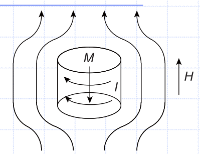
Superconductivity

Zero resistance

Meissner effect (Perfect diamagnetism)



$B = H + 4\pi M$ (cgs) $\qquad B = \mu_0(H + M)$ (MKS)

→ $B = 0$ Complete screening of the magnetic field.

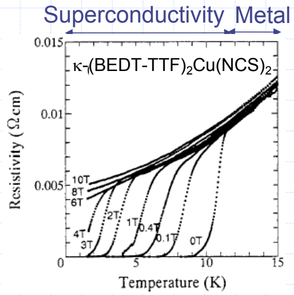
$M = -1/4\pi H \qquad M = -H$

or $\chi = -1/4\pi \qquad \chi = -1$

Zero resistance is a requisite of the perfect diamagnetism.

Only zero resistance → Cooled from $T > T_c$ to $T < T_c$ → No diamagnetism

Perfect diamagnetism is a stronger condition than perfect conductance.



London equation

Equation of motion

$$m^*\frac{\partial v}{\partial t} = e^*\varepsilon \xrightarrow{J=n_s e^* v} \frac{\partial J}{\partial t} = \frac{n_s e^{*2}}{m^*}\varepsilon \xrightarrow{\nabla\times} \nabla\times\varepsilon = -\frac{1}{c}\frac{\partial B}{\partial t}$$

[MKS: *c* → 1]

$$\nabla\times\frac{\partial J}{\partial t} = \frac{n_s e^{*2}}{m^*}\nabla\times\varepsilon = \frac{n_s e^{*2}}{cm^*}\left(-\frac{\partial B}{\partial t}\right) \longrightarrow \frac{\partial}{\partial t}\left(\nabla\times J + \frac{n_s e^{*2}}{cm^*}B\right) = 0$$

We shall assume ( ) is always zero in a superconductor.

$$\nabla\times J = -\frac{1}{c\Lambda}B$$ **where** $\Lambda = m^*/n_s e^{*2}$ **(London equation)**

$$\nabla\times B = \frac{4\pi}{c}J \xrightarrow[\nabla\times\nabla\times B \to -\nabla^2 B]{\nabla\times} \lambda^2\nabla^2 B = B$$ **where** $\lambda^2 = \dfrac{c^2 m^*}{4\pi n_s e^{*2}}$

$\nabla\times$

In a superconductor, *B* decays like $B = H\exp(-x/\lambda)$

$\lambda^2\nabla^2 J = J$ **Similarly, *J* decays like** $J = J_0\exp(-x/\lambda)$

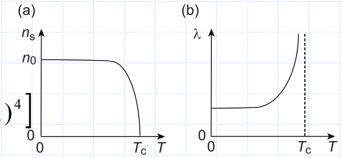
λ: Penetration depth 40 ~ 200 nm

Perfect diamagnetism

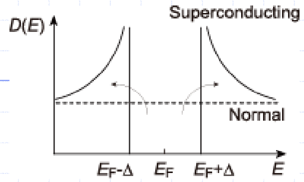
⇕

London equation

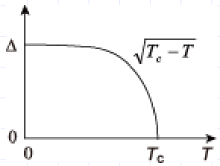
$$n_s = n_0\left[1-(T/T_c)^4\right]$$

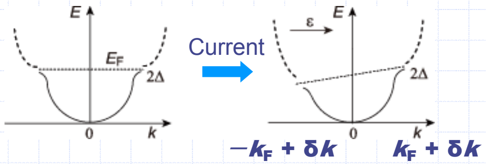


Superconductor has an energy gap.



$$\Delta \propto \sqrt{T_c - T}$$





Standing wave

↓

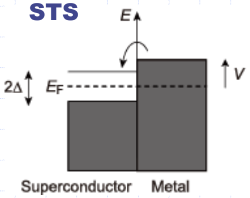
Gap moves

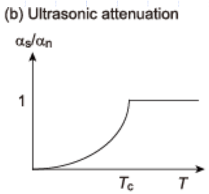
↓

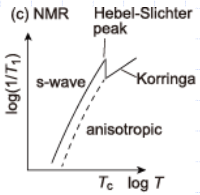
Zero resistance

Gap appears in

STS

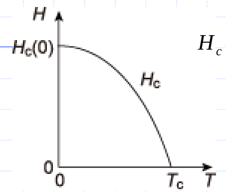






Critical field

Magnetic filed destroys superconductivity.

