2018 2Q
Wireless Communication Engineering

#4 Digital Modulation and Pulse Shaping

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Course Schedule (1)

	Date	Text	Contents
#1	June 11	1, 7	Introduction to wireless communication systems
#2	June 14	2, 5, etc	Link budget design of wireless access
#3	June 18		Up/down conversion and equivalent baseband system
#4	June 21	3.3, 3.4	Digital modulation and pulse shaping
#5	June 25	3.5	Demodulation and matched filter
#6	June 28		Collaborative exercise for better understanding 1
#7	July 2	3.5	Detection and error due to noise
#8	July 5	4.4	Channel fading and diversity combining

From Previous Lecture

- Equivalent baseband system

$$y_{\mathrm{B}}(t) = \int h_{\mathrm{B}}(\tau) s_{\mathrm{B}}(t-\tau)\mathrm{d}\tau$$

$$y_{\mathrm{B}}(t) = y_{\mathrm{A}}(t)e^{-j2\pi f_0 t} \qquad y_{\mathrm{A}}(t) = y(t) + j\,\mathrm{hilb}\big(y(t)\big)$$

- Power spectrum of transmit signal

$$S^{\mathrm{s}}(f) = \frac{1}{4}\left(S_{\mathrm{B}}^{\mathrm{s}}(f - f_0) + S_{\mathrm{B}}^{\mathrm{s}}(-f - f_0)\right)$$

