2018 2Q
Wireless Communication Engineering

#13 Array Signal Processing and MIMO Communications

Kei Sakaguchi
sakaguchi@mobile.ee.

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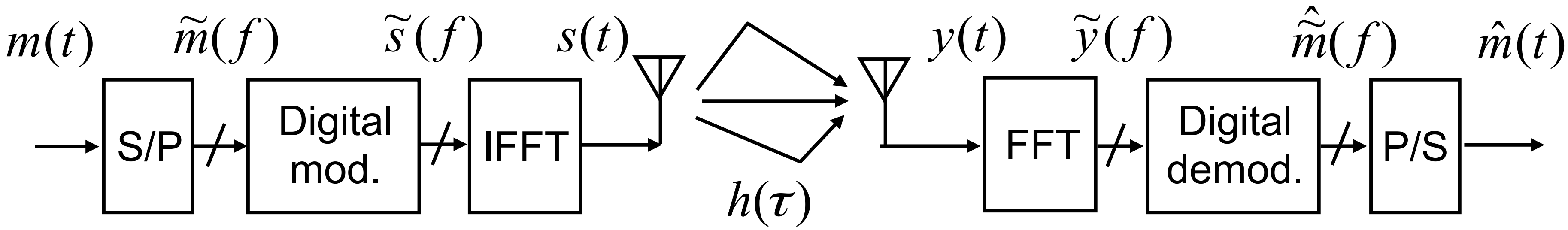
Course Schedule (2)

	Date	Text	Contents
#9	July 9	4.6	Error correction coding
	June 12		No class
#10	July 19		Adaptive modulation coding
#11	July 23	4.3	Inter symbol interference and adaptive equalizer
#12	July 26	3.5	Orthogonal frequency division multiplexing (OFDM)
#13	July 30		Array signal processing and MIMO communications
#14	Aug 2		Collaborative exercise for better understanding 2
#15	Aug 6	All	Final examination

From Previous Lecture

- Orthogonal Frequency Division Multiplexing (OFDM)



OFDM mod.

$$s(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \tilde{s}(n) \exp\left(j2\pi k \frac{n}{N} \right)$$

OFDM demod. (FDE)

$$\tilde{y}(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} y(k) \exp\left(-j2\pi n \frac{k}{N} \right)$$

$$= \tilde{h}(n)\tilde{s}(n) + \tilde{n}(n)$$

$$\hat{\tilde{s}}(n) = \tilde{y}(n) \big/ \tilde{h}(n)$$

- AMC over OFDM

Coding: Error correction of subcarriers at fading dip
Adaptive mod.: Adaptive data rate control per subcarrier

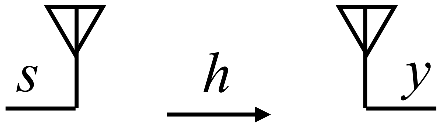
Contents

- Channel capacity of SISO
- Channel capacity of SIMO/MISO
- Channel capacity of MIMO

SISO System

Received signal

$$y(t) = hs(t) + n(t)$$



Output SNR

$$\gamma = \frac{\mathrm{E}[|hs(t)|^2]}{\mathrm{E}[|n(t)|^2]} = \frac{|h|^2 P}{\sigma^2}$$

PDF of output SNR in Rayleigh fading

$$f(\gamma) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right)$$

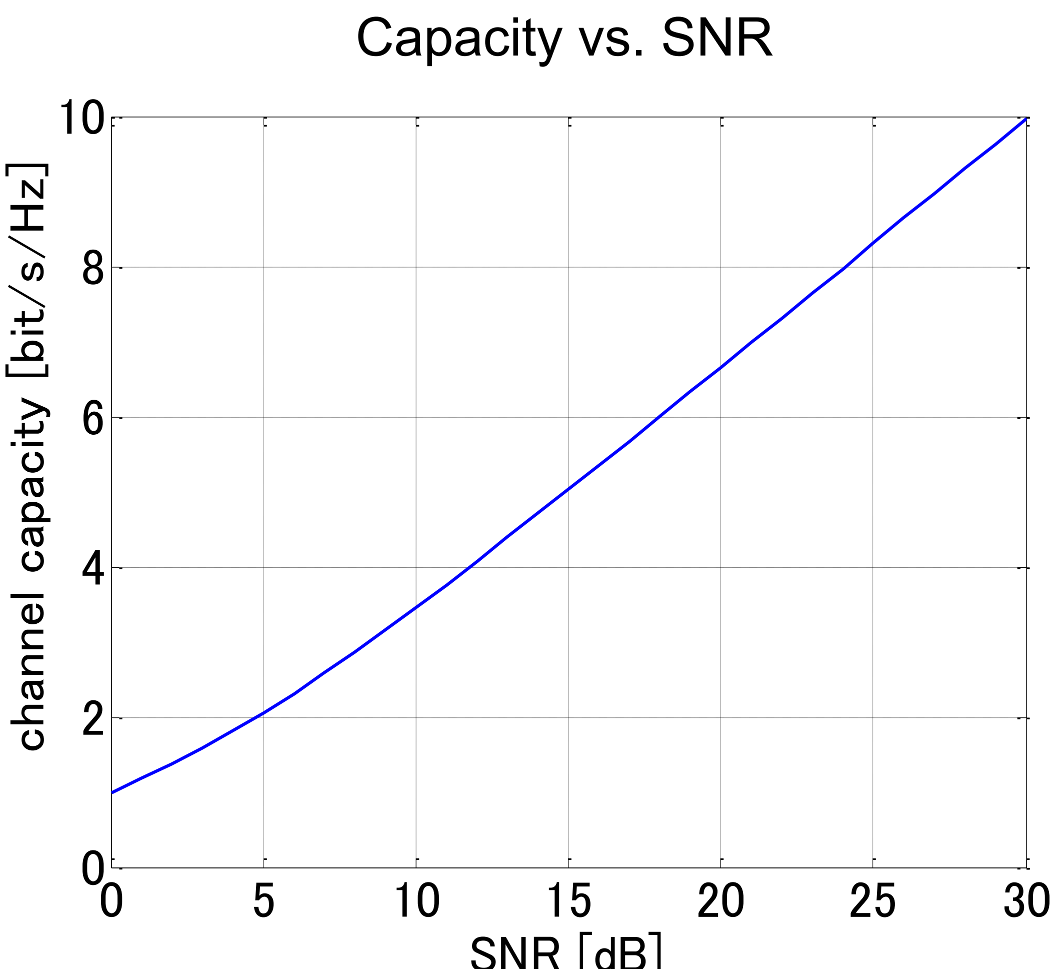
Channel capacity

$$C_{\mathrm{SISO}}(\gamma) = \log_2\left(1 + \frac{|h|^2 P}{\sigma^2}\right)$$

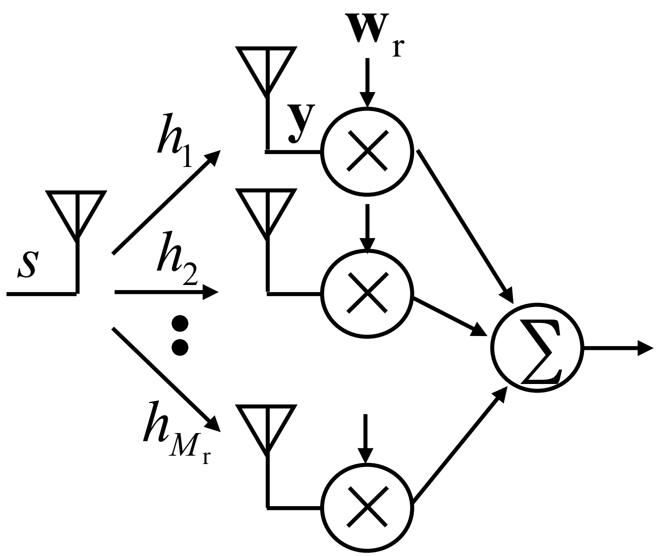
Average channel capacity

$$\overline{C}_{\mathrm{SISO}}(\gamma) = \int f(\gamma) C_{\mathrm{SISO}}(\gamma) \mathrm{d}\gamma$$

