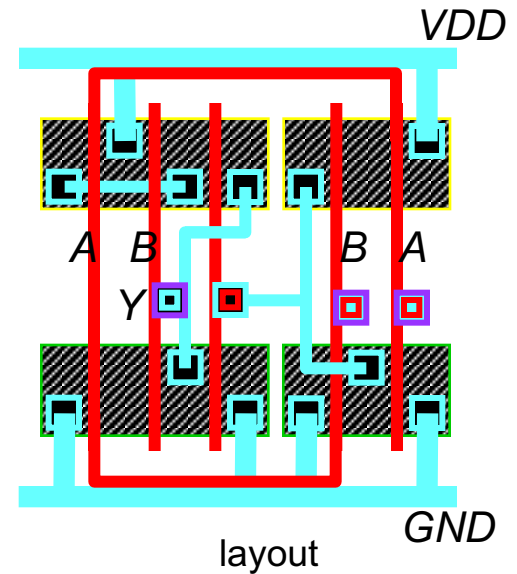
CMOS Layout

A) Generate layout data from stick diagram

- Technology-dependent design rule :
 - Minimum width
 - Minimum spacing



stick diagram



layout

Arithmetic Logic Synthesis

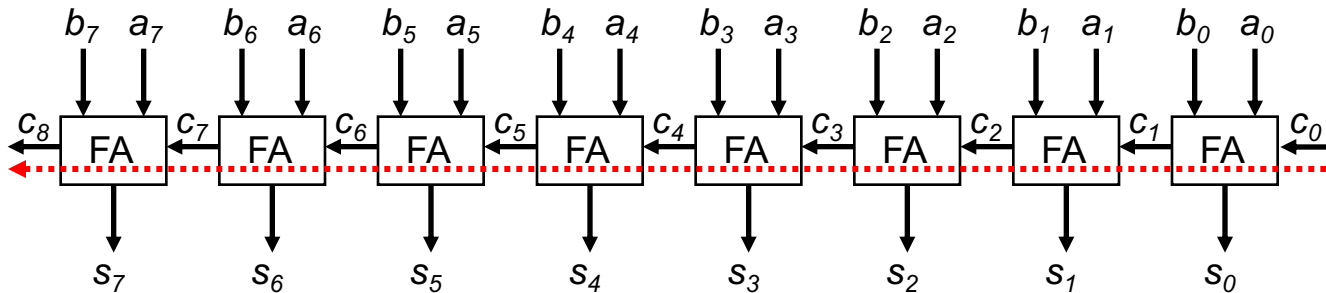
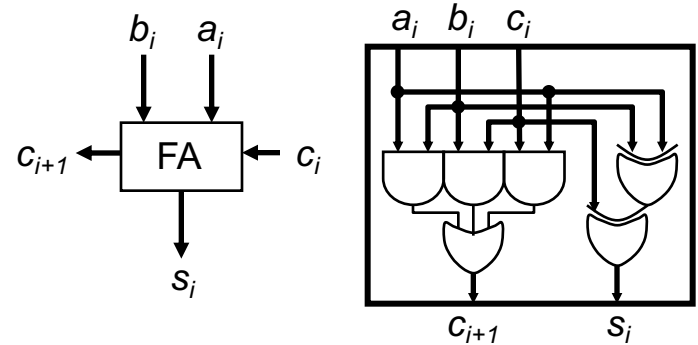
- Circuit implementation techniques for arithmetic logic modules (adder/subtractor, multiplier, divider, square root, trigonometric functions, etc.) have been extensively studied.
 - Transistor-level structures
 - Gate-level structures
 - RTL structures
- Logic optimization algorithms do not work well with arithmetic logic
 - Logic synthesis tools generate predesigned logic structures for a limited types of arithmetic logic (adder, subtractor, multiplier)
 - Some standard cell libraries include cells dedicated for arithmetic logic (such as carry computation)

Adders

- Addition is the most basic arithmetic operation
- Variety of adder structures
 - ✓ Ripple-carry adder
 - ✓ Carry-lookahead adder
 - ✓ Carry-skip adder
 - ✓ Carry-select adder
 - ✓ Carry-save adder (for intermediate accumulation)
- Tradeoff in circuit area, speed (power)

Ripple-Carry Adder

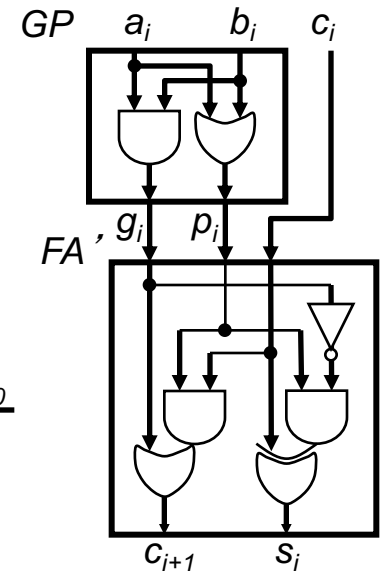
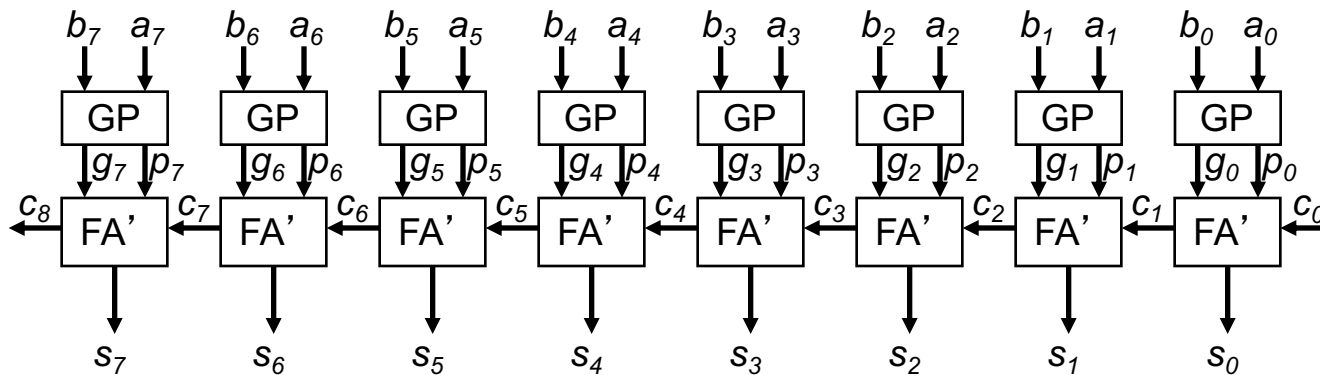
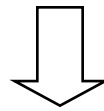
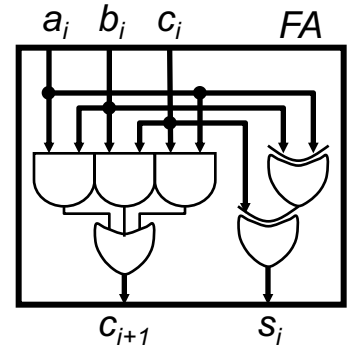
- $c_{i+1} = a_i \cdot b_i + a_i \cdot c_i + b_i \cdot c_i$
- $s_i = a_i \oplus b_i \oplus c_i$



