2018 2Q

Wireless Communication Engineering

#8 Channel Fading and Diversity Combining

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Course Schedule (1)

	Date	Text	Contents
#1	June 11	1, 7	Introduction to wireless communication systems
#2	June 14	2, 5, etc	Link budget design of wireless access
#3	June 18		Up/down conversion and equivalent baseband system
#4	June 21	3.3, 3.4	Digital modulation and pulse shaping
#5	June 25	3.5	Demodulation and matched filter
#6	June 28		Collaborative exercise for better understanding 1
#7	July 2	3.5	Detection and error due to noise
#8	July 5	4.4	Channel fading and diversity combining

From Previous Lectures

- Channel estimation & coherent detection

$$\hat{h}_{\mathrm{B}} = \frac{1}{K}\sum_{k=1}^{K} y(k) \Big/ s_{\mathrm{TR}}(k) \quad \longrightarrow \quad \hat{s}(k) = y(k)/h_{\mathrm{B}} = s(k) + n(k)/h_{\mathrm{B}}$$

- Error rate of BPSK signal

$$p_{\mathrm{eb}} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{P_{\mathrm{s}}\left|h_{\mathrm{B}}\right|^2}{\sigma^2}}\right) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\gamma}\right)$$

- Error rate of QAM signal

$$p_{\mathrm{eb}} = \frac{2}{\log_2 M}\left(1 - \frac{1}{\sqrt{M}}\right)\mathrm{erfc}\left(\sqrt{\frac{3\gamma}{2\left(M-1\right)}}\right)$$

Contents

- Narrow band system
- Gain of propagation channel
- Rayleigh fading & probability distribution
- Error rate in fading channel
- Diversity technologies
- Maximum ratio combining diversity

Narrow Band System

Time invariant narrow band system

$$y_{\mathrm{B}}(t) = \int h_{\mathrm{B}}(\tau)\tilde{s}_{\mathrm{B}}(t-\tau)\mathrm{d}\tau = h_{\mathrm{B}} s_{\mathrm{B}}(t)$$

$$h_{\mathrm{B}} = h_{\mathrm{B}}(\tau_0) = h(\tau_0) e^{-j2\pi f_0 \tau_0}$$

Time variant narrow band system

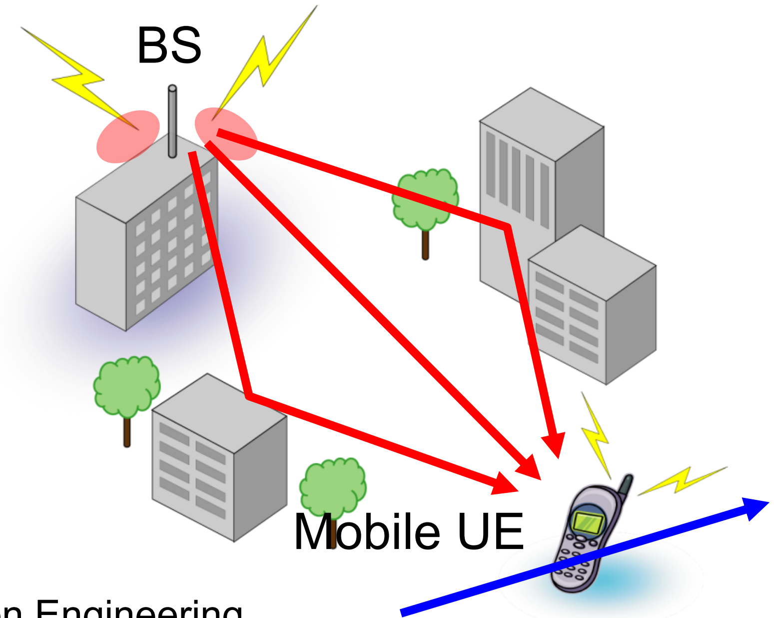
$$h_{\mathrm{B}} \rightarrow h_{\mathrm{B}}(d, \phi_{\mathrm{s}}, \phi_{\mathrm{r}}, t)$$

d : Distance between Tx & Rx

ϕ : Tx & Rx antenna angle

t : Mobility of UE

Mobile communication



Gain of Propagation Channel

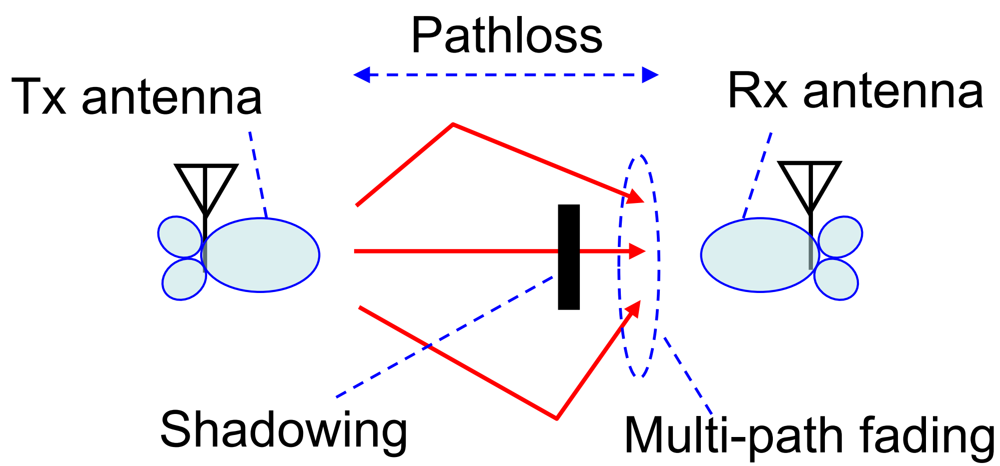
Gain of narrow band propagation channel

$$G_{\mathrm{prop}} = \left| h_{\mathrm{B}}\left(d, \phi_{\mathrm{s}}, \phi_{\mathrm{r}}, t\right) \right|^2$$