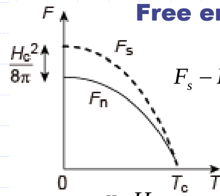


$$H_c(T) = H_c(0)[1-(\frac{T}{T_c})^2]$$

⬇

$$H_{c2}(0) = -0.5T_c\left(\frac{dH_{c2}}{dT}\right)_{T_c}$$

Free energy



$$F_s - F_n = -\frac{H_c^2}{8\pi}$$

Magnetization energy $dE = MdH$

Perfect diamagnetism $M = -1/4\pi H$ → $F_s - F_n = -\int_0^{H_c}\frac{H}{4\pi}dH$

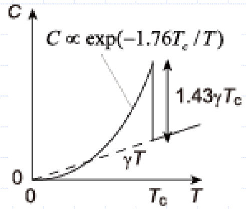
Superconducting transition: 2nd order @ *H*=0

1st order @ *H*≠0

Superconducting gap

⬇

Heat capacity decays



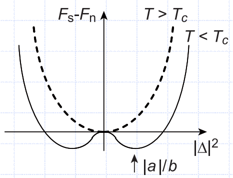
Ginzburg-Landau Expansion

Near T_c, Free energy is expanded by Δ

$$F_s - F_n = a|\Delta|^2 + \frac{b}{2}|\Delta|^4 + \cdots$$ **Ginzburg-Landau expansion**

Δ that minimizes F:

$$\frac{\partial(F_s - F_n)}{\partial|\Delta|^2} = a + b|\Delta|^2 = 0$$ **and assume $a = a'(T - T_c)$**



$T > T_c \rightarrow a > 0 \rightarrow |\Delta|^2 = 0$
$T < T_c \rightarrow a < 0 \rightarrow |\Delta|^2 = |a|/b$

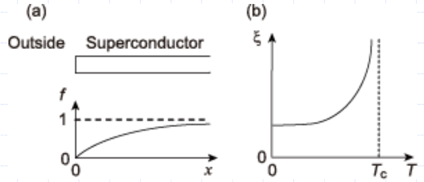
$$\Rightarrow |\Delta| = \sqrt{\frac{a'(T_c - T)}{b}}$$

Rewrite GL expansion using $\psi = (\sqrt{2m}/\hbar^2)\Delta$
and adding kinetic energy and magnetization energy

$$F_s - F_n = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m^*}\left|\left(\frac{\hbar}{i}\nabla - \frac{e^*}{c}\mathbf{A}\right)\psi\right|^2 + \frac{H^2}{8\pi}$$

$$\delta\psi^* \rightarrow \alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m^*}\left(\frac{\hbar}{i}\nabla - \frac{e^*}{c}\mathbf{A}\right)^2\psi = 0$$

$$\psi = (|\alpha|/\beta)^{1/2} f,\quad \xi^2 = \hbar^2/2m^*|\alpha| \rightarrow \xi^2\frac{d^2 f}{dx^2} + f - f^3 = 0 \longrightarrow f = \tanh\left(\frac{x}{\sqrt{2}\xi}\right)$$



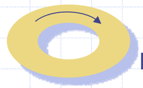
ξ: Coherence length 2~200 nm
How quick SC is restored.

$$\xi \propto \left(1 - \frac{T}{T_c}\right)^{-1/2} \leftarrow \alpha \propto (T - T_c)$$

SC is not restored at T_c.

$$\delta\mathbf{A} \rightarrow \mathbf{J} = \frac{\hbar e^*}{2m^* i}(\psi^*\nabla\psi - \psi\nabla\psi^*) - \frac{e^{*2}}{cm^*}\psi^*\psi\mathbf{A}$$

$$\psi = \psi_0 e^{i\theta} \rightarrow \mathbf{J} = \frac{\hbar e^*}{m^*}|\psi|^2\nabla\theta - \frac{e^{*2}}{cm^*}|\psi|^2\mathbf{A}$$ **X 1st term → London equation**