- Small area
- Slow because of carry propagation

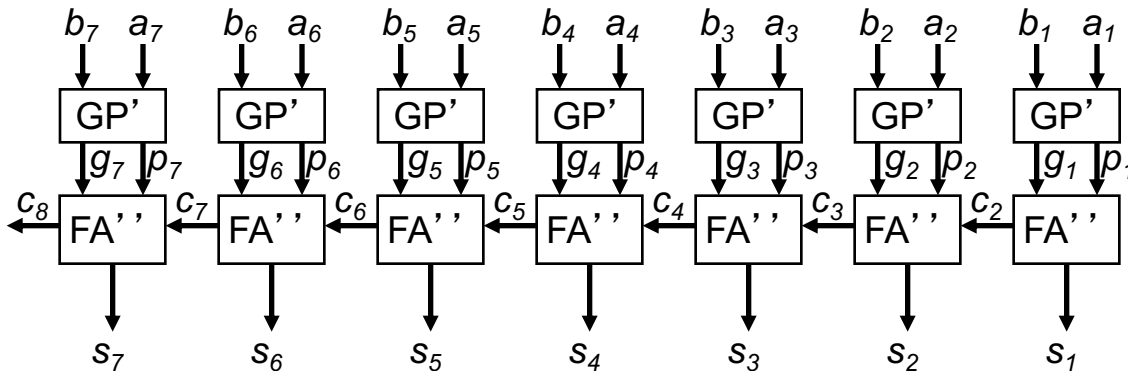
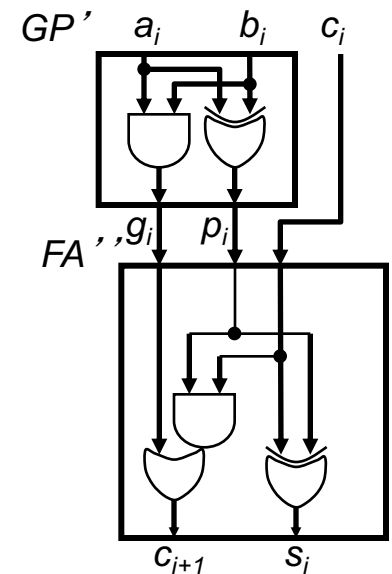
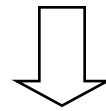
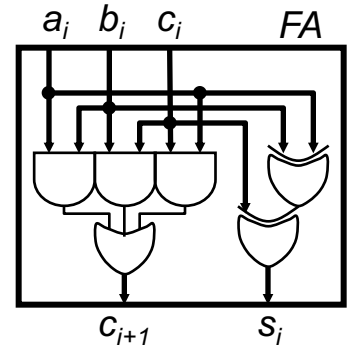
Full Adder with Carry-Generate and Carry-Propagate (1)

- *Generate* : $g_i = a_i \cdot b_i$
- *Propagate* : $p_i = a_i + b_i$
- $c_{i+1} = a_i \cdot b_i + a_i \cdot c_i + b_i \cdot c_i$
 $= a_i \cdot b_i + (a_i + b_i) \cdot c_i = g_i + p_i \cdot c_i$
- $s_i = a_i \oplus b_i \oplus c_i$
 $= (\bar{a}_i \cdot b_i + a_i \cdot \bar{b}_i) \oplus c_i = (p_i \cdot \bar{g}_i) \oplus c_i$



Full Adder with Carry-Generate and Carry-Propagate (2)

- *Generate* : $g_i = a_i \cdot b_i$
 - *Propagate* : $p_i = a_i \oplus b_i$
- $c_{i+1} = a_i \cdot b_i + a_i \cdot c_i + b_i \cdot c_i$
- $$= a_i \cdot b_i + a_i \cdot (b_i + \bar{b}_i) \cdot c_i + b_i \cdot (a_i + \bar{a}_i) \cdot c_i$$
- $$= a_i \cdot b_i + a_i \cdot b_i \cdot c_i + (a_i \cdot \bar{b}_i + \bar{a}_i \cdot b_i) \cdot c_i$$
- $$= a_i \cdot b_i + (a_i \oplus b_i) \cdot c_i = g_i + p_i \cdot c_i$$
- $s_i = a_i \oplus b_i \oplus c_i = p_i \oplus c_i$