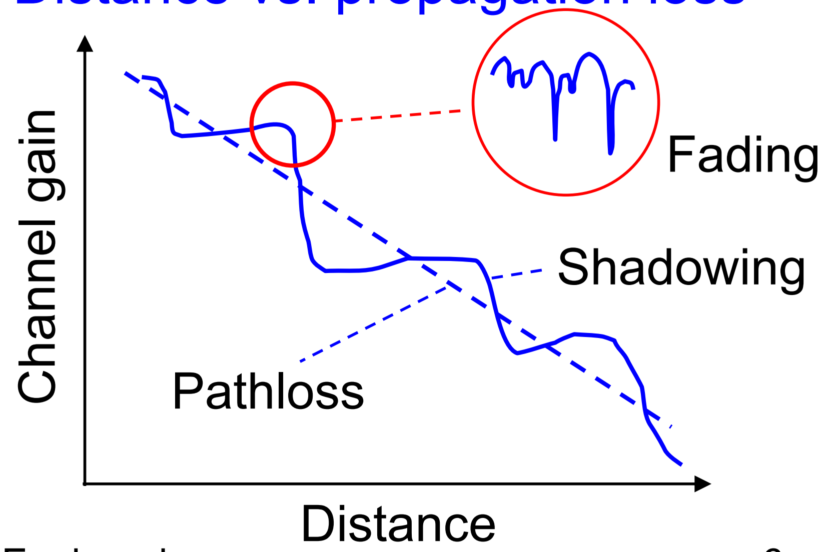
$$= G_{\mathrm{rant}}\left(\phi_{\mathrm{r}}\right) G_{\mathrm{fading}}\left(t\right) G_{\mathrm{shadow}}\left(t\right) G_{\mathrm{pl}}\left(d\right) G_{\mathrm{sant}}\left(\phi_{\mathrm{s}}\right)$$

Rx antenna gain
Multi-path fading
Shadowing
Pathloss
Tx antenna gain

Narrow band propagation channel



Distance vs. propagation loss



Standing Wave & Fading

Multi-path propagation channel

$$h_{\mathrm{B}}(t) = \sum_i h_{\mathrm{B}i}(t)$$

$$h_{\mathrm{B}i}(t) = h_{\mathrm{pl}}(d)\, e^{-j2\pi f_0 \tau_i(t)}$$

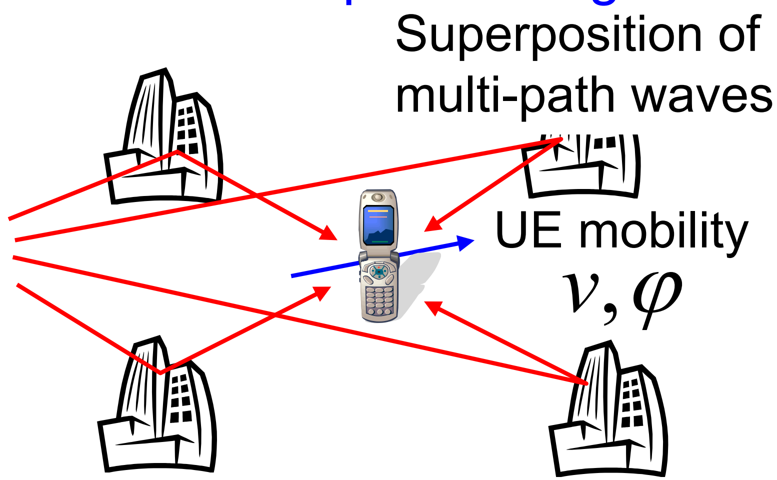
Doppler shift (phase shift due to mobility)

$$\tau_i(t) = \tau_0 + \frac{vt\cos\varphi_i}{c}$$

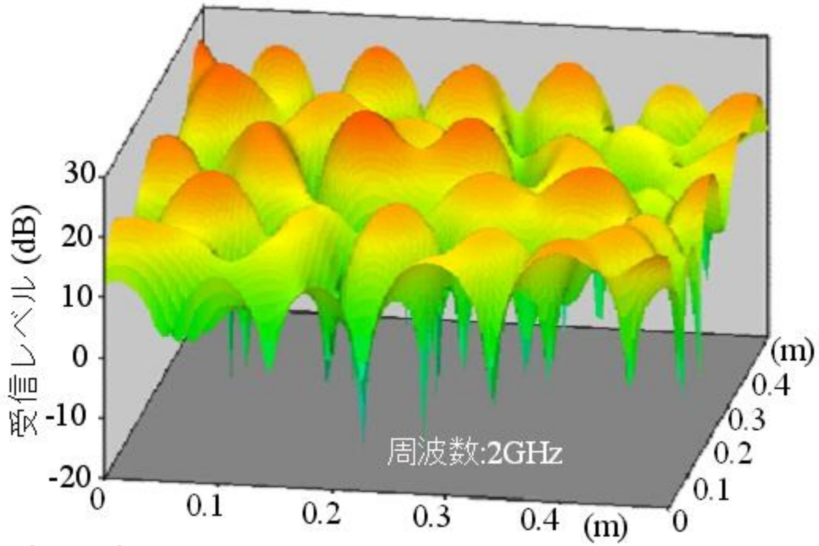
$$h_{\mathrm{B}i}(t) = h_{\mathrm{pl}}(d)\, e^{j\theta_0 - 2\pi f_{\mathrm{d}i} t}$$