SISO Channel Capacity



SIMO System



Received signal vector

$$\mathbf{y}(t) = \mathbf{h}s(t) + \mathbf{n}(t)$$

Output SNR

$$\gamma = \max_{\mathbf{w}_\mathrm{r}} \frac{\mathrm{E}[|\mathbf{w}_\mathrm{r}^H \mathbf{h}s|^2]}{\mathrm{E}[|\mathbf{w}_\mathrm{r}^H \mathbf{n}|^2]} = \sum_{i=1}^{M_r} |h_i|^2 \frac{P}{\sigma^2}$$

PDF of OSNR in independent Rayleigh fading

$$f(\gamma) = \frac{1}{(M-1)!\bar{\gamma}^{M_\mathrm{r}}} \gamma^{M_\mathrm{r}-1} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right)$$

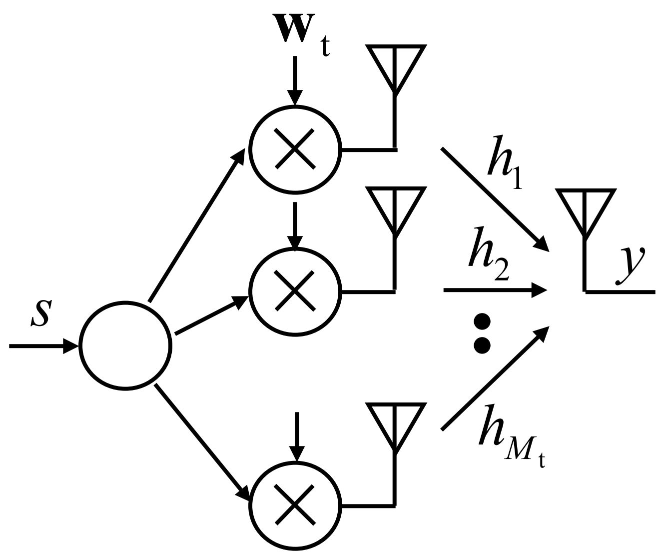
Channel capacity

$$C_\mathrm{SIMO}(\gamma) = \log_2\left(1 + \sum_{i=1}^{M_\mathrm{r}} |h_i|^2 \frac{P}{\sigma^2}\right)$$

Average channel capacity

$$\overline{C}_\mathrm{SIMO}(\gamma) = \int f(\gamma) C_\mathrm{SIMO}(\gamma) \mathrm{d}\gamma$$

MISO System



Received signal

$$y(t) = \mathbf{h}^T \mathbf{w}_\mathrm{t} s(t) + n(t)$$

Output SNR

$$\gamma = \max_{\mathbf{w}_\mathrm{t}} \frac{\mathrm{E}[\left| \mathbf{h}^T \frac{\mathbf{w}_\mathrm{t}}{|\mathbf{w}_\mathrm{t}|} s \right|^2]}{\mathrm{E}[|n|^2]} = \sum_{i=1}^{M_\mathrm{t}} |h_i|^2 \frac{P}{\sigma^2}$$

PDF of OSNR in independent Rayleigh fading

$$f(\gamma) = \frac{1}{(M-1)!\bar{\gamma}^{M_\mathrm{t}}} \gamma^{M_\mathrm{t}-1} \exp\left(-\frac{\gamma}{\bar{\gamma}} \right)$$