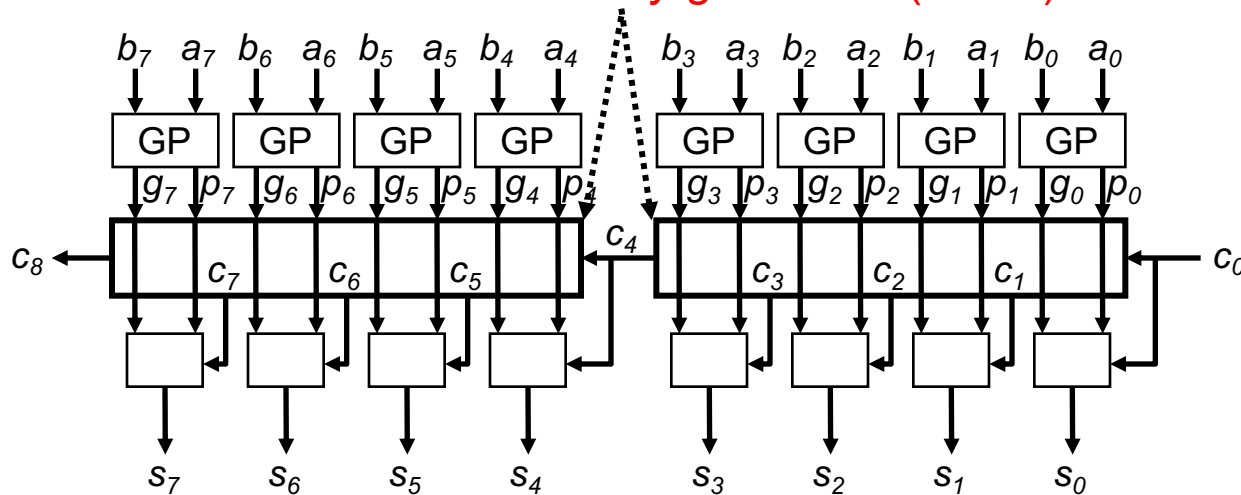


Carry-Lookahead Adder

- Unfolding carry propagation

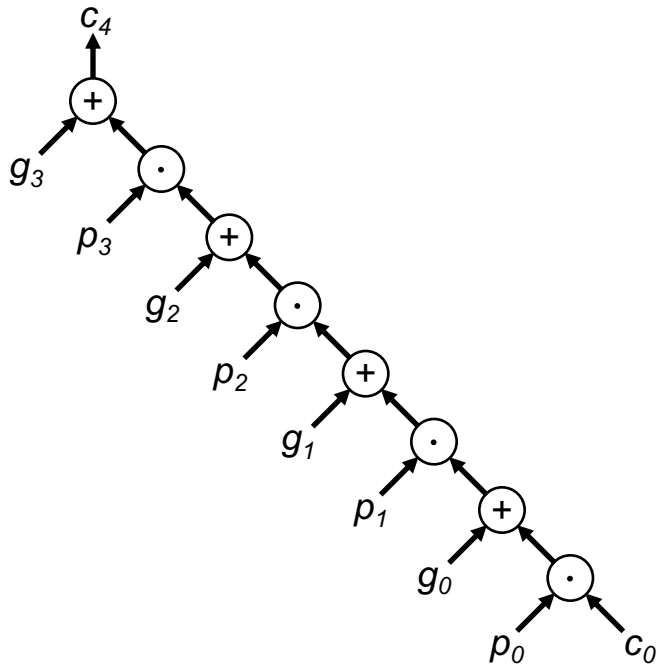
→ 4-bit lookahead carry generator (LAC4)

- $c_{i+1} = g_i + p_i c_i$
 - $c_{i+2} = g_{i+1} + p_{i+1} c_{i+1} = g_{i+1} + p_{i+1}(g_i + p_i c_i)$
 - $c_{i+3} = g_{i+2} + p_{i+2} c_{i+2} = g_{i+2} + p_{i+2}(g_{i+1} + p_{i+1}(g_i + p_i c_i))$
 - $c_{i+4} = g_{i+3} + p_{i+3} c_{i+3} = g_{i+3} + p_{i+3}(g_{i+2} + p_{i+2}(g_{i+1} + p_{i+1}(g_i + p_i c_i)))$
- 4-bit lookahead carry generator (LAC4)*



CMOS Implementation of Lookahead Carry Generator (1)

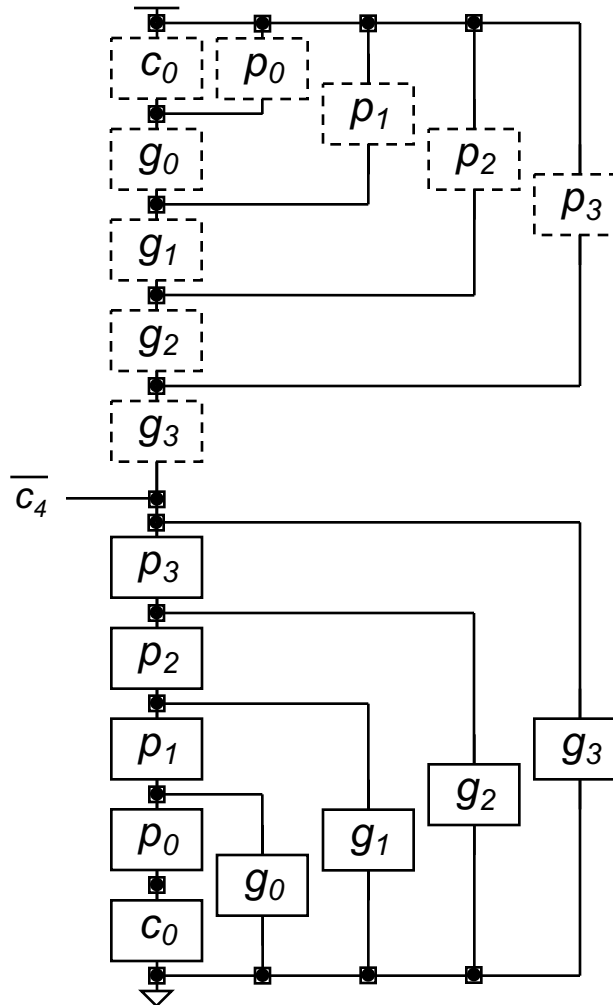
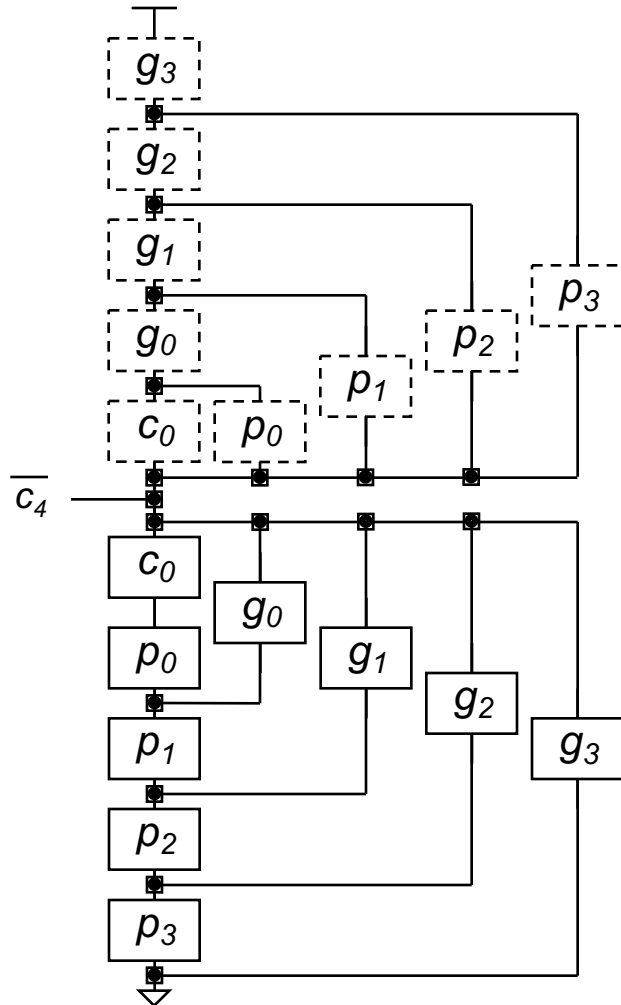
- $$c_4 = g_3 + p_3(g_2 + p_2(g_1 + p_1(g_0 + p_0c_0)))$$



	nMOS	pMOS
	conducts if $x = 1$	conducts if $x = 0$

CMOS Implementation of Lookahead Carry Generator (2)