Ring current $\oint\nabla\theta dl = 2\pi n$

Stokes theorem $\oint\mathbf{A}dl = \int\nabla\times\mathbf{A}dl = \int HdS = \Phi$

$$\rightarrow \frac{m^* c}{e^{*2}}\oint Jdl + \Phi = n\Phi_0 \qquad \Phi_0 = \frac{2\pi\hbar c}{e^*} = \frac{hc}{e^*} = \frac{hc}{2e} = 2\times10^{-7}\ \text{Gauss/cm}^2$$

Magnetic flux: H created by a Cooper pair

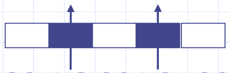
Type II Superconductor

λ: magnetic field is screened
ξ: SC gap is restored

SC has $H_c^2/8\pi$ lower energy

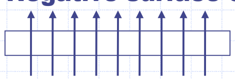
$\longrightarrow H_c^2(\xi - \lambda)/8\pi$ **Sueface energy**

$\xi > \lambda$: Positive surface energy → No Φ → Type I superconductor

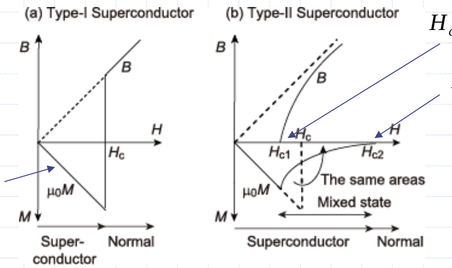


Infinite SC Plate → H has to pass
→ Intermidiate state
(Domains of SC and Non SC regions)

$\xi < \lambda$: Negative surface energy → Φ → Type II superconductor



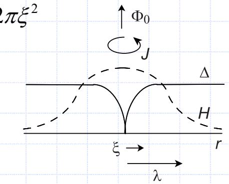
Many fluxes → Mixed state



$M = -1/4\pi$

$$H_{c1} = \frac{\Phi_0}{4\pi\lambda^2}\ln\frac{\lambda}{\xi} \qquad H_{c2} = \frac{\Phi_0}{2\pi\xi^2}$$

$$H_{c1} = H_c/\sqrt{2}\kappa \qquad H_{c2} = \sqrt{2}\kappa H_c \qquad \kappa = \lambda/\xi$$



Material	T_c /K	ξ /nm	λ /nm	$\kappa = \xi/\lambda$	2Δ/K	$2\Delta/k_BT_c$
Al	1.18	1550	45	0.03	4.1	3.5
Pb	7.2	87	39	0.48	31	4.3
Nb	9.25	39	52	1.3	35	3.8
NbN	16	5	200	40	59	3.7
Nb_3Sn	18	3	65	22	77	4.3
Nb_3Ge	23.2	3	90	30	77	4.3
$PbMo_6S_8$	14	2.2	215	98	97	4.2
$LiTi_2O_4$	11				42	3.8
$La_{2-x}Sr_xCuO_4$	37	2	200	100	151	4.1
$YBa_2Cu_3O_7$	89	// 3.4	36	11	348	4.0
		⊥0.7	125	179		
$Tl_2Ba_2Ca_2Cu_3O_{10}$	123	//	173		510	4.1
		⊥	480			
$HgBa_2Ca_2Cu_3O_8$	133	// 1.5	130	87	557	4.2
		⊥0.19	3500	1.8×10^4		
MgB_2	39	// 6.5	100	15	140	3. 6
		⊥2. 5				
$SmFeAsO_{1-x}F_x$	55		200		100	

Type I (Al, Pb, Nb)

Type II (NbN – $SmFeAsO_{1-x}F_x$)

Material	T_c /K	ξ /nm	λ /nm	$\kappa = \xi/\lambda$	2Δ/K	$2\Delta/k_BT_c$
K_3C_{60}	19	2.6	240	92	57	3.6
Rb_3C_{60}	29.6	2	247	124	87	3.0
$(TMTSF)_2ClO_4$	1. 2	*a* 77			3.8	3.2
		b 36				
		c 2				
κ-$(ET)_2Cu(NCS)_2$	10.4	// 7	1400	200	30	2.9
		⊥ 0.5	40000	8×10^4		
κ-$(ET)_2Cu[N(NC)_2]Br$	11.3	// 3.7	1500	400	57	5.0
		⊥ 0.6	38000	6×10^4		