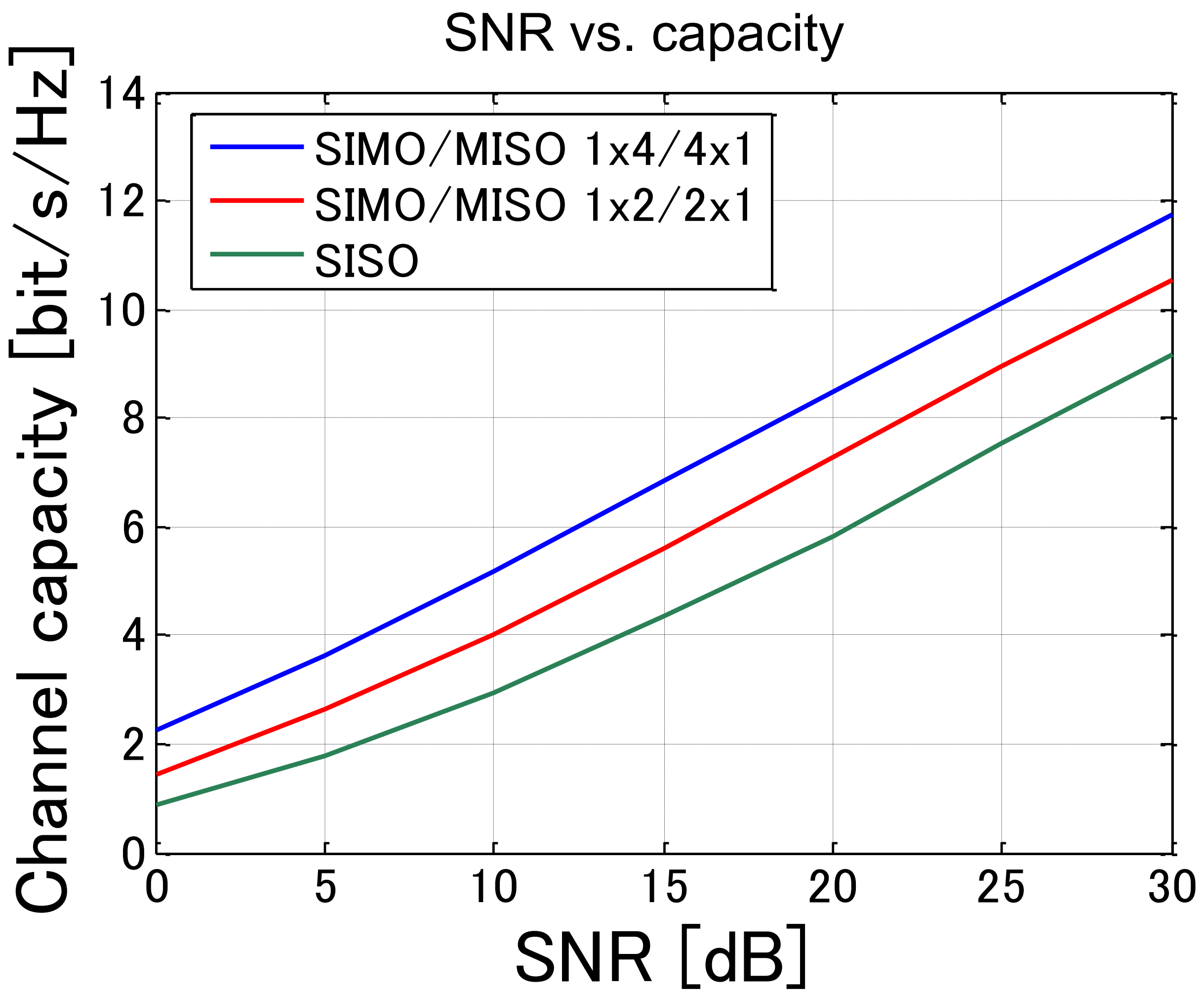
Channel capacity

$$C_{\mathrm{MISO}}(\gamma) = \log_2\left(1 + \sum_{i=1}^{M_\mathrm{t}} |h_i|^2 \frac{P}{\sigma^2} \right)$$

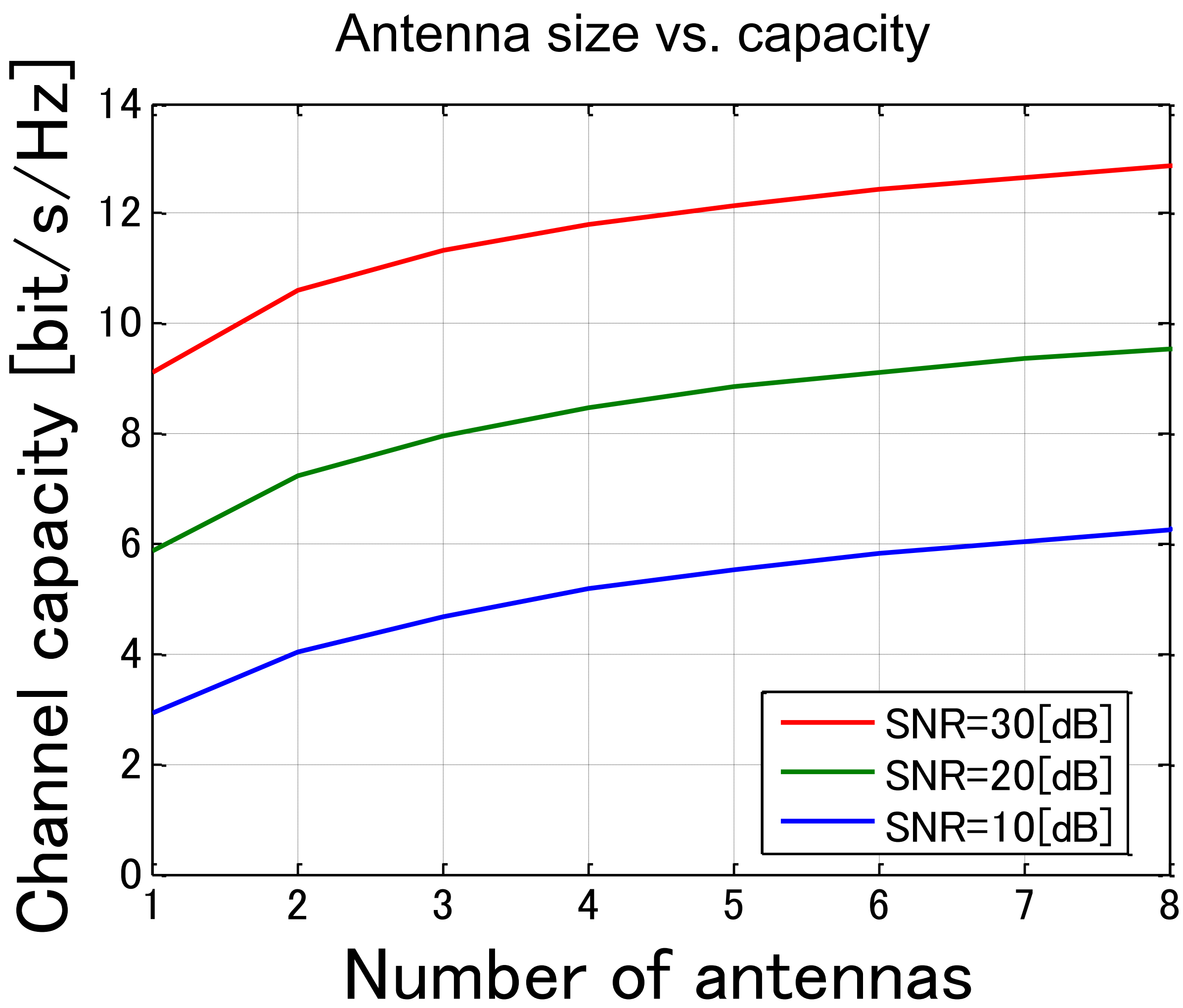
Average channel capacity

$$\overline{C}_{\mathrm{MISO}}(\gamma) = \int f(\gamma) C_{\mathrm{MISO}}(\gamma) \mathrm{d}\gamma$$

SIMO/MISO Channel Cpacity



SIMO/MISO Channel Capacity



Singular Value Decomposition

- Singular Value Decomposition (SVD)

$$\mathbf{H} = \mathbf{U}\,\boldsymbol{\Sigma}\,\mathbf{V}^H = \sqrt{\lambda_1}\mathbf{u}_1\mathbf{v}_1^H + \sqrt{\lambda_2}\mathbf{u}_2\mathbf{v}_2^H + \cdots + \sqrt{\lambda_m}\mathbf{u}_m\mathbf{v}_m^H$$

$$\lambda_1 = \max_{\mathbf{u},\mathbf{v}} \left|\mathbf{u}^H\mathbf{H}\mathbf{v}\right|^2 \qquad \text{s.t.} \qquad |\mathbf{u}|^2 = 1,\ |\mathbf{v}|^2 = 1$$

Singular values: $\boldsymbol{\Sigma} = \mathrm{diag}\begin{bmatrix} \sqrt{\lambda_1} & \sqrt{\lambda_2} & \cdots & \sqrt{\lambda_m} \end{bmatrix}$ $\qquad m = \mathrm{rank}(\mathbf{H})$

Singular matrices: $\mathbf{U}^H\mathbf{U} = \mathbf{I} \qquad \mathbf{V}^H\mathbf{V} = \mathbf{I}$

- Relationship with Eigenvalue Decomposition

$$\mathbf{R}_r = \mathbf{H}\mathbf{H}^H = \mathbf{U}\,\boldsymbol{\Sigma}\,\mathbf{V}^H\mathbf{V}\,\boldsymbol{\Sigma}\,\mathbf{U}^H = \mathbf{U}\,\boldsymbol{\Lambda}\,\mathbf{U}^H$$

$$\mathbf{R}_t = \mathbf{H}^H\mathbf{H} = \mathbf{V}\,\boldsymbol{\Sigma}\,\mathbf{U}^H\mathbf{U}\,\boldsymbol{\Sigma}\,\mathbf{V}^H = \mathbf{V}\,\boldsymbol{\Lambda}\,\mathbf{V}^H$$