- $$c_4 = g_3 + p_3(g_2 + p_2(g_1 + p_1(g_0 + p_0c_0)))$$

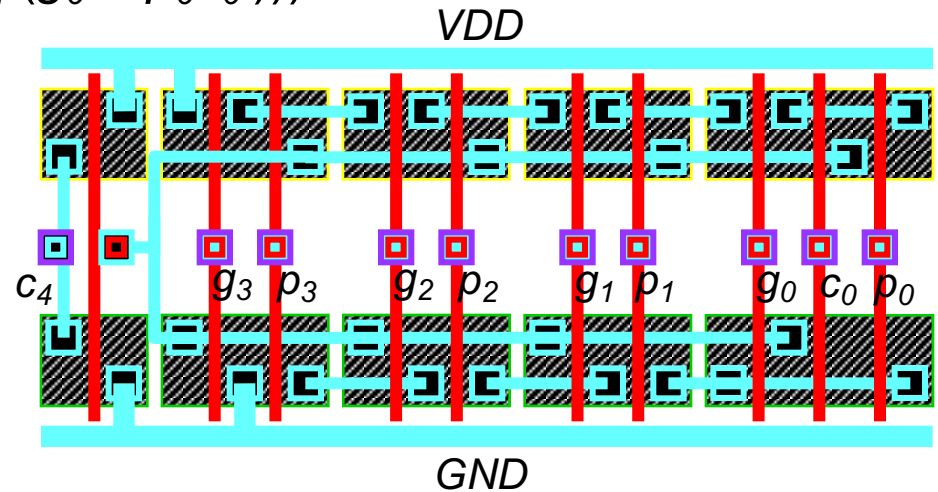
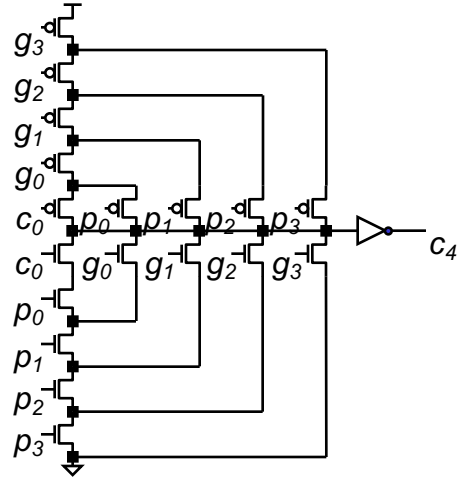


There are a total of 2^{10} combinations of network topologies

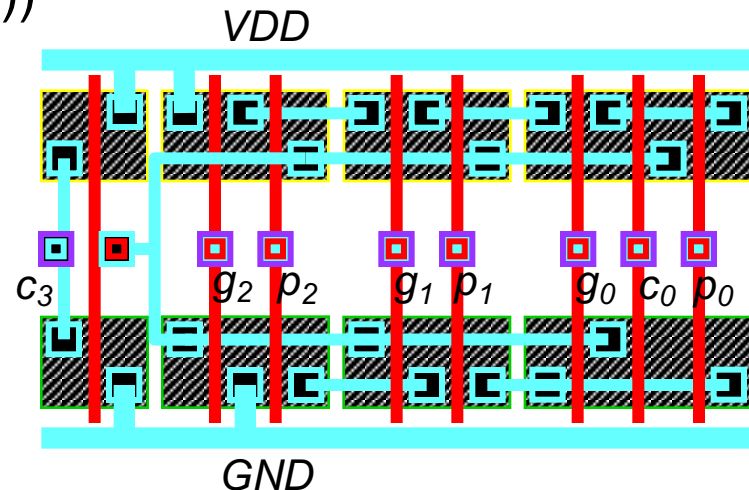
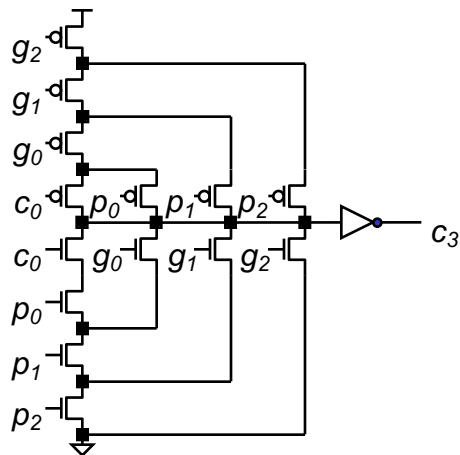
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CMOS Implementation of Lookahead Carry Generator (3)

- $$c_4 = g_3 + p_3(g_2 + p_2(g_1 + p_1(g_0 + p_0c_0)))$$

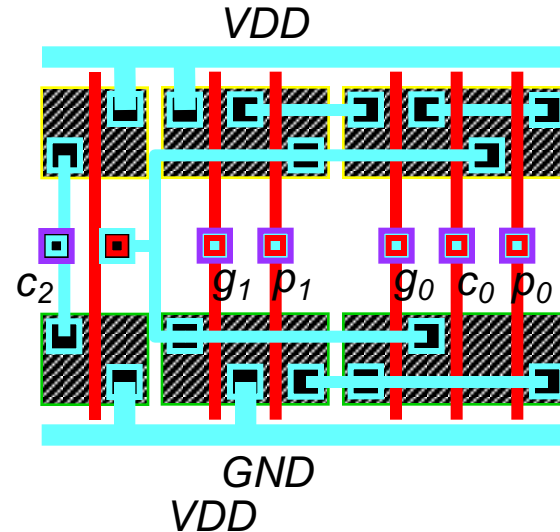
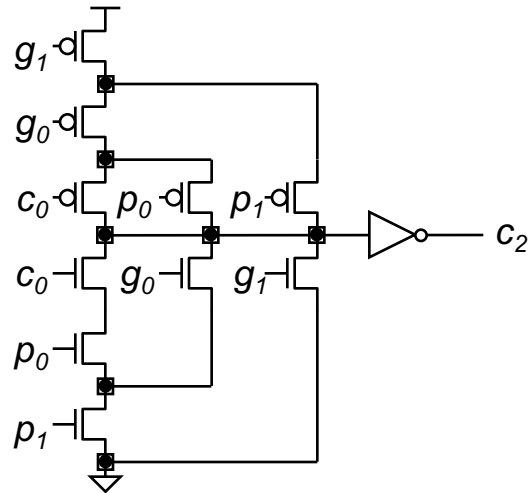