Type II

Pauli Limit

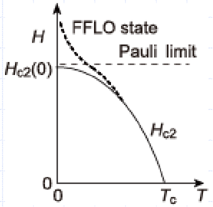
Δ < Zeemann energy → $\Delta = \sqrt{2}\mu_B H_{c2}$ → $2\Delta = 3.53 k_B T_c$

H_{c2} < (1.86 T/K) T_c

$$H_{c2}(0) = -0.5 T_c \left(\frac{dH_{c2}}{dT}\right)_{T_c} \longrightarrow H_{c2}(0) = -0.69 T_c \left(\frac{dH_{c2}}{dT}\right)_{T_c}$$

+ Spatially inhomogeneous superconductivity

FFLO (Fulde-Ferrell- Larkin-Ovchinnikov) state



Fourier transform of the Coulomb repulsion

$$V(r) = \frac{e^2}{4\pi\varepsilon_0 r} \longrightarrow V(k) = \int V(r) e^{ikr} \frac{dk}{(2\pi)^3} = \frac{e^2}{\varepsilon_o k^2}$$

→Yukawa potential

$$V(r) = \frac{e^2}{4\pi\varepsilon_0 r} e^{-k_0 r} \longrightarrow V(k) = \frac{e^2}{\varepsilon_o (k^2 + k_0^2)}$$

Plasma oscillation → Exchanging electrons and ions

$$\varepsilon(\omega) = 1 - \frac{\Omega_p^2}{\omega^2} \quad \text{where} \quad \Omega_p = \frac{nZ^2 e^2}{\varepsilon M}$$

$$\longrightarrow V(k) = \frac{e^2}{\varepsilon_o (k^2 + k_0^2)} \frac{\omega^2}{\omega^2 - \Omega_p^2}$$

→ Attraction for $\omega < \Omega_p$ ~ Debye temperature

Electrons in ion⁺ feel attraction @ low energy due to retarded screening of the ions.

→ Cooper pairs → Superconductivity

Bardeen-Cooper-Schrieffer Theory

Interacting electrons

$$H = \sum_{i \neq j} t_{ij} c_{i\sigma}^+ c_{j\sigma} + \sum_{k,k'} V_{k,k'} c_{k'\uparrow}^+ c_{-k'\downarrow}^+ c_{-k\downarrow} c_{k\uparrow}$$

Mean field for $< c_{k\uparrow}^+ c_{-k\downarrow}^+ >$ → Cooper pairs [not for $<n_{i\sigma}> = <c_{i\sigma}^+ c_{i\sigma}>$]

$$V c_{k'\uparrow}^+ c_{-k'\downarrow}^+ c_{-k\downarrow} c_{k\uparrow} = V < c_{k'\uparrow}^+ c_{-k'\downarrow}^+ > c_{-k\downarrow} c_{k\uparrow} + V c_{k'\uparrow}^+ c_{-k'\downarrow}^+ < c_{-k\downarrow} c_{k\uparrow} > - V < c_{k'\uparrow}^+ c_{-k'\downarrow}^+ >< c_{-k\downarrow} c_{k\uparrow} >$$

Ground state

$$|\,\mathrm{BCS}> = \prod_k \left(\cos\theta_k + \sin\theta_k c_{k\uparrow}^+ c_{-k\downarrow}^+\right)|0> \qquad \text{cf.} \quad |\,\mathrm{Normal}> = \prod_k c_{k\uparrow}^+ c_{-k\downarrow}^+ |0>$$

Using

$$\Delta_k = -\sum_{k'} V_{k,k'} < c_{-k'\downarrow} c_{k'\uparrow} >$$

$$\Delta_k^* = -\sum_{k'} V_{k,k'} < c_{k'\uparrow}^+ c_{-k'\downarrow}^+ >$$

$\sin\theta_k = 1$ for $k < k_F$

$\sin\theta_k = 0$ for $k > k_F$

$$\longrightarrow H = 2\sum_k \xi_k c_{k\sigma}^+ c_{k\sigma} - \sum_k (\Delta_k^* c_{-k\downarrow} c_{k\uparrow} + \Delta_k c_{k\uparrow}^+ c_{-k\downarrow}^+ - \Delta_k < c_{k\uparrow}^+ c_{-k\downarrow}^+ >)$$