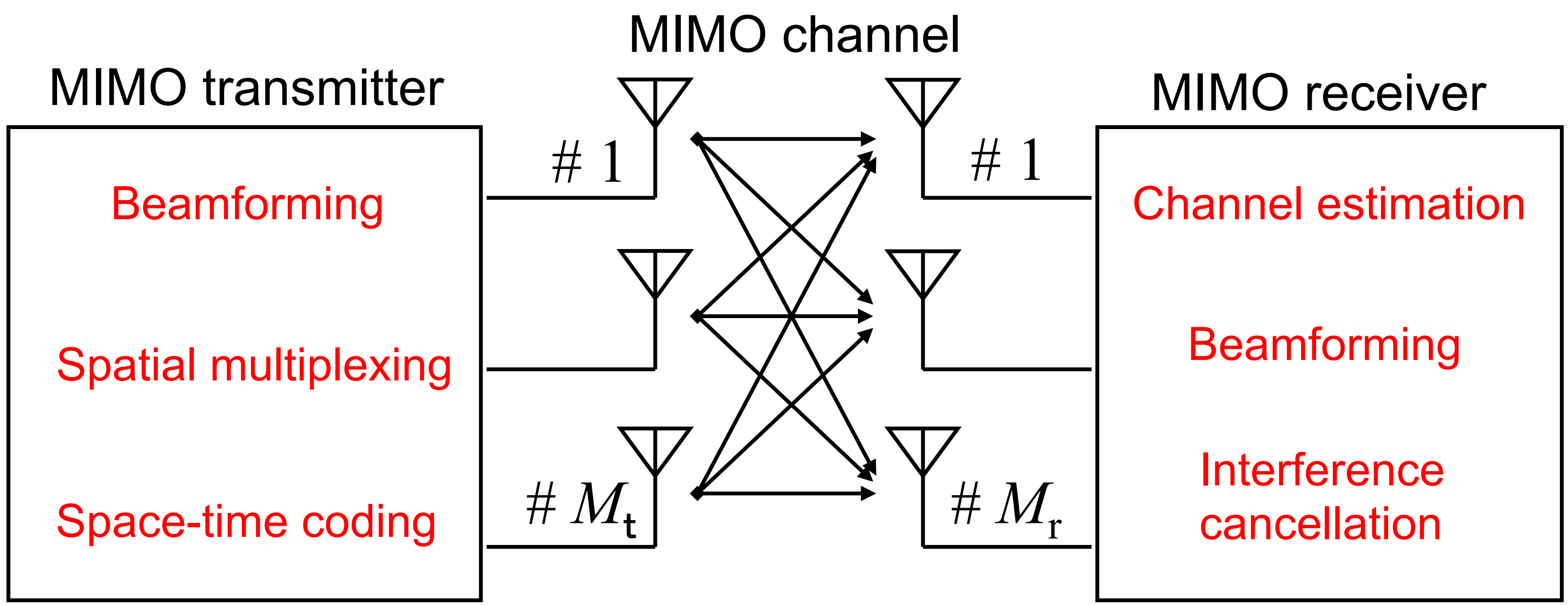
Eigenvalues: $\boldsymbol{\Lambda} = \mathrm{diag}\begin{bmatrix} \lambda_1 & \lambda_2 & \cdots & \lambda_m \end{bmatrix}$

MIMO Communication System

– Multi-Input & Multi-Output (MIMO) at the same channel

– Utilization of rank $m = \min(M_t, M_r)$ effective channels
(ex. SISO → 1, SIMO → 1)

– Benefits of MIMO = increase of throughput & area coverage



Signal Model for MIMO System

Received signal vector

Transmit signal vector

$$\mathbf{y}(t) = \mathbf{H}\mathbf{s}(t) + \mathbf{n}(t)$$

MIMO channel matrix

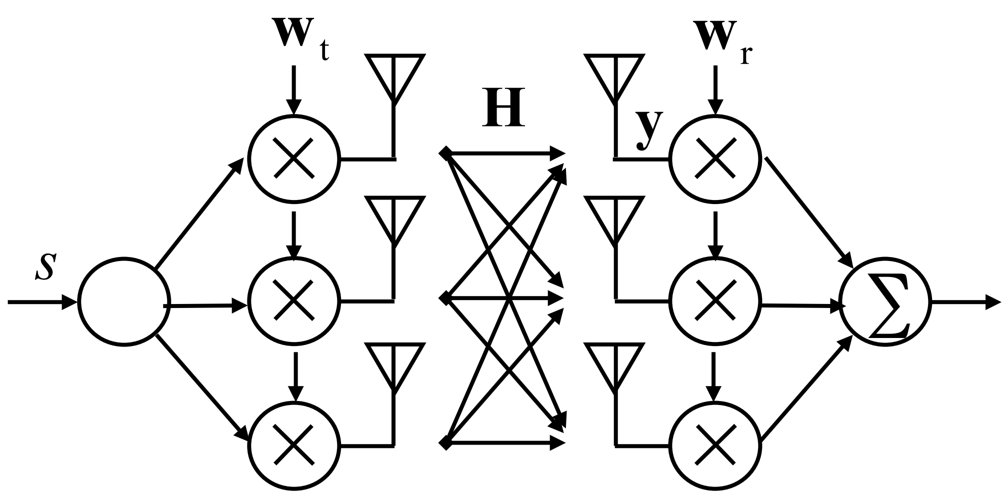
Noise vector

$$\begin{bmatrix} y_1 \\ \vdots \\ y_{M_\mathrm{r}} \end{bmatrix} = \begin{bmatrix} h_{11} & \cdots & h_{1M_\mathrm{t}} \\ \vdots & \ddots & \vdots \\ h_{M_\mathrm{r}1} & \cdots & h_{M_\mathrm{r}M_\mathrm{t}} \end{bmatrix} \begin{bmatrix} s_1 \\ \vdots \\ s_{M_\mathrm{t}} \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_{M_\mathrm{r}} \end{bmatrix}$$

MIMO Diversity

Received signal vector

$$\mathbf{y}(t) = \mathbf{H}\mathbf{w}_{\mathrm{t}} s(t) + \mathbf{n}(t)$$



Output SNR

$$\gamma = \max_{\mathbf{w}_{\mathrm{r}},\mathbf{w}_{\mathrm{t}}} \frac{\mathrm{E}[|\mathbf{w}_{\mathrm{r}}^H \mathbf{H}\mathbf{w}_{\mathrm{t}} s|^2]}{\mathrm{E}[|\mathbf{w}_{\mathrm{r}}^H \mathbf{n}|^2]}$$

$$= \frac{|\mathbf{u}^H \mathbf{H}\mathbf{v}|^2 P}{\sigma^2 |\mathbf{u}|^2} = \frac{\lambda P}{\sigma^2}$$