$$f_{\mathrm{d}i} = \frac{v\cos\varphi_i}{\lambda_0}$$

Multi-path fading



Standing wave



Rayleigh Fading

Multi-path channel

$$h_{\mathrm{B}}(t)=\sum_i h_{\mathrm{pl}}(d)e^{j\theta_i(t)}=x+jy$$

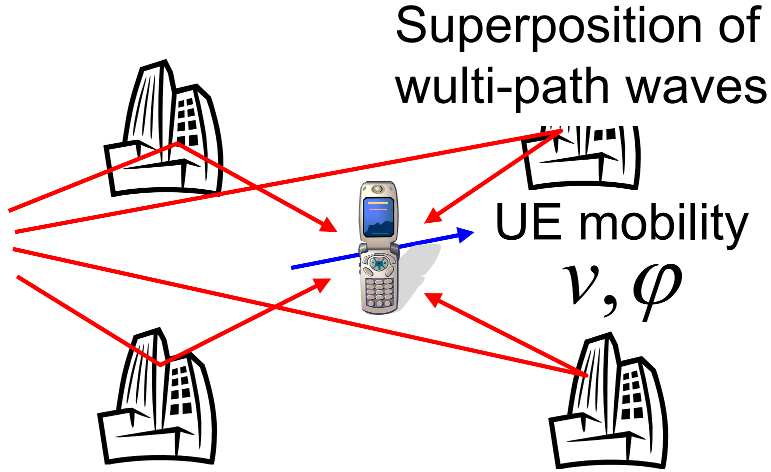
Central limit theorem

Sum of independent random variables

↓

Gaussian distribution

Multi-path fading

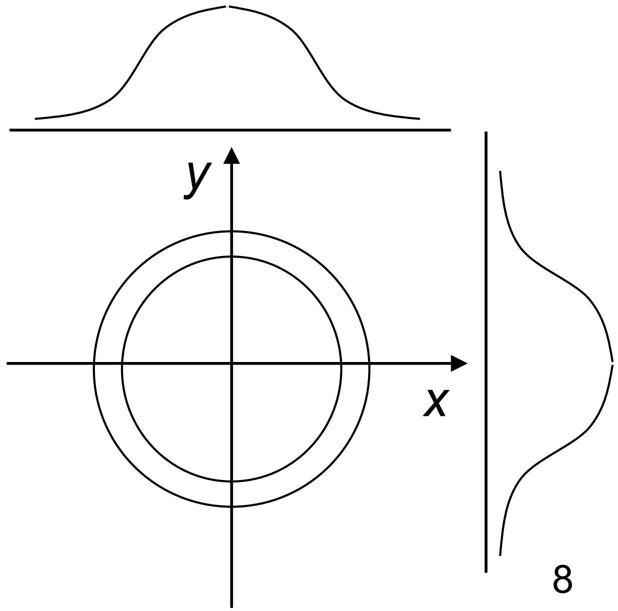


Complex Gaussian distribution

$$f(x)=\frac{1}{\sqrt{\pi\sigma_{\mathrm{h}}^2}}\exp\left(-\frac{x^2}{\sigma_{\mathrm{h}}^2}\right)$$

$$f(x,y)=\frac{1}{\pi\sigma_{\mathrm{h}}^2}\exp\left(-\frac{x^2+y^2}{\sigma_{\mathrm{h}}^2}\right)$$

Complex Gaussian dist.



Probability of Fading

Cartesian to polar conversion

$$f(x,y)=\frac{1}{\pi\sigma_{\mathrm{h}}^2}\exp\left(-\frac{x^2+y^2}{\sigma_{\mathrm{h}}^2}\right)$$

$$x=r\cos\phi \quad y=r\sin\phi$$

$$f(r,\phi)=f(x,y)\left|\det\mathbf{J}\right|=\frac{r}{\pi\sigma_{\mathrm{h}}^2}\exp\left(-\frac{r^2}{\sigma_{\mathrm{h}}^2}\right)$$

Jacobian

$$\mathrm{d}x\,\mathrm{d}y=\begin{vmatrix}\frac{\partial x}{\partial r} & \frac{\partial x}{\partial \phi}\\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \phi}\end{vmatrix}\mathrm{d}r\,\mathrm{d}\phi$$

$$=\left|\det\mathbf{J}\right|\mathrm{d}r\,\mathrm{d}\phi$$

Uniform phase distribution

Rayleigh distribution (amplitude & power)

$$f(r)=\int_0^{2\pi}f(r,\phi)\,\mathrm{d}\phi=\frac{2r}{\sigma_{\mathrm{h}}^2}\exp\left(-\frac{r^2}{\sigma_{\mathrm{h}}^2}\right)$$

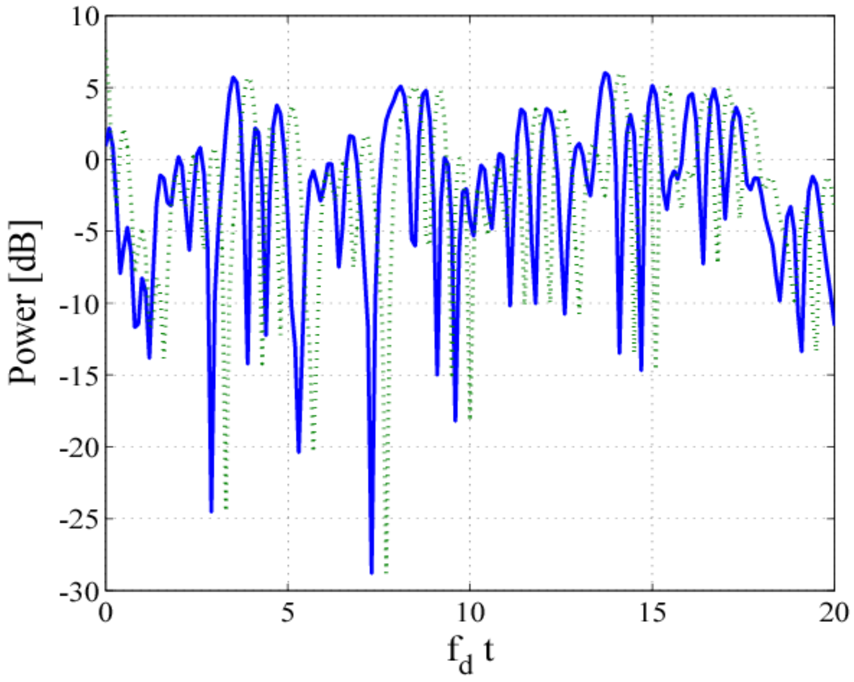
$$r=\sqrt{g}$$

$$f(g)=\frac{1}{2\sqrt{g}}f(r)=\frac{1}{\sigma_{\mathrm{h}}^2}\exp\left(-\frac{g}{\sigma_{\mathrm{h}}^2}\right)$$