$\xi_k = E(k) - \mu$

Diagonalization (Bogoliubov transformation)

$$c_{k\uparrow}^+ = \cos\theta_k \gamma_{k\uparrow}^+ + \sin\theta_k \gamma_{-k\downarrow} \qquad c_{k\uparrow} = \cos\theta_k \gamma_{k\uparrow} + \sin\theta_k \gamma_{-k\downarrow}^+$$

$$c_{-k\downarrow} = \cos\theta_k \gamma_{-k\downarrow} - \sin\theta_k \gamma_{k\uparrow}^+ \qquad c_{-k\downarrow}^+ = \cos\theta_k \gamma_{-k\downarrow}^+ - \sin\theta_k \gamma_{k\uparrow}$$

Creation and annihilation operators are "hybridized".

Nondiagonal term:

$$(2\xi_k \sin\theta_k \cos\theta_k + \Delta_k^* \sin^2\theta_k - \Delta_k \cos^2\theta_k)(\gamma_{k\uparrow}^+ \gamma_{-k\downarrow}^+ + \gamma_{-k\downarrow}\gamma_{k\uparrow})$$

is zero for

$$\tan 2\theta_k = \frac{\Delta_k}{\xi(k)} \longrightarrow \cos^2 2\theta_k = \frac{1}{1 + \tan^2 2\theta_k} = \frac{\xi_k^2}{\xi_k^2 + \Delta_k^2}$$

$$\longrightarrow \cos^2\theta_k = \frac{1 + \cos 2\theta_k}{2} = \frac{1}{2}\left(1 + \frac{\xi_k}{\sqrt{\xi_k^2 + \Delta_k^2}}\right) \qquad \sin^2\theta_k = \frac{1}{2}\left(1 - \frac{\xi_k}{\sqrt{\xi_k^2 + \Delta_k^2}}\right)$$

Diagonal term:

$$\xi_k(\cos^2\theta_k - \sin^2\theta_k) + 2\Delta_k \cos\theta_k \sin\theta_k \longrightarrow E = \sum_k \sqrt{\xi_k^2 + \Delta_k^2}$$

Bogoliubov trans. to Δ_k

$$\Delta_k = -\sum_{k'} V_{k,k'} \sin\theta_{k'} \cos\theta_{k'} (1 - < \gamma_{-k'\downarrow}^+ \gamma_{-k'\downarrow} > - < \gamma_{k'\uparrow}^+ \gamma_{k'\uparrow} >)$$

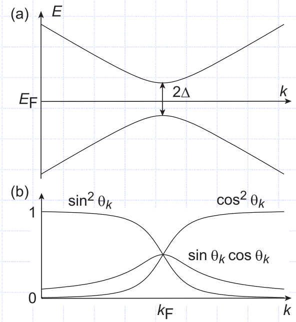
$$\longleftarrow \gamma_{-k'\downarrow} \gamma_{-k'\downarrow}^+ = 1 - \gamma_{-k'\downarrow}^+ \gamma_{-k'\downarrow}$$

$< \gamma_{-k'\downarrow} \gamma_{k'\uparrow} >$ $< \gamma_{k'\uparrow}^+ \gamma_{-k'\downarrow}^+ >$ → 0

Here $\sin\theta_{k'} \cos\theta_{k'} = (1/2)\sin 2\theta_{k'} = \Delta_{k'} / 2\sqrt{\xi_{k'}^2 + \Delta_{k'}^2}$

$< \gamma_{-k'\downarrow}^+ \gamma_{-k'\downarrow} >$ $< \gamma_{k'\uparrow}^+ \gamma_{k'\uparrow} >$ →

$$f(E) = 1/(\exp(E/k_B T) + 1)$$



Gap equation (determines the gap.)