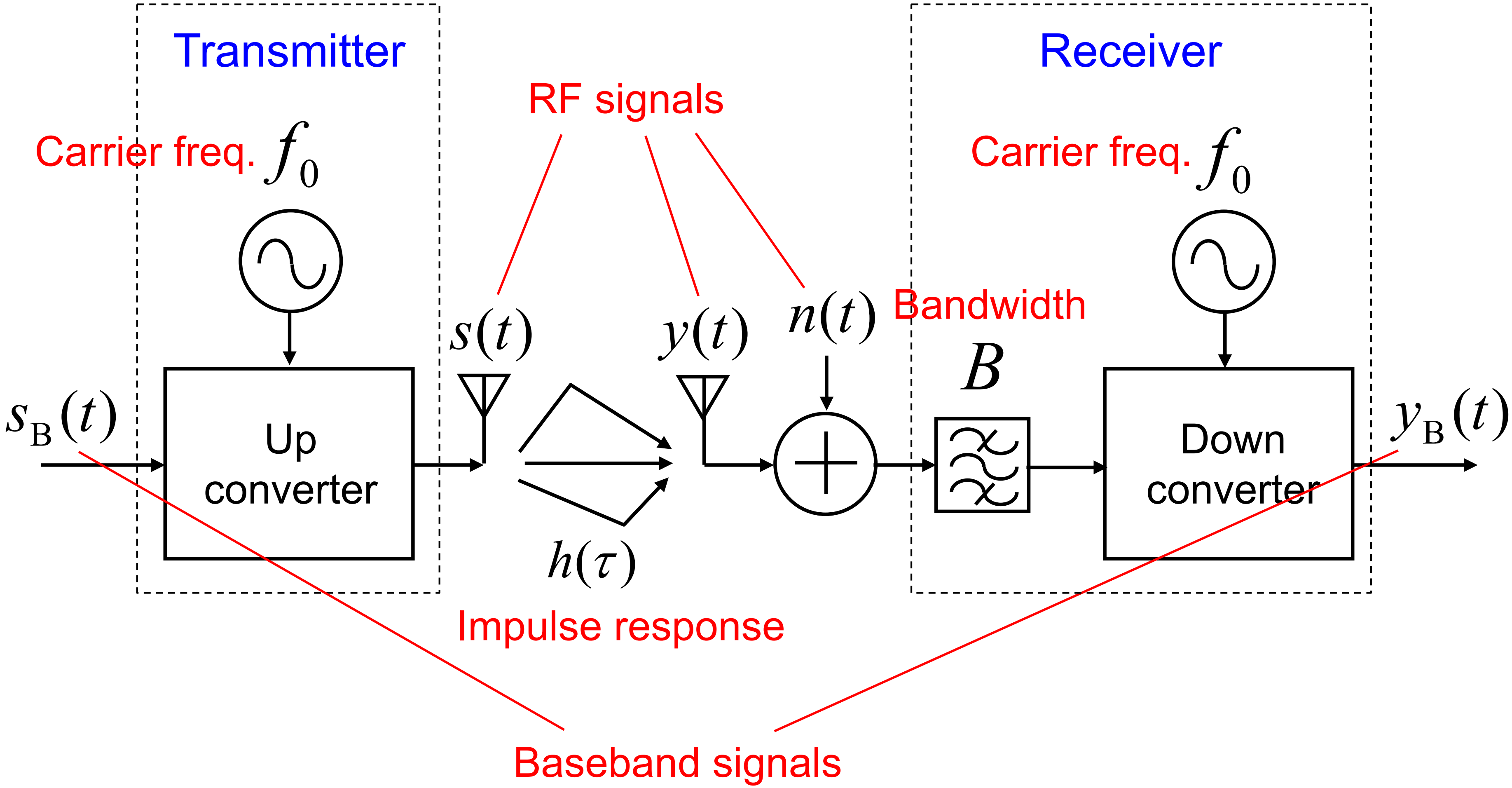
- Power spectrum of receive signal

$$S_{\mathrm{B}}^{\mathrm{y}}(f) = \left|H_{\mathrm{B}}(f)\right|^2 S_{\mathrm{B}}^{\mathrm{s}}(f)$$

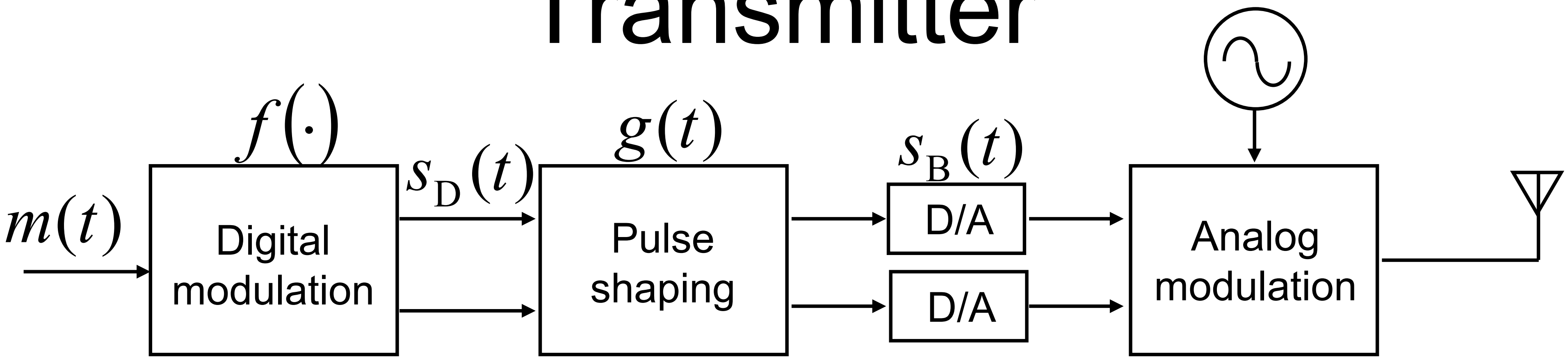
Contents

- Structure of transmitter
- Amplitude shift keying
- Phase shift keying
- Frequency shift keying
- Pulse shaping
- Experiments

Transceiver System



Transmitter



Message signal

Message sequence

Modulation order & bit rate

$$m(t) = \sum_n a_n \delta(t - nT_s) \qquad a_n = \begin{cases} 1, & \text{if symbol 1} \\ 0, & \text{if symbol 0} \end{cases} \longrightarrow \begin{array}{l} M = 2 \\ R = \log_2 M = 1 \end{array}$$