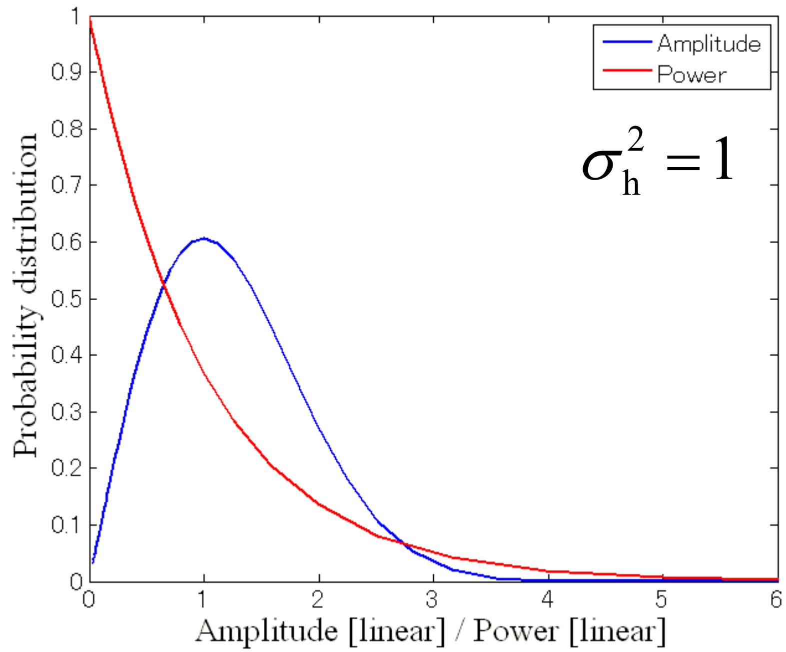
Rayleigh distribution

Rayleigh Distribution

Fading variation



Probability distribution



Cumulative Distribution

Rayleigh distribution

$$f(g) = \frac{1}{\sigma_{\mathrm{h}}^2} \exp\left(-\frac{g}{\sigma_{\mathrm{h}}^2} \right)$$

Cumulative probability distribution (CDF)

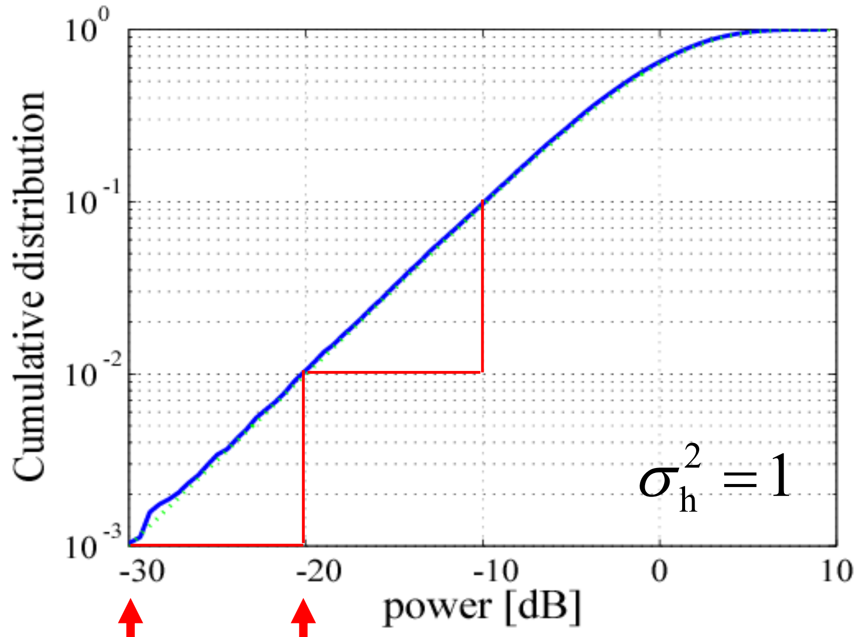
$$\tilde{f}(g) = \int_0^{\tilde{g}} f(g) \mathrm{d}g = 1 - \exp\left(-\frac{\tilde{g}}{\sigma_{\mathrm{h}}^2} \right)$$

Taylor expansion

$$\tilde{f}(p) = \tilde{f}(0) + g\tilde{f}'(0) + \frac{g^2}{2}\tilde{f}''(0) + \cdots$$

$$= \frac{g}{\sigma_{\mathrm{h}}^2} + \frac{g^2}{2\left(\sigma_{\mathrm{h}}^2\right)^2} + \cdots$$

CDF



Power attenuation more than 20dB with probability 1/100

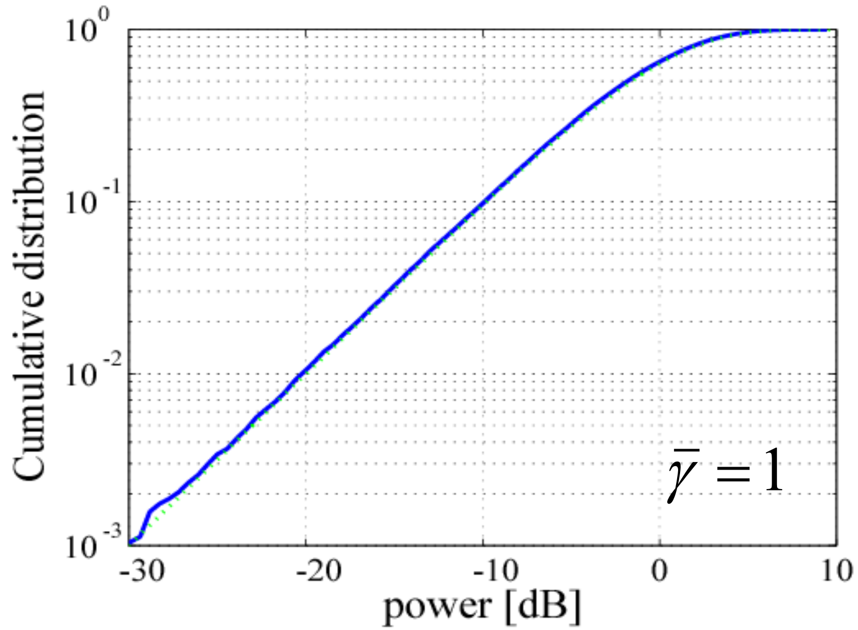
Power attenuation more than 30dB with probability 1/1000