- $$c_3 = g_2 + p_2(g_1 + p_1(g_0 + p_0c_0))$$

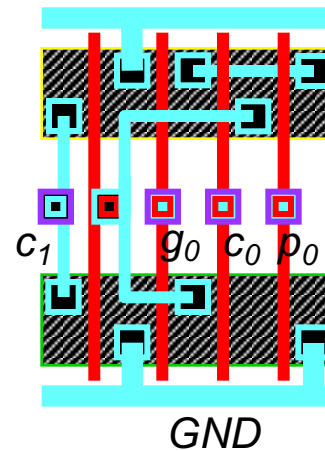
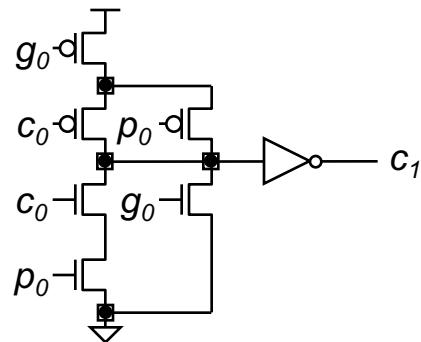


CMOS Implementation of Lookahead Carry Generator (4)

- $c_1 = g_0 + p_0 c_0$



- $c_2 = g_1 + p_1 (g_0 + p_0 c_0)$



Hierarchical Carry-Lookahead (1)

- 4-bit generate-propagate :

$$\begin{aligned}
 \checkmark \quad c_{i+4} &= g_{i+3} + p_{i+3} (g_{i+2} + p_{i+2} (g_{i+1} + p_{i+1} (g_i + p_i c_i))) \\
 &= \underline{g_{i+3} + p_{i+3} (g_{i+2} + p_{i+2} (g_{i+1} + p_{i+1} g_i))} + \underline{(p_{i+3} p_{i+2} p_{i+1} p_i) c_i} \\
 &= g'_{i+4} + p'_{i+4} c_i
 \end{aligned}$$

$$\checkmark \quad c_{i+8} = g'_{i+4} + p'_{i+4} c_{i+4} = g'_{i+4} + p'_{i+4} (g'_{i+4} + p'_{i+4} c_i)$$

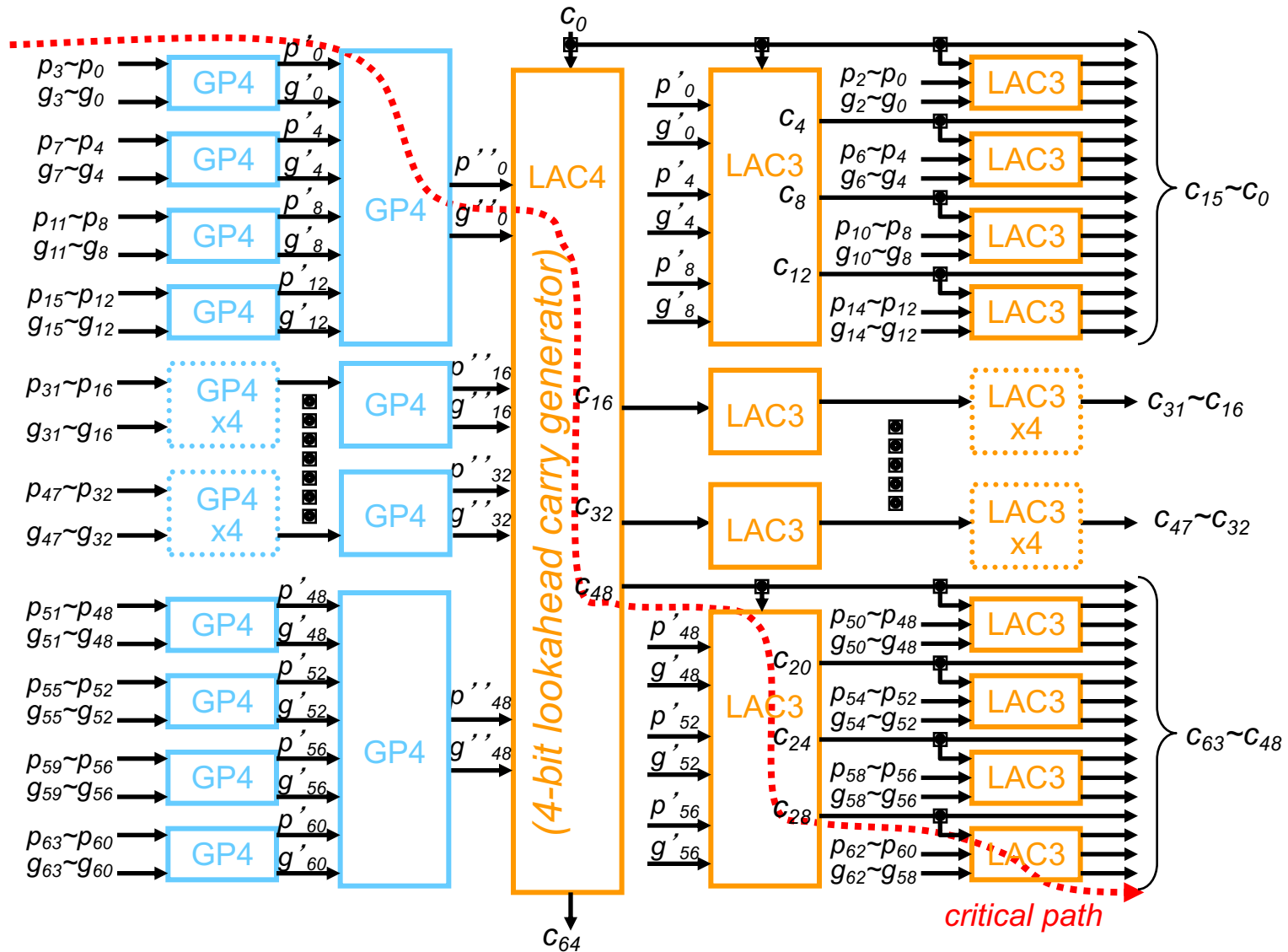
$$\checkmark \quad c_{i+12} = g'_{i+8} + p'_{i+8} c_{i+8} = g'_{i+8} + p'_{i+8} (g'_{i+4} + p'_{i+4} (g'_{i+4} + p'_{i+4} c_i))$$

$$\begin{aligned}
 \checkmark \quad c_{i+16} &= g'_{i+12} + p'_{i+12} c_{i+12} = \\
 &= g'_{i+12} + p'_{i+12} (g'_{i+8} + p'_{i+8} (g'_{i+4} + p'_{i+4} (g'_{i+4} + p'_{i+4} c_i)))
 \end{aligned}$$

- 16-bit (4²-bit) generate-propagate :

$$\begin{aligned}
 \checkmark \quad c_{i+16} &= \underline{g'_{i+12} + p'_{i+12} (g'_{i+8} + p'_{i+8} (g'_{i+4} + p'_{i+4} g'_{i+4}))} \\
 &\quad + \underline{(p'_{i+12} p'_{i+8} p'_{i+4} p'_i) c_i} \\
 &= g''_{i+16} + p''_{i+16} c_i
 \end{aligned}$$

Hierarchical Carry-Lookahead (2)



Hierarchical Carry-Lookahead (3)