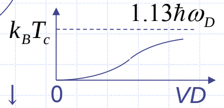
$$\Delta_k = -\sum_{k'} V_{k,k'} \frac{\Delta_{k'}}{2\sqrt{\xi_{k'}^2+\Delta_{k'}^2}}(1-2f(E))$$

$V_{k,k'} \to -V \quad \xi < \omega_D$; $\;0 \quad \xi > \omega_D$ $\longrightarrow$ $\Delta_k \to \Delta \quad \xi < \omega_D$; $\;0 \quad \xi > \omega_D$ $\longrightarrow$ $\Delta = V\sum_{k'} \frac{\Delta}{2\sqrt{\xi_{k'}^2+\Delta^2}}$

$$\longrightarrow 1 = V\int_0^{\hbar\omega_D} \frac{D d\xi}{\sqrt{\xi^2+\Delta^2}} = VD\sinh^{-1}\frac{\hbar\omega_D}{\Delta} \longrightarrow \Delta = 2\hbar\omega_D \exp(-1/VD)$$

$1-2f(E)$ =tanh(E/2) $\longrightarrow$ $\frac{1}{DV} = \int_0^{\hbar\omega_D} \frac{\tanh(\sqrt{\xi^2+\Delta^2}/2k_BT)}{\sqrt{\xi^2+\Delta^2}} d\xi$ $\longrightarrow$ $\frac{2\Delta}{k_BT_c} = \frac{2\pi}{e^\gamma} = 3.53$

Δ→0 @ T_c $\quad k_BT_c = 1.13\hbar\omega_D \exp(-1/VD)$



Δ_k	**Isotropic**	**s-wave Singlet**	↑↓
	Anisotripic	**Possible for V > 0 (Repulsion)**	
$\Delta_p = \Delta_0 \sin ka$		**p-wave Triplet**	↑↑
$\Delta_d = \Delta_0(\cos ka - \cos kb)$		**d_{x2-y2}-wave Singlet**	↑↓

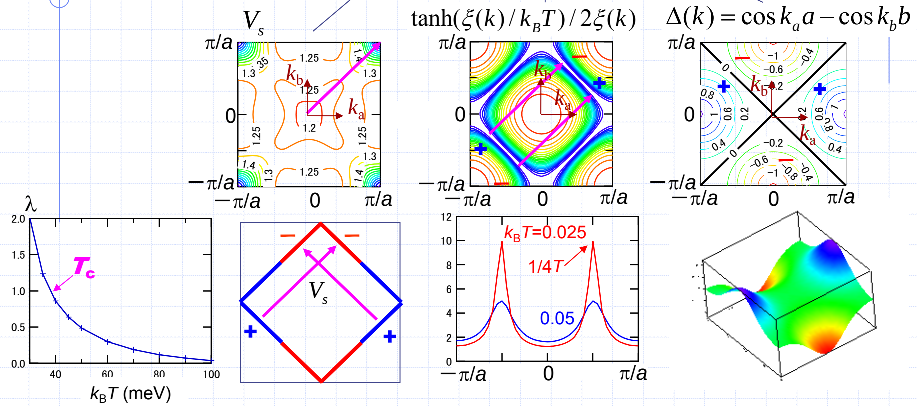
To investigate anisotropic SC, Δ→0 in √

$$\lambda\Delta(k) = -\sum_{k'} V_s(k-k') \frac{\tanh(\xi(k')/2k_BT)}{2\xi(k')}\Delta(k') \qquad \lambda \to 1 \text{ @ } T_c$$

Linearized gap equation of Eliashberg equation

Half-filled square lattice — Eliashberg equation

$$\lambda\Delta(k) = -\sum_{k'} V_s(k-k') \frac{\tanh(\xi(k')/2k_BT)}{2\xi(k')}\Delta(k')$$



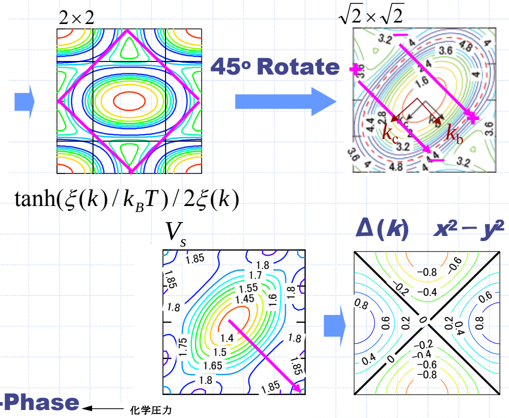
V_s connects +Δ(k') and −Δ(k)

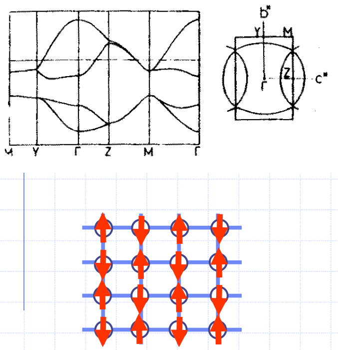
→ Attracting SC (λ >0) from Repulsive V_s > 0

SDW happens prior to SC in the ordinary square lattice.