CDF of SNR

Signal-to-Noise Ratio (SNR)

$$\gamma = \frac{P\left|h_B(t)\right|^2}{\sigma^2} = \frac{Pg(t)}{\sigma^2}$$

CDF



Rayleigh fading

$$f(g) = \frac{1}{\sigma_h^2}\exp\left(-\frac{g}{\sigma_h^2}\right)$$

$$\sigma_h^2 = \mathrm{E}\left[\left|h_B(t)\right|^2\right] = \bar{g}$$

PDF of SNR

$$f(\gamma) = \frac{1}{\bar{\gamma}}\exp\left(-\frac{\gamma}{\bar{\gamma}}\right)$$

$$\bar{\gamma} = \frac{P\bar{g}}{\sigma^2}$$

Average SNR

Error Rate in Fading Channel

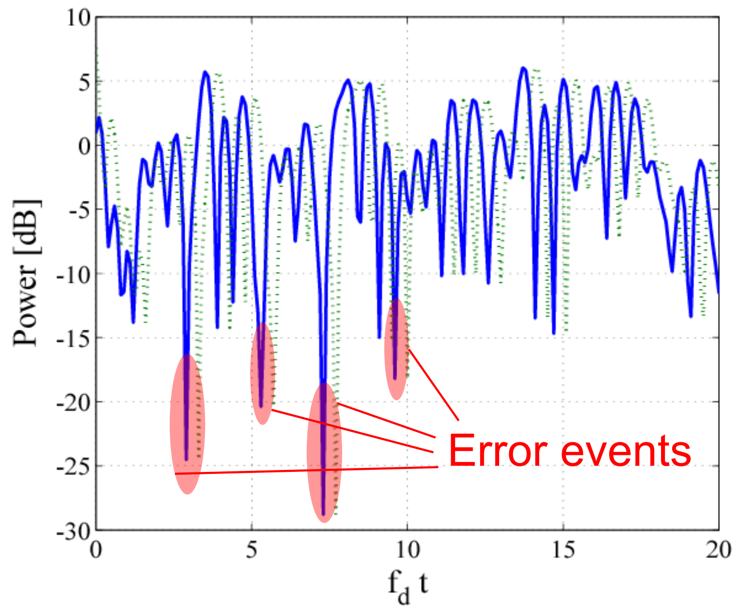
BER of BPSK

$$p_{\mathrm{eb}}(\gamma) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\gamma}\right) \qquad \gamma = \frac{P|h|^2}{\sigma^2}$$

Rayleigh fading channel

$$f(\gamma) = \frac{1}{\bar{\gamma}}\exp\left(-\frac{\gamma}{\bar{\gamma}}\right) \qquad \bar{\gamma} = \mathrm{E}\left[\frac{P|h(t)|^2}{\sigma^2}\right]$$

Fading channel



Average BER

$$\bar{p}_{\mathrm{eb}}(\bar{\gamma}) = \int p_{\mathrm{eb}}(\gamma) f(\gamma)\mathrm{d}\gamma$$

$$= \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right)$$

Mathematical formulas

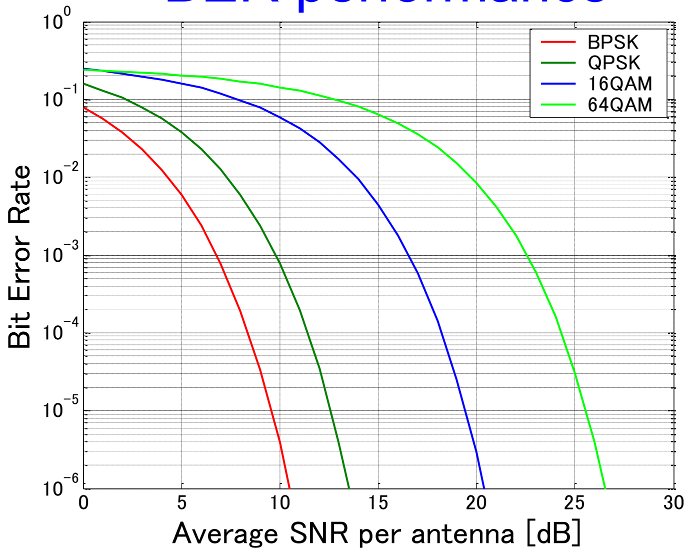
$$\int f(x)g'(x)\mathrm{d}x = f(x)g(x) - \int f'(x)g(x)\mathrm{d}x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{erfc}(x) = -\frac{2}{\sqrt{\pi}}\exp\left(-x^2\right)$$