Maximum singular value

$$\max_{\mathbf{u},\mathbf{v}} |\mathbf{u}^H \mathbf{H}\mathbf{v}|^2 = \lambda$$

Channel capacity

$$C_{\mathrm{MD}}(\gamma) = \log_2\left(1 + \frac{\lambda P}{\sigma^2}\right)$$

Spatial Multiplexing

Received signal vector

$$\mathbf{y}(t) = \sum_{i=1}^{M_t} \mathbf{h}_i \frac{s_i(t)}{\sqrt{M_t}} + \mathbf{n}(t)$$

Interference cancellation

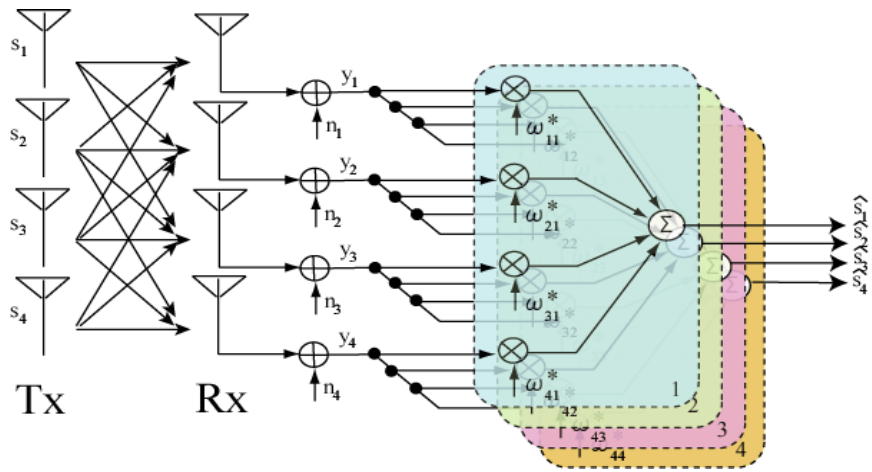
$$\mathbf{w}_{ri} = \mathbf{H}_{\backslash i}^{\perp}$$

Output SNR

$$\gamma_i = \frac{E\left[\left|\mathbf{w}_{ri}^H \mathbf{h}_i \frac{s_i(t)}{\sqrt{M_t}}\right|^2\right]}{E\left[\left|\mathbf{w}_{ri}^H \mathbf{n}\right|^2\right]}$$

Channel capacity

$$C_{MD}(\gamma) = \sum_{i=1}^{M_t} \log_2(1+\gamma_i)$$



Null subspace interference cancellation

$$\mathbf{H}_{\backslash i} = \left[\mathbf{h}_1, \cdots, \mathbf{h}_{i-1}, \mathbf{h}_{i+1} \cdots, \mathbf{h}_{M_t}\right]$$

$$\mathbf{H}_{\backslash i}\mathbf{H}_{\backslash i}^H = \mathbf{E}\Lambda\mathbf{E}^H$$

$$\mathbf{E} = \left[\mathbf{e}_1, \cdots, \mathbf{e}_{M_t}\right]$$

$$\mathbf{H}_{\backslash i}^{\perp} = \mathbf{e}_{M_t}$$

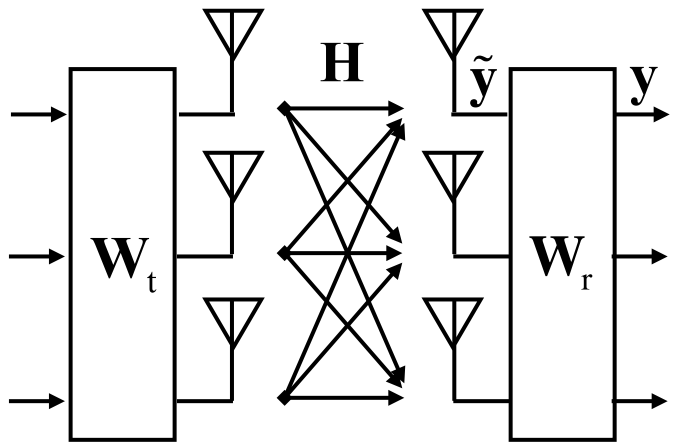
SVD-MIMO

Singular value decomposition

$$\mathbf{H} = \mathbf{U}\,\Sigma\,\mathbf{V}^H$$

$$= \sqrt{\lambda_1}\mathbf{u}_1\mathbf{v}_1^H + \sqrt{\lambda_2}\mathbf{u}_2\mathbf{v}_2^H + \cdots + \sqrt{\lambda_m}\mathbf{u}_m\mathbf{v}_m^H$$

$$\Sigma = \mathrm{diag}\left[\sqrt{\lambda_1},\cdots,\sqrt{\lambda_m}\right] \quad m = \min\left(M_t, M_r\right)$$



SVD-MIMO

$$\mathbf{y}(t) = \mathbf{W}_r^H\mathbf{H}\mathbf{W}_t\mathbf{s}(t) + \mathbf{W}_r^H\mathbf{n}(t)$$

$$= \mathbf{U}^H\mathbf{H}\mathbf{V}\mathbf{s}(t) + \mathbf{U}^H\mathbf{n}(t) = \mathbf{U}^H\mathbf{U}\Sigma\mathbf{V}^H\mathbf{V}\mathbf{s}(t) + \tilde{\mathbf{n}}(t)$$

$$= \Sigma\mathbf{s}(t) + \tilde{\mathbf{n}}(t)$$

Parallel SISO channel

$$\begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} \sqrt{\lambda_1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sqrt{\lambda_m} \end{bmatrix} \begin{bmatrix} s_1 \\ \vdots \\ s_m \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \vdots \\ \tilde{n}_m \end{bmatrix}$$