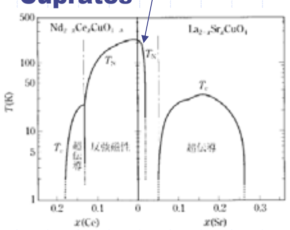
In cuprates, doping → SC instead of AF.

κ-(BEDT-TTF)$_2$Cu(NCS)$_2$

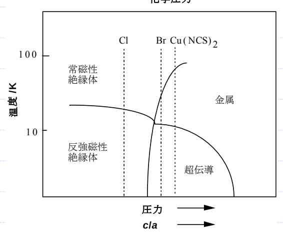




Cuprates



κ-Phase ← 化学圧力

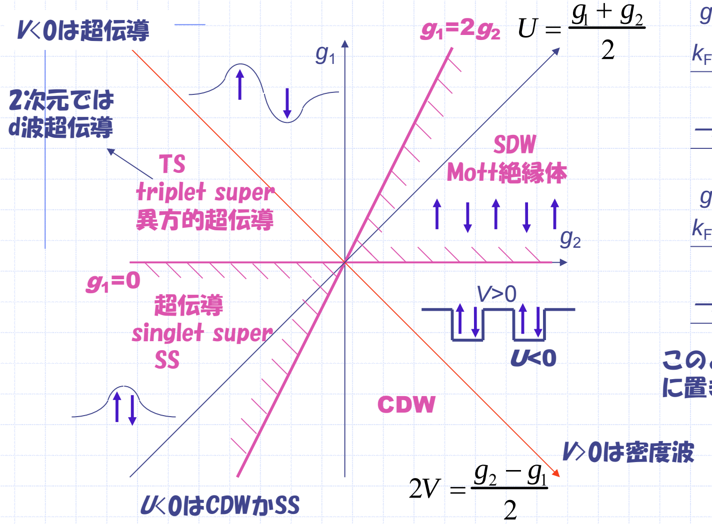


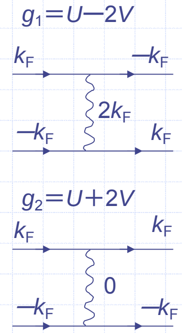
SC boadering on AF

d_{x2-y2}-wave SC due to spin fructuation

拡張Hubbardモデルの1次元で基底状態（g-ology）

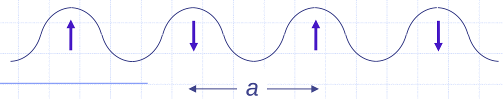
$$H = \sum_{i\neq j} t_{ij} a_i^\dagger a_j + U\sum_i n_{i\uparrow}n_{i\downarrow} + V\sum_{i\neq j} n_i n_j$$

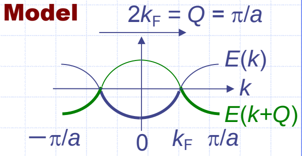




このような散乱確率に置き換えて計算

Hartree-Fock Approximation of the Hubbard Model





1D Half-filled metal → Antiferromagnetic Insulator (Spin Density Wave)

$E(k)$ and $E(k+Q)$ crosses at k_F (perfect nesting) $E(k) = -E(k+Q)$

Hubbard model

$$H = \sum_i t c_{i\sigma}^+ c_{i+1\sigma} + \sum_i U n_{i\uparrow} n_{i\downarrow}$$

Insert $c_{i\sigma} = \sum_i e^{ikx} c_{k\sigma}$ **in** $\sum_i t c_{i\sigma}^+ c_{i+1\sigma}$

$$\sum_i t \left(\sum_k e^{-ikx} c_{k\sigma}^+\right)\left(\sum_{k'} e^{ik'(x+a)} c_{k'\sigma}\right) = t\left(e^{-ika} + e^{ika}\right) c_{k\sigma}^+ c_{k\sigma} = \underbrace{(2t\cos ka)}_{E(k)} n_{k\sigma}$$

Mean-field approximation to the 2nd term

$$U n_\uparrow n_\downarrow = U < n_\uparrow > n_\downarrow + U n_\uparrow < n_\downarrow > - U < n_\uparrow >< n_\downarrow >$$