Symbol period

Digital modulation

$$s_D(t) = s_{DI}(t) + j s_{DQ}(t) = f(m(t))$$

Mapping function

- Amplitude
- Phase
- Frequency

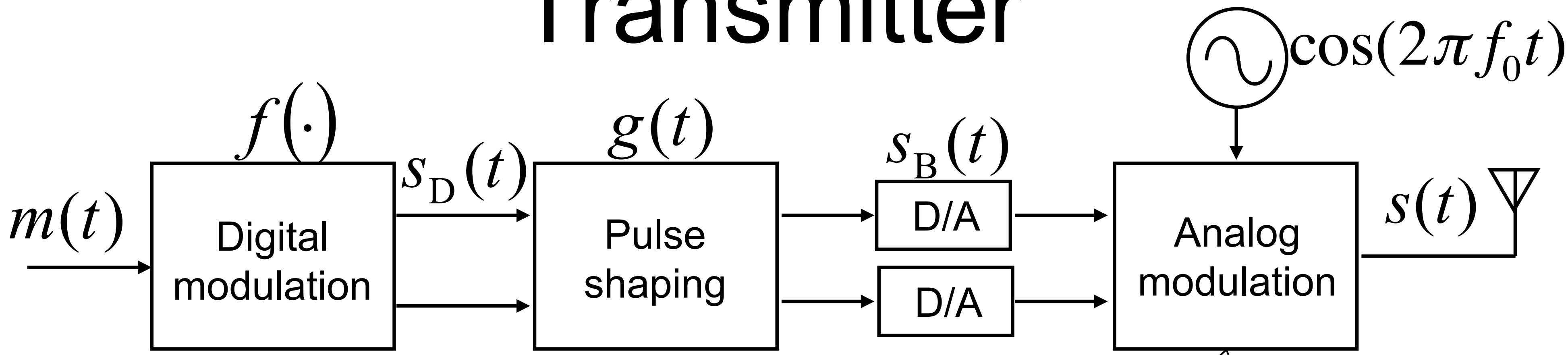
Pulse shaping

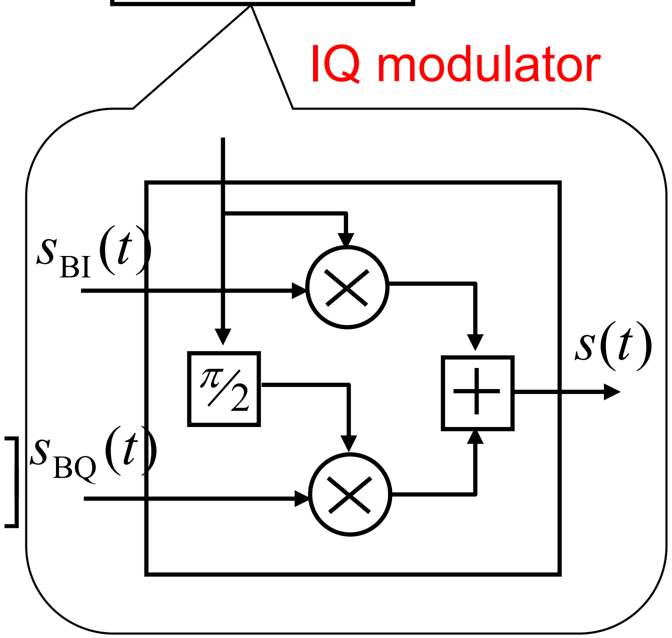
$$s_B(t) = s_{BI}(t) + j s_{BQ}(t) = \int g(\tau) s_D(t - \tau) \mathrm{d}\tau$$

Pulse shape

- Rectangular
- Nyquist
- Gaussian

Transmitter





Analog modulation

Carrier freq.

$$s(t) = s_{BI}(t)\cos(2\pi f_0 t) - s_{BQ}(t)\sin(2\pi f_0 t)$$

In-phase (I) Quadrature (Q)

$$= \mathrm{Re}\left[\left(s_{BI}(t) + j s_{BQ}(t)\right)\left(\cos(2\pi f_0 t) + j\sin(2\pi f_0 t)\right)\right]$$

Complex baseband signal

$$= \mathrm{Re}\left[r(t)\exp\left(j\theta(t)\right)\exp\left(j2\pi f_0 t\right)\right] = r(t)\cos(2\pi f_0 t + \theta(t))$$

Amplitude Frequency Phase

Design of Modulation & Pulse Shaping

Carrier freq. f_0	⟺	Coverage $d_0 = \sqrt{\dfrac{A_t P_t}{4\pi\alpha N_0 f_0 \gamma_0}}$
Symbol period T_s	⟺	Bandwidth $B \propto 1/T_s$
Modulation order M	⟺	Data rate $\dfrac{\log_2 M}{T_s}$
Modulation $f(\cdot)$	⟺	Power efficiency, complexity, error rate p_e
Pulse shape $g(t)$	⟺	Power leakage $S_B^s(f) = \lvert G(f)\rvert^2 S_D^s(f)$

Amplitude Shift Keying (ASK)