Tx weight: $\mathbf{W}_t = \mathbf{V}$

Rx weight: $\mathbf{W}_r = \mathbf{U}$

SVD-MIMO

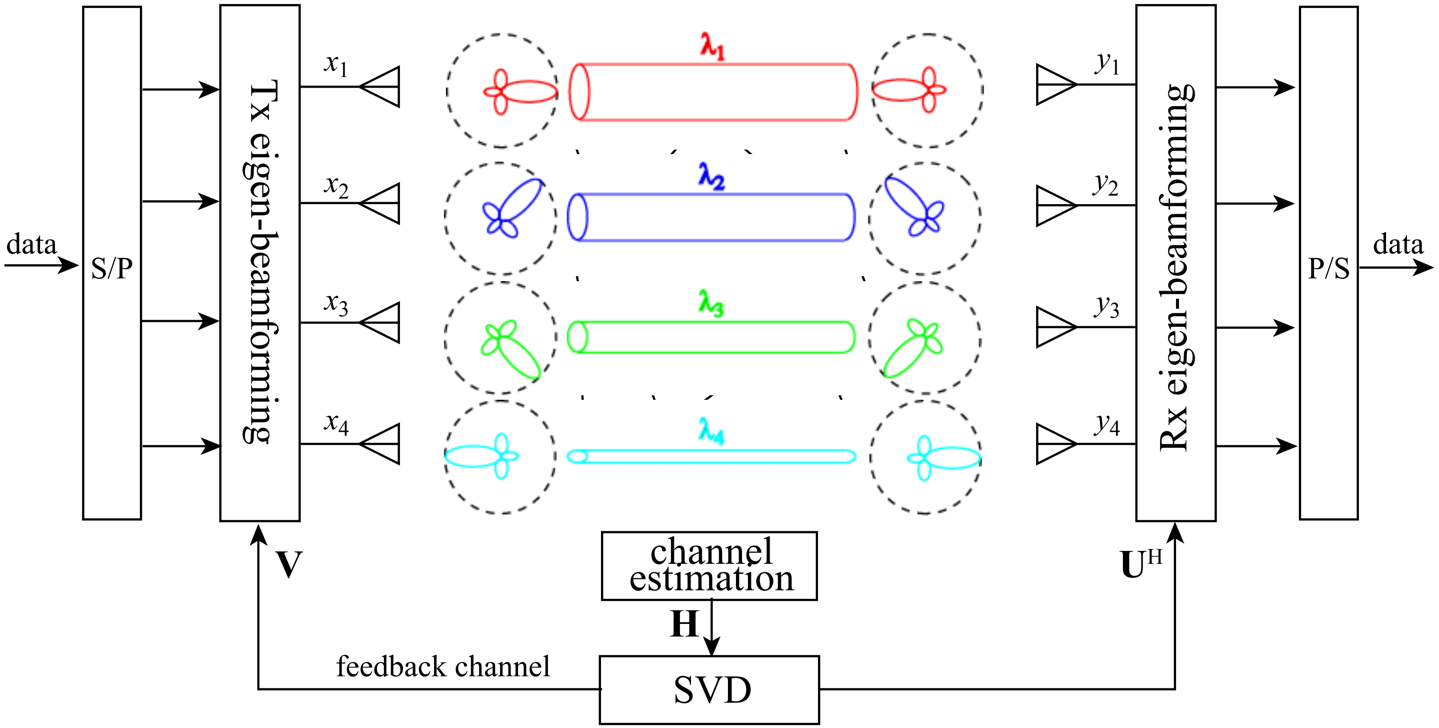
Singular value decomposition

$$\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H$$

$$\Sigma = \mathrm{diag}\left[\sqrt{\lambda_1}, \cdots, \sqrt{\lambda_m}\right]$$

MIMO channel capacity

$$C_{\mathrm{MIMO}}(\gamma) = \sum_{i=1}^{m} \log_2\left(1 + \frac{P}{\sigma^2 m}\lambda_i\right)$$



PDF of SVD-MIMO

PDF of singular values (Wishart distribution)

$$f(\lambda_1,\cdots,\lambda_m)=\frac{1}{K_{m,n}}\prod_{i=1}^{m}\exp(-\lambda_i)\lambda_i^{n-m}\prod_{i<j}^{m}(\lambda_i-\lambda_j)^2$$

$$K_{m,n}=\frac{\pi^{m(m-1)}}{\Gamma_m(n)\Gamma_m(m)}$$

$$n=\max\left(M_{\mathrm{t}},M_{\mathrm{r}}\right)$$

$$m=\min\left(M_{\mathrm{t}},M_{\mathrm{r}}\right)$$

Marginal probability

$$f(\lambda_1)=\int\cdots\int f(\lambda_1,\cdots,\lambda_m)d\lambda_2\cdots d\lambda_m$$

PDF of SVD-MIMO (Example)

2x2 MIMO

$$M_{\mathrm{t}} = 2 \qquad M_{\mathrm{r}} = 2$$

Joint distribution (Wishart distribution)

$$f(\lambda_1, \lambda_2) = e^{-\lambda_1 - \lambda_2}(\lambda_1 - \lambda_2)^2$$

Marginal probability

$$f(\lambda_1) = -2e^{-2\lambda_1} + e^{-\lambda_1}\left(2 - 2\lambda_1 + \lambda_1^2\right) \longrightarrow \mathrm{E}[\lambda_1] = 3.5$$

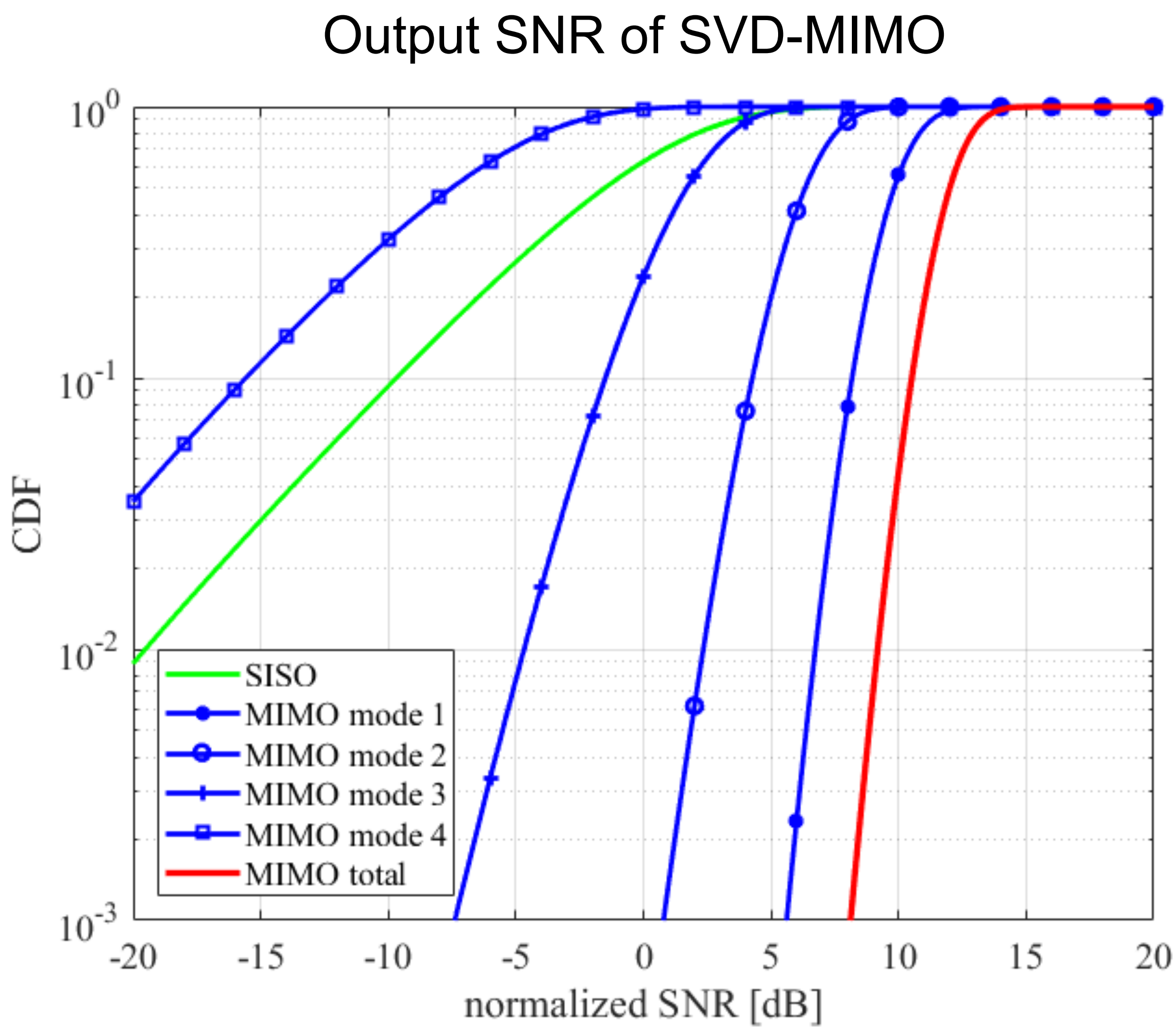
$$f(\lambda_2) = 2e^{-2\lambda_2} \longrightarrow \mathrm{E}[\lambda_2] = 0.5$$