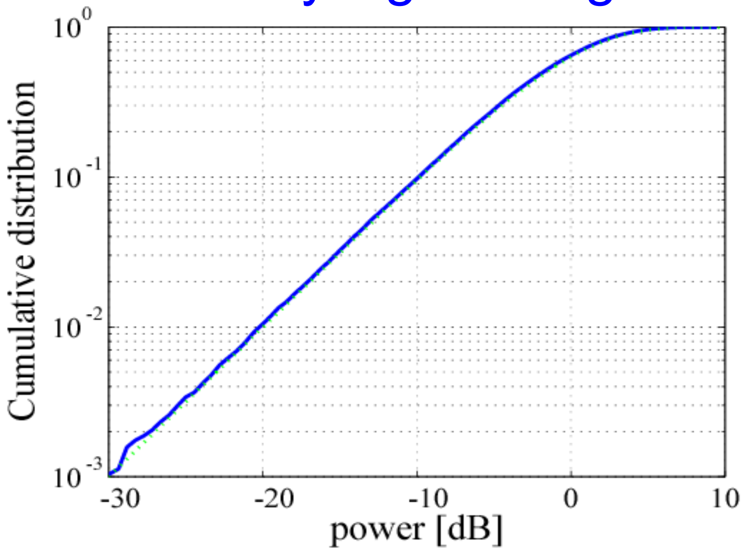
$$\int_0^\infty \exp\left(-ax^2\right)\mathrm{d}x = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

BER Performance in Fading Channel

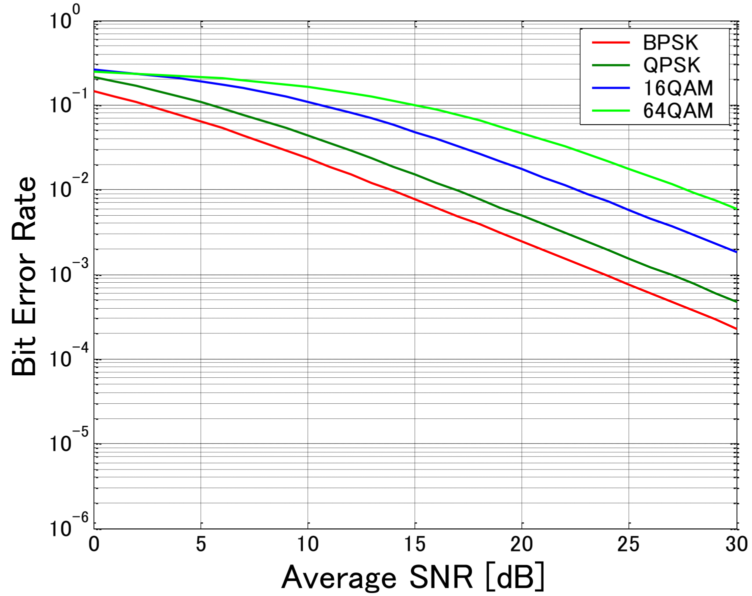
BER performance



Rayleigh fading



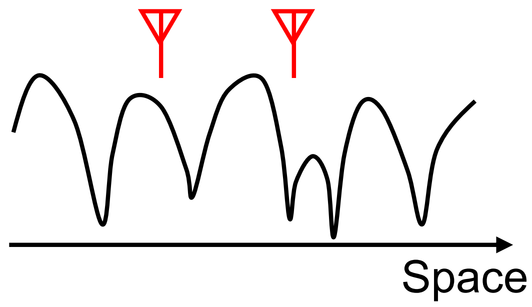
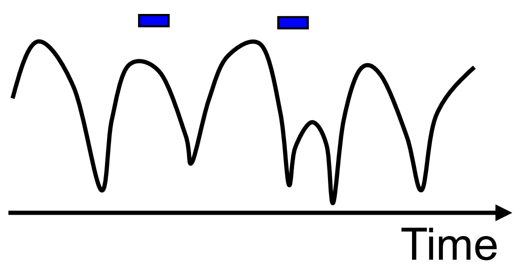
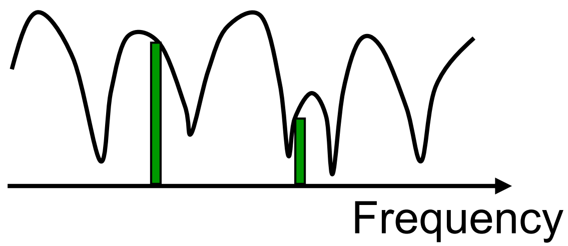
Average BER performance



One of biggest problem
in wireless communications

Diversity Technologies

Fading combatting schemes using more than 2 {Antenna, Symbol, Subcarrier}

Antenna	Array antenna In multi-path env.	
Time	Repetition, Interleaver In Doppler spread env.	
Frequency	OFDM In delay spread env.	

Antenna Signal Processing

Receive signal model

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_M \end{bmatrix} s + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_M \end{bmatrix}$$

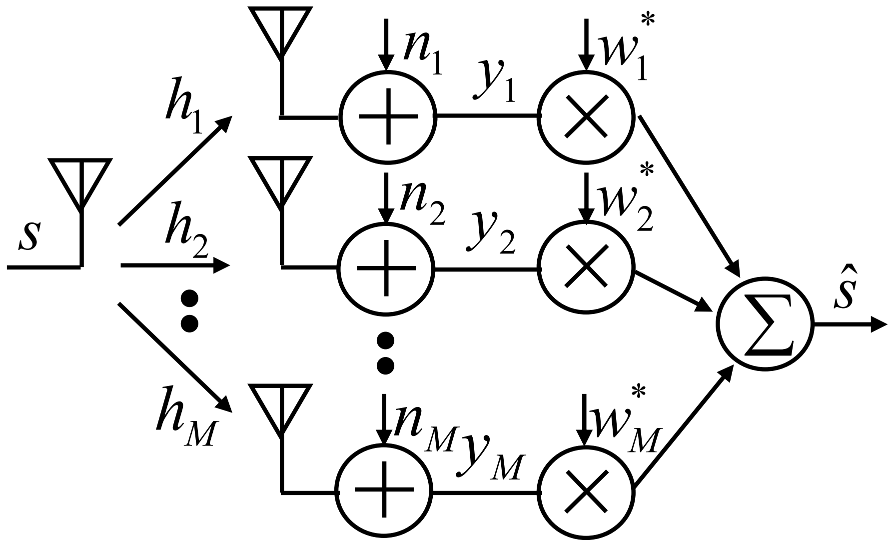
$$\mathbf{y} = \mathbf{h}s + \mathbf{n}$$