- Delay estimation (assume the below normalized gate delays):

- $D_{g1} = 1.0 : g_i = a_i \cdot b_i$
- $D_{p1} = 2.0 : p_i = a_i \oplus b_i$
- $D_s = 2.0 : s_i = p_i \oplus c_i$
- $D_{c1} = 1.0 : c_{i+1} = g_i + p_i c_i$
- $D_{c2} = 1.5 : c_{i+2} = g_{i+1} + p_{i+1}(g_i + p_i c_i)$
- $D_{c3} = 2.0 : c_{i+3} = g_{i+2} + p_{i+2}(g_{i+1} + p_{i+1}(g_i + p_i c_i))$
- $D_{c4} = 2.5 : c_{i+4} = g_{i+3} + p_{i+3}(g_{i+2} + p_{i+2}(g_{i+1} + p_{i+1}(g_i + p_i c_i)))$
- $D_{g4} = 2.0 : g'_i = g_{i+3} + p_{i+3}(g_{i+2} + p_{i+2}(g_{i+1} + p_{i+1}g_i))$
- $D_{p4} = 2.0 : p'_i = p_{i+3}p_{i+2}p_{i+1}p_i$

➤ **64-bit ripple-carry adder :**

- $D(a_0 \rightarrow c_{64}) = \max\{D_{g1}, D_{p1}\} + 64 * D_{c1} = 2.0 + 64.0 = 66.0$
- $D(a_0 \rightarrow s_{63}) = \max\{D_{g1}, D_{p1}\} + 63 * D_{c1} + D_s = 2.0 + 63.0 + 2.0 = 67.0$

➤ **64-bit carry-lookahead adder using 4²-bit carry-lookahead:**

- $D(a_0 \rightarrow c_{64}) = \max\{D_{g1}, D_{p1}\} + 2 * \max\{D_{g4}, D_{p4}\} + D_{c4} = 2.0 + 4.0 + 2.5 = 8.5$
- $D(a_0 \rightarrow s_{63}) = \max\{D_{g1}, D_{p1}\} + 2 * \max\{D_{g4}, D_{p4}\} + 3 * D_{c3} + D_s$
 $= 2.0 + 4.0 + 6.0 + 2.0 = 14.0$

Hierarchical Carry-Lookahead (4)

- Area estimation (assume the following normalized areas):

- $A_{g1} = 1.0 : g_i = a_i \cdot b_i$
- $A_{p1} = 1.75 : p_i = a_i \oplus b_i$
- $A_s = 1.75 : s_i = p_i \oplus c_i$
- $A_{c1} = 1.25 : c_{i+1} = g_i + p_i c_i$
- $A_{c2} = 2.00 : c_{i+2} = g_{i+1} + p_{i+1}(g_i + p_i c_i)$
- $A_{c3} = 2.75 : c_{i+3} = g_{i+2} + p_{i+2}(g_{i+1} + p_{i+1}(g_i + p_i c_i))$
- $A_{c4} = 3.50 : c_{i+4} = g_{i+3} + p_{i+3}(g_{i+2} + p_{i+2}(g_{i+1} + p_{i+1}(g_i + p_i c_i)))$
- $A_{g4} = 2.75 : g'_i = g_{i+3} + p_{i+3}(g_{i+2} + p_{i+2}(g_{i+1} + p_{i+1}g_i))$
- $A_{p4} = 1.5 : p'_i = p_{i+3}p_{i+2}p_{i+1}p_i$