Cumulative probability

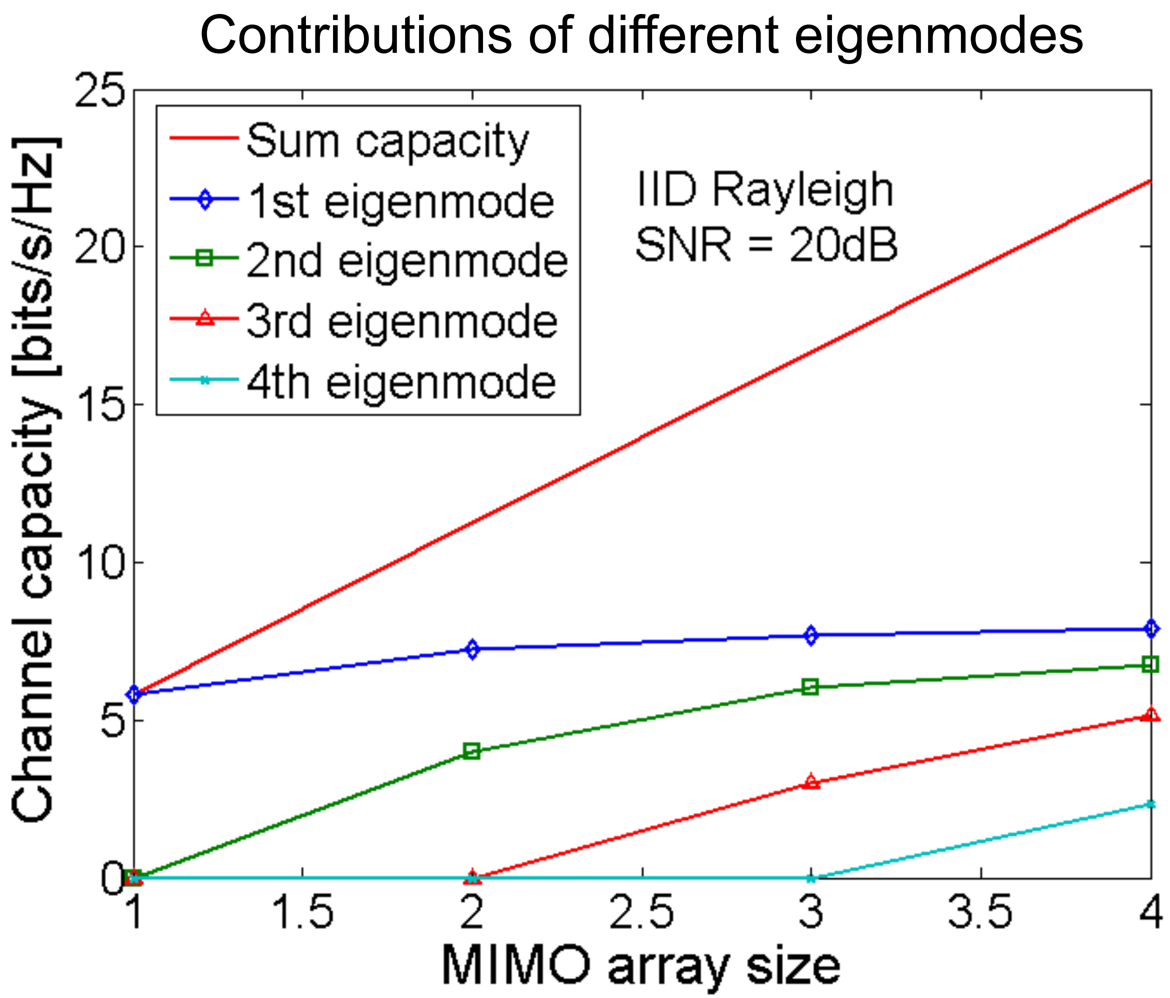
$$\tilde{f}\left(\tilde{\lambda}_1\right) = \int_0^{\tilde{\lambda}_1} f(\lambda_1)\mathrm{d}\lambda_1 = 1 - e^{-\tilde{\lambda}_1}\left(\tilde{\lambda}_1^{\,2} + 2\right) + e^{-2\tilde{\lambda}_1} \approx \tilde{\lambda}_1^{\,4} / 12 + \cdots$$

$$\tilde{f}\left(\tilde{\lambda}_2\right) = \int_0^{\tilde{\lambda}_2} f(\lambda_2)\mathrm{d}\lambda_2 = 1 - e^{-2\tilde{\lambda}_2} \approx 2\tilde{\lambda}_2 - 2\tilde{\lambda}_2^{\,2} + \cdots$$