Digital modulation

$$s_{\mathrm{D}}(t) = s_{\mathrm{DI}}(t) = m(t)$$

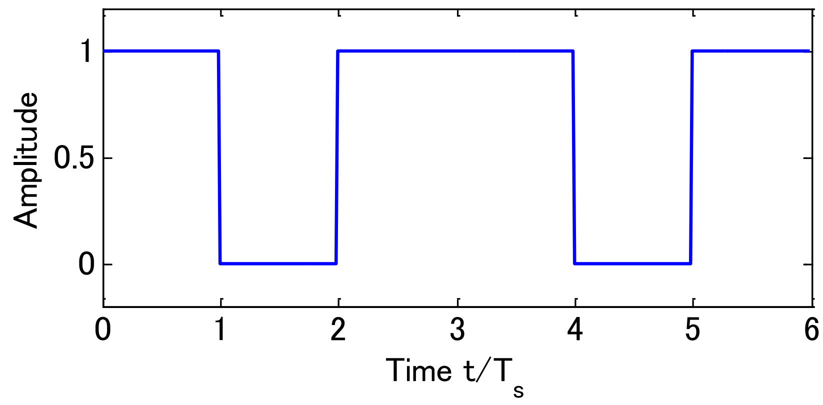
Pulse shaping (rectangular)

$$s_{\mathrm{B}}(t) = \int g_{\mathrm{rect}}(\tau) s_{\mathrm{D}}(t-\tau)\,\mathrm{d}\tau = \sum_{n} a_n g_{\mathrm{rect}}(t - nT_{\mathrm{s}})$$

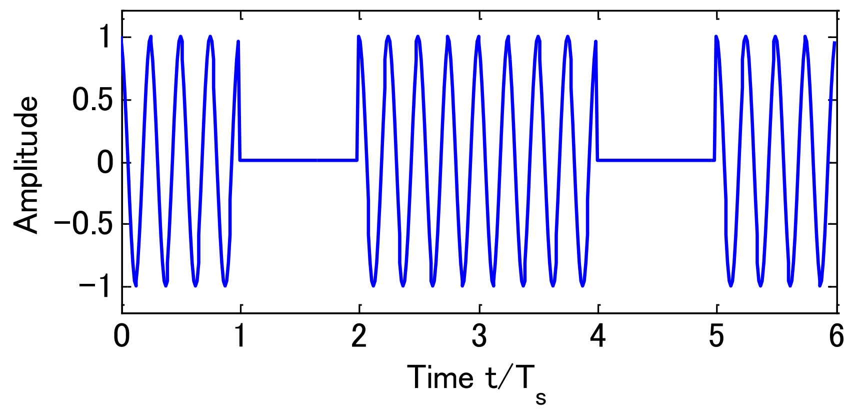
Analog modulation

$$s(t) = s_{\mathrm{B}}(t)\cos(2\pi f_0 t) = \begin{cases} \cos(2\pi f_0 t), & \text{if } a_n = 1 \\ 0, & \text{if } a_n = 0 \end{cases}$$

Message sequence



RF signal



Main features

Error rate = Fair, Power efficiency = Fair, Complexity = Excellent

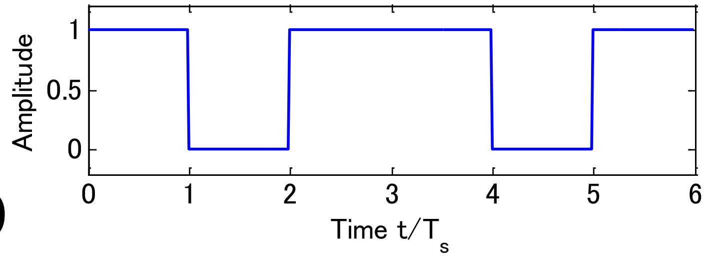
Binary Phase Shift Keying (BPSK)

Digital modulation

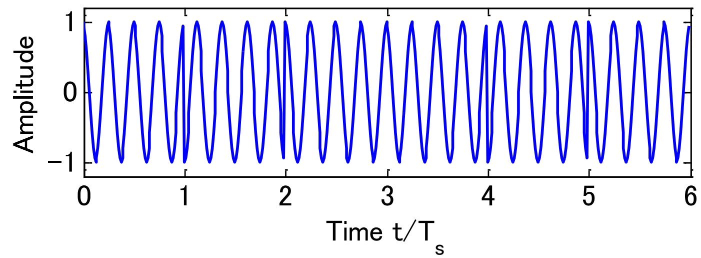
$$