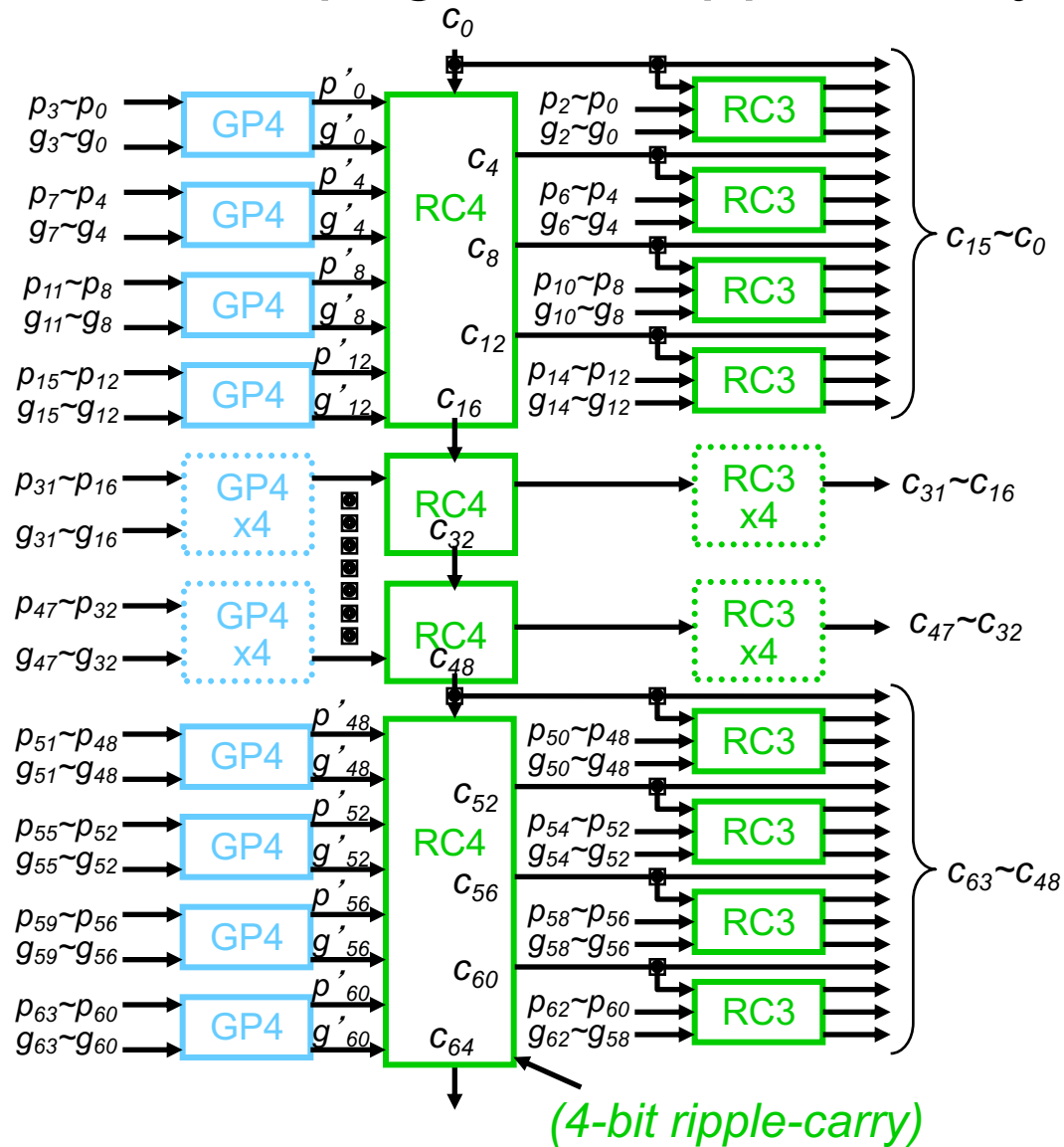
➤ *64-bit ripple-carry adder :*

- $A = 64 * (A_{PG} + A_s + A_{c1}) = 64 * (2.75 + 1.75 + 1.25) = 368$

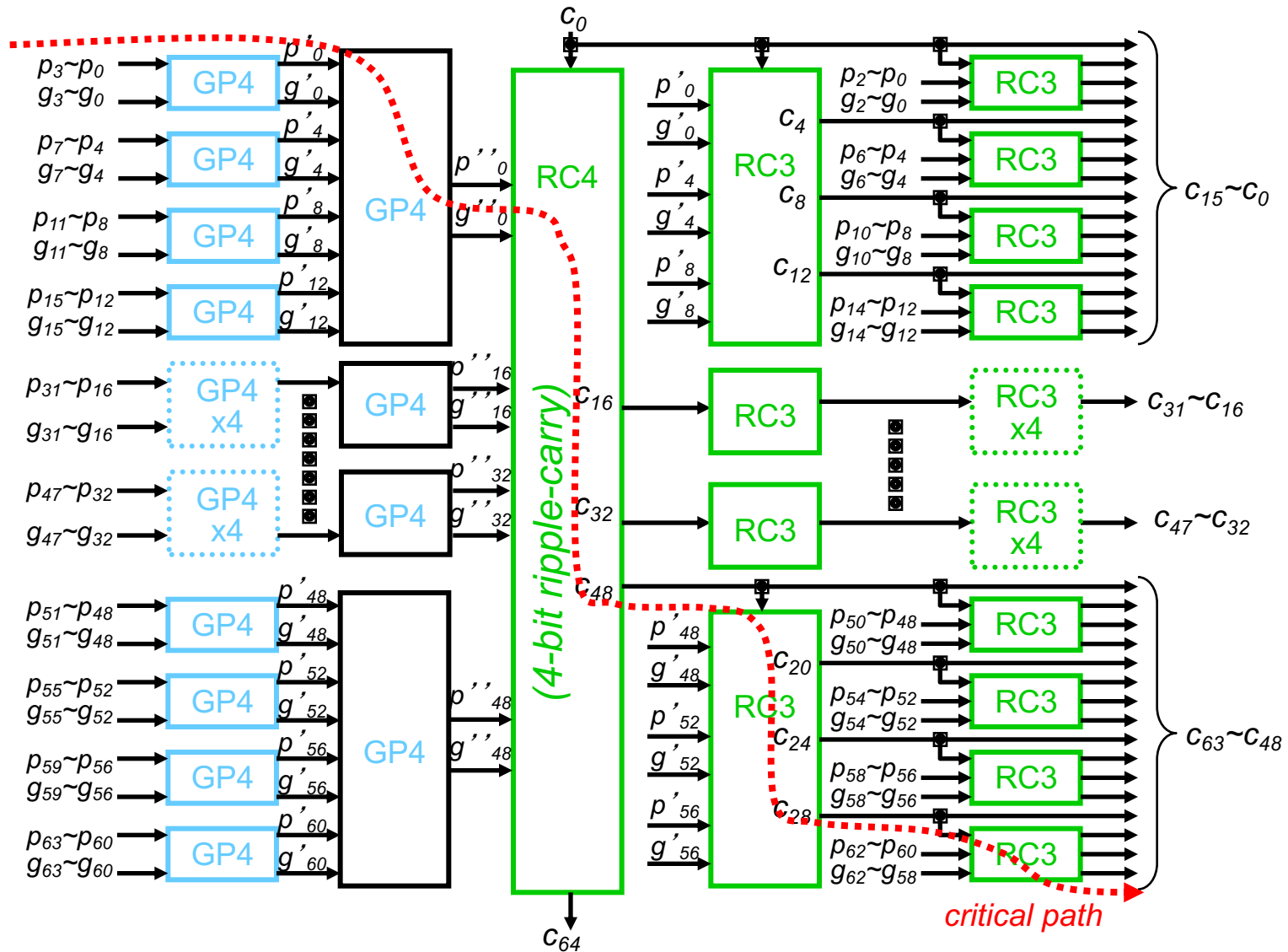
➤ *64-bit carry-lookahead adder using 4²-bit carry-lookahead:*

- $A = 64 * (A_{PG} + A_s) + (16 + 4) * A_{PG4} + A_{LCA4} + (16 + 4) * A_{LCA3}$
 $= 64 * (2.75 + 1.75) + (16 + 4) * 4.25 + 9.5 + (16 + 4) * 6.0 = 502.5$