Using $< n_\uparrow > = \frac{n+m}{2}$ $\quad < n_\downarrow > = \frac{n-m}{2}$ **(←Stoner model)**

$$U n_\uparrow n_\downarrow = U\frac{n+m}{2} n_\downarrow + U\frac{n-m}{2} n_\uparrow - \underline{U\frac{n^2-m^2}{4}}$$

constant

$i \to k$ $\quad n_{i\sigma} = \left(\sum_k e^{-ikx} c_{k\sigma}^+\right)\left(\sum_{k'} e^{ik'x} c_{k'\sigma}\right) = c_{k\sigma}^+ c_{k+Q\sigma}$

$\lambda_\sigma = \pm 1$ for ↑ and ↓

In summary

$$H = \sum_k \left[E(k) c_{k\sigma}^+ c_{k\sigma} + E(k+Q) c_{k+Q\sigma}^+ c_{k+Q\sigma} - \frac{Um\lambda_\sigma}{2}\left(c_{k+Q\sigma}^+ c_{k\sigma} + c_{k\sigma}^+ c_{k+Q\sigma}\right)\right] + \frac{U(n^2+m^2)}{4}$$

In order to diagonalize the nondiagonal term containing *U*, we put

$$c_{k\sigma} = \cos\phi_k a_{k\sigma} - \sin\phi_k b_{k\sigma}$$
$$c_{k+Q\sigma} = \sin\phi_k a_{k\sigma} + \cos\phi_k b_{k\sigma}$$

[a, b : anihilation operators of A, B sublattices]

The nondiagonal term becomes

$$\left[-(E(k) - E(k+Q))\cos\phi_k \sin\phi_k + \frac{Um}{2}(\cos^2\phi_k - \sin^2\phi_k)\right](a_{k\sigma}^+ b_{k\sigma} + b_{k\sigma}^+ a_{k\sigma})$$

ϕ_k that makes this zero is $\tan 2\phi_k = \frac{Um}{E(k) - E(k+Q)}$

$$\to \cos^2 2\phi_k = \frac{1}{1+\tan^2 2\phi_k} = \frac{(E(k)-E(k+Q))^2}{(E(k)-E(k+Q))^2 + (Um)^2}$$

$$\to \cos^2\phi_k = \frac{1+\cos 2\phi_k}{2} = \frac{1}{2}\left(1 + \frac{E(k)-E(k+Q)}{\sqrt{(E(k)-E(k+Q))^2+(Um)^2}}\right)$$

$$\sin^2\phi_k = 1 - \cos^2\phi_k = \frac{1}{2}\left(1 - \frac{E(k)-E(k+Q)}{\sqrt{(E(k)-E(k+Q))^2+(Um)^2}}\right)$$

The diagonal term becomes

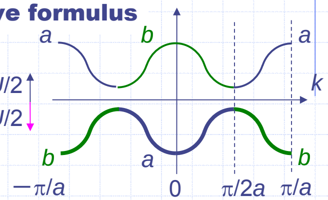
$$(E(k)\cos^2\phi_k + E(k+Q)\sin^2\phi_k + 2Um\cos\phi_k\sin\phi_k) a_{k\sigma}^+ a_{k\sigma}$$
$$+ (E(k)\cos^2\phi_k + E(k+Q)\sin^2\phi_k - 2Um\cos\phi_k\sin\phi_k) b_{k\sigma}^+ b_{k\sigma}$$

$\cos^2\theta_k$, $\sin^2\phi_k$ are inserted in the above formulus

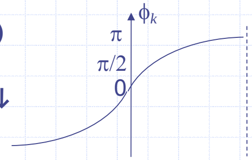
$$W(k) = \frac{E(k)+E(k+Q) \pm \sqrt{(E(k)-E(k+Q))^2 + (mU)^2}}{2}$$

$E(k) = -E(k+Q) \to$

$$W(k) = \frac{1}{2}\sqrt{(E(k)-E(k+Q))^2 + (mU)^2}$$



Gap opens for small *U*
→ Antiferromagnetic insulator (SDW)



BZ half → 2x lattice → Periodicity of ↑ and ↓

F. Gebhard *The Mott Metal-Insulator Transition*, Springer (1997)