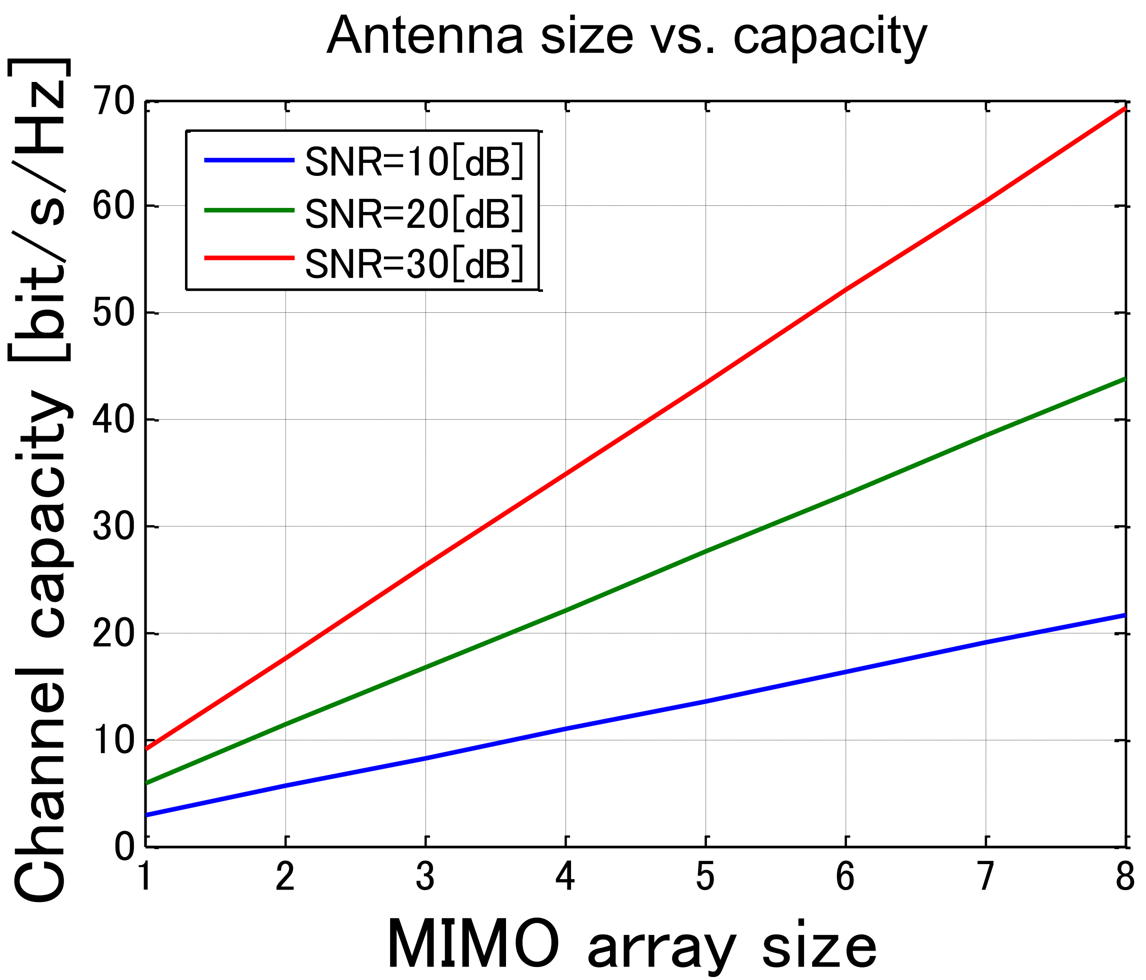
CDF of SVD-MIMO



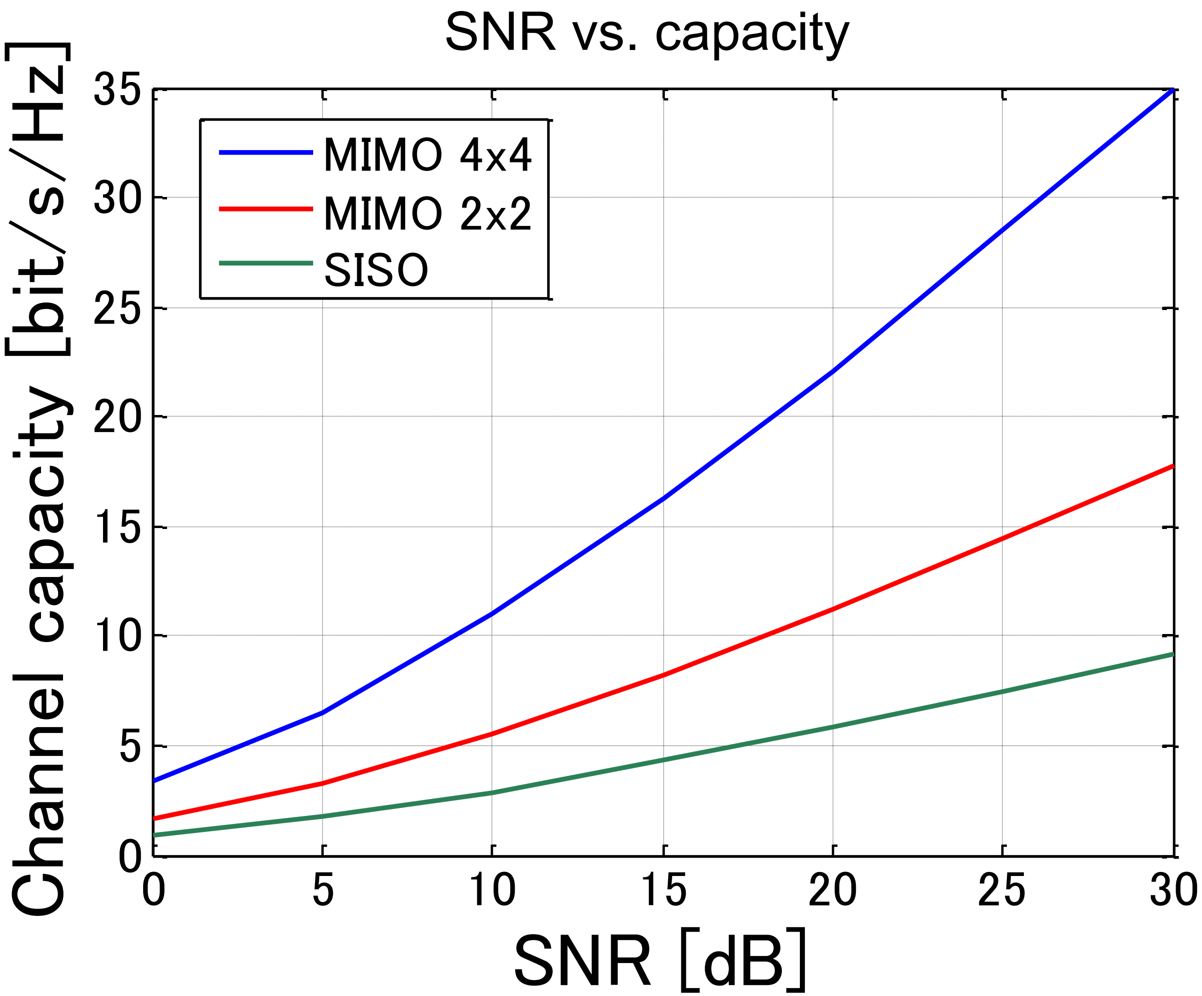
MIMO Channel Capacity



MIMO Channel Capacity



MIMO Channel Capacity



Summary

- SISO system

$$y(t) = hs(t) + n(t) \qquad C_{\mathrm{SISO}}(\gamma) = \log_2\left(1 + \frac{|h|^2 P}{\sigma^2}\right)$$

- SIMO/MISO system

$$\mathbf{y}(t) = \mathbf{h}s(t) + \mathbf{n}(t) \qquad C_{\mathrm{SIMO}}(\gamma) = \log_2\left(1 + \sum_{i=1}^{M_r} |h_i|^2 \frac{P}{\sigma^2}\right)$$

- MIMO system

$$\mathbf{y}(t) = \mathbf{Hs}(t) + \mathbf{n}(t) \qquad C_{\mathrm{MIMO}}(\gamma) = \sum_{i=1}^{m} \log_2\left(1 + \frac{P}{\sigma^2 m}\lambda_i\right)$$

$$\mathbf{H} = \mathbf{U}\,\boldsymbol{\Sigma}\,\mathbf{V}^H = \sqrt{\lambda_1}\mathbf{u}_1\mathbf{v}_1^H + \sqrt{\lambda_2}\mathbf{u}_2\mathbf{v}_2^H + \cdots + \sqrt{\lambda_m}\mathbf{u}_m\mathbf{v}_m^H$$