Weighted combining

$$\hat{s} = \mathbf{w}^H \mathbf{y} = \mathbf{w}^H \mathbf{h} s + \mathbf{w}^H \mathbf{n}$$

$$\mathbf{w} = \begin{bmatrix} w_1 & w_2 & \cdots & w_M \end{bmatrix}^T$$

Array antenna receiver



Maximum Ratio Combining

SNR after weighted combining

$$\gamma = \frac{\mathrm{E}[|\mathbf{w}^H \mathbf{h} s|^2]}{\mathrm{E}[|\mathbf{w}^H \mathbf{n}|^2]} = \frac{|\mathbf{w}^H \mathbf{h}|^2 \, \mathrm{E}[|s|^2]}{\mathbf{w}^H \, \mathrm{E}[\mathbf{n}\mathbf{n}^H]\mathbf{w}} = \frac{|\mathbf{w}^H \mathbf{h}|^2 P}{|\mathbf{w}|^2 \sigma^2}$$

Vector inner product

Vector outer product

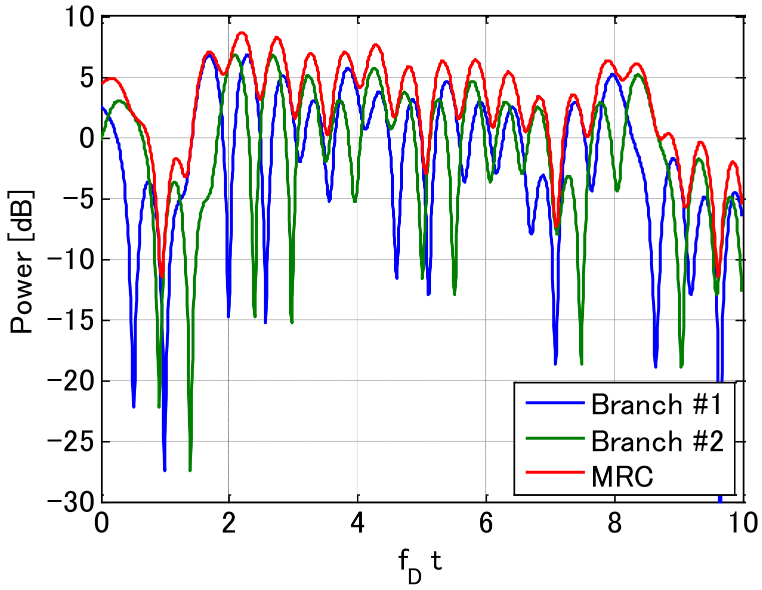
$$\mathrm{E}[\mathbf{n}\mathbf{n}^H] = \sigma^2 \mathbf{I}_M$$ ← Uncorrelated noise

Maximum Ratio Combining (MRC)

$$\mathbf{w}_{\mathrm{opt}} = \arg\max \gamma = \alpha \mathbf{h}$$

$$\gamma_{\mathrm{opt}} = \frac{|\mathbf{h}|^2 P}{\sigma^2} = \frac{\sum_{i=1}^{M} |h_i|^2 P}{\sigma^2} = \sum_{i=1}^{M} \gamma_i$$ ← Sum of branch (antenna) SNR

Performance of MRC



PDF & Characteristic Function

PDF of sum of random variables

Independent random variables x y

$$f(x) \quad f(y) \quad f(x,y) = f(x)f(y)$$

$z = x + y$ ← Sum of random variables

$f(z) = \int f(x)f(z-x)\mathrm{d}x$ ← Convolution

Characteristic function of PDF

$$f(\gamma) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\phi(t)\exp(-j\gamma t)\,\mathrm{d}t \longleftrightarrow \varphi(t) = \int_{0}^{\infty} f(\gamma)\exp(j\gamma t)\,\mathrm{d}\gamma$$

Theorem on Fourier transformation

$$\gamma = \sum_i \gamma_i \longleftrightarrow \varphi(t) = \prod_i \varphi_i(t)$$

CDF of SNR after MRC

SNR after MRC

$$\gamma = \sum_{i=1}^{M} \gamma_i$$

Characteristic function of each antenna

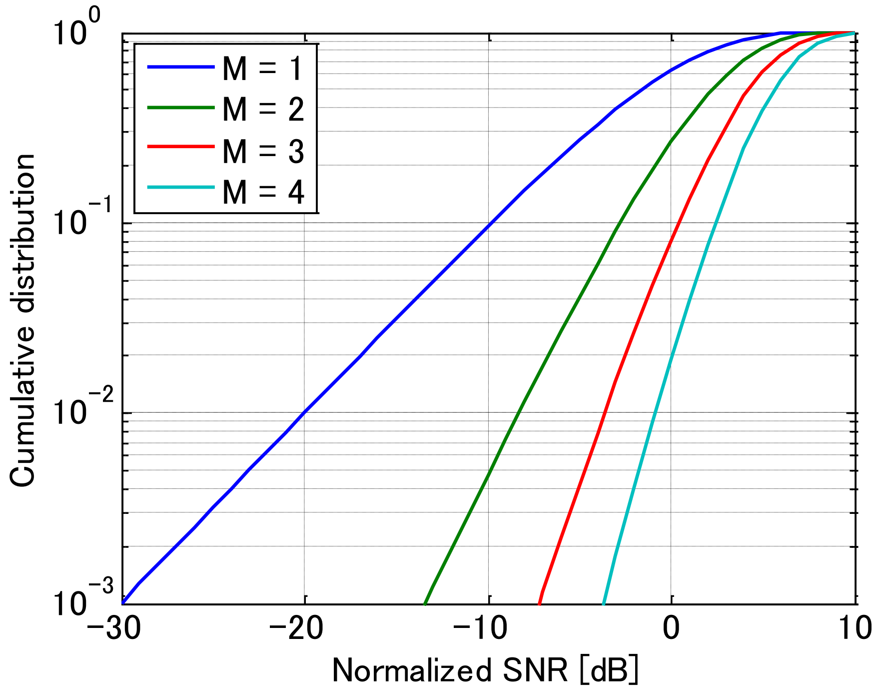
$$f(\gamma_i) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma_i}{\bar{\gamma}}\right) \leftrightarrow \varphi_i(t) = \frac{1}{1 - j\bar{\gamma}t}$$

PDF of SNR after MRC

$$\varphi(t) = \prod_{i=1}^{M} \varphi_i(t) = \left(\frac{1}{1 - j\bar{\gamma}t}\right)^M$$

$$f(\gamma) = \frac{1}{(M-1)!\bar{\gamma}^M} \gamma^{M-1} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right)$$ ← Gamma dist. (χ square dist.)