4-bit Generate-Propagate + Ripple-Carry Adder

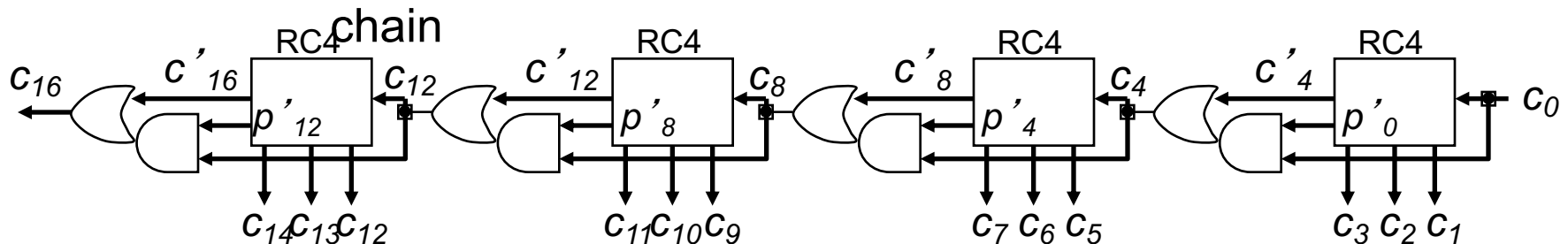


4²-bit Generate-Propagate + Ripple-Carry Adder



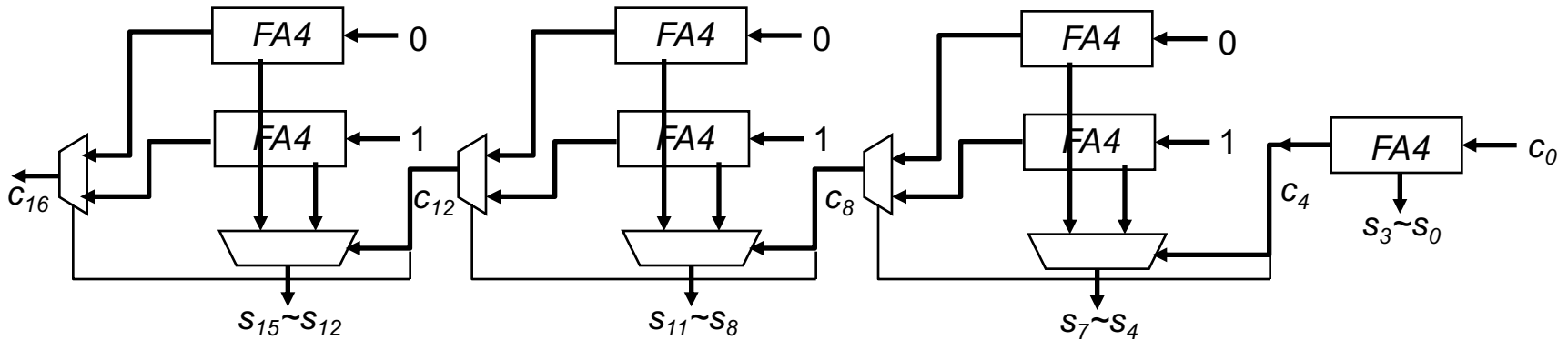
Carry-Skip (Carry-Bypass) Adder

- “propagate” $\rightarrow$ carry-skip condition
 - $c_{i+1} = g_i + p_i c_i$
 - $c_{i+2} = g_{i+1} + p_{i+1} c_{i+1}$
 - $c_{i+3} = g_{i+2} + p_{i+2} c_{i+2}$
 - $c'_{i+4} = g_{i+3} + p_{i+3} c_{i+3}$
 - $p'_i = p_{i+3} p_{i+2} p_{i+1} p_i$
 - $c_{i+4} = c'_{i+4} + p'_i c_i$
- Carry input with value ‘1’ will instantly propagate to the carry output if $p'_i = 1$, bypassing the ripple-carry chain



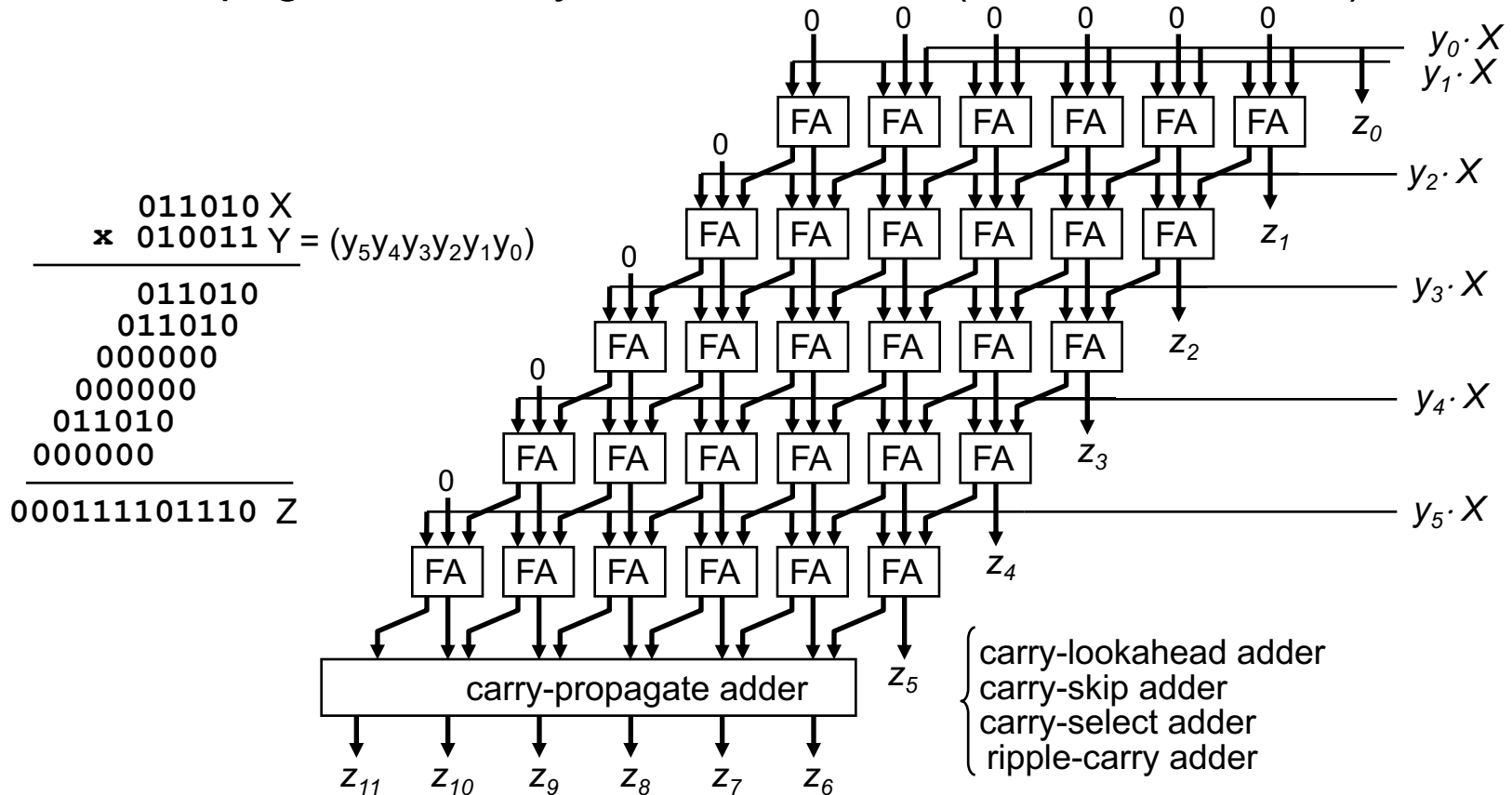
Carry-Select Adder

- Calculate the sum values and carry values in parallel on the assumptions of carry input values (0 and 1). Select the two values by the carry input value
- Some optimization on conditional adders is possible (but not much)



Carry-Save Adder

- Used in multiplier for accumulating a large number of partial products.
- Propagate the carry to the next level (not to the next bit)



High-Level Synthesis and Its Design Environment

- High-level synthesis converts the design described at algorithm-level to RTL
 - Needs a rich module library consisting of high-level circuit blocks (adders, multipliers, ALUs, decoders, RAM, ROM, etc.) as well as high quality standard cell library
 - Numerous circuit implementation techniques for arithmetic logic are captured into these libraries, and high-level synthesis provides a path for utilizing these design resource efficiently and intelligently.