CDF



BER of MRC Diversity

Average BER

$$\bar{p}_{\mathrm{e}}(\bar{\gamma}) = \int p_{\mathrm{e}}(\gamma) f(\gamma) \mathrm{d}\gamma$$

BER in AWGN (Gaussian Noise)

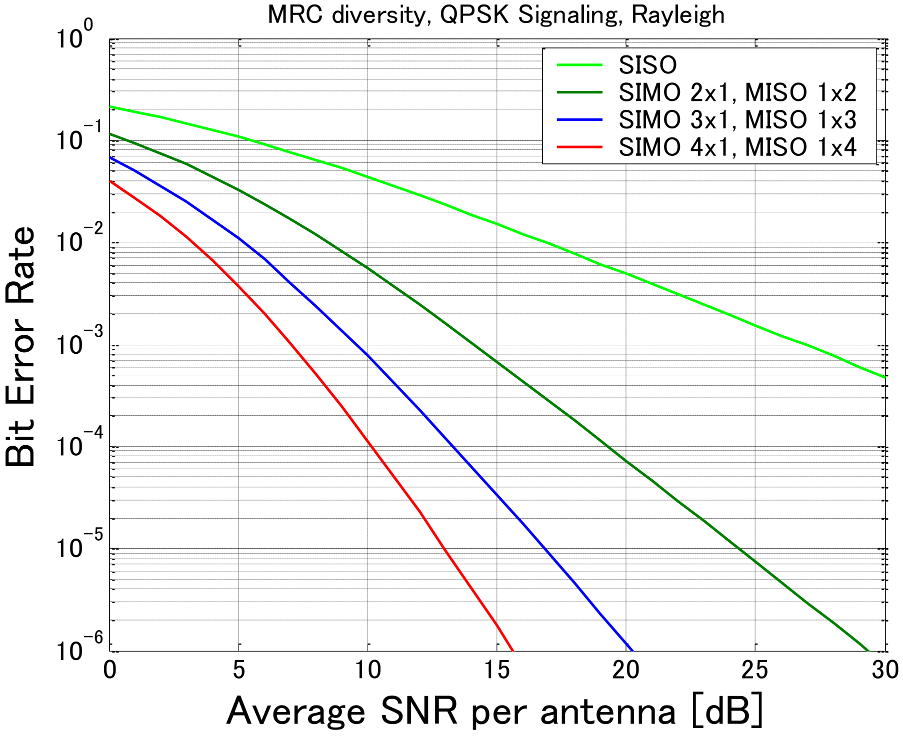
$$P_{\mathrm{e}} = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\gamma}\right)$$

PDF of SNR after MRC

$$f(\gamma) = \frac{1}{(M-1)!\,\bar{\gamma}^{M}} \gamma^{M-1} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right)$$

$$\bar{p}_{\mathrm{e}}(\bar{\gamma}) = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right)^{M} \sum_{i=0}^{M-1} \binom{M-1+i}{i} \left(\frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right)\right)^{i}$$

BER performance



Summary

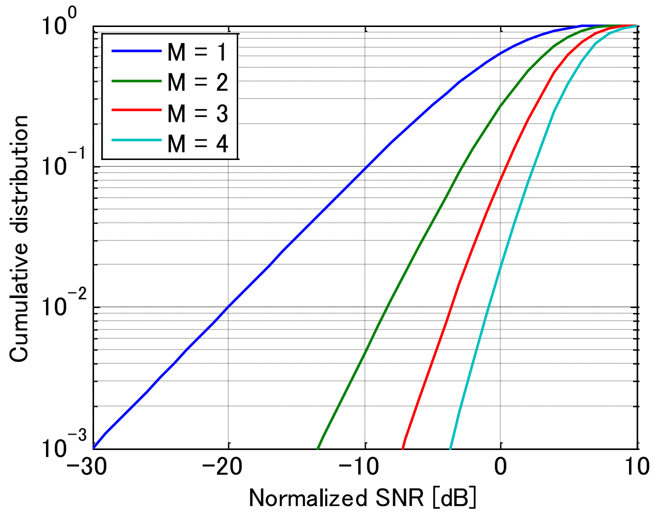
■ Error rate in fading channel

$$p_{\mathrm{eb}}(\gamma) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\gamma}\right) \Rightarrow \bar{p}_{\mathrm{eb}}(\bar{\gamma}) = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right)$$

■ Antenna signal processing

$$\mathbf{y} = \mathbf{h}s + \mathbf{n}$$

$$\hat{s} = \mathbf{w}^H\mathbf{y} = \mathbf{w}^H\mathbf{h}s + \mathbf{w}^H\mathbf{n}$$



■ Performance of MRC diversity

$$\gamma_{\mathrm{opt}} = \frac{|\mathbf{h}|^2 P}{\sigma^2} = \frac{\sum_{i=1}^{M}|h_i|^2 P}{\sigma^2} = \sum_{i=1}^{M}\gamma_i \Rightarrow f(\gamma) = \frac{1}{(M-1)!\,\bar{\gamma}^M}\gamma^{M-1}\exp\left(-\frac{\gamma}{\bar{\gamma}}\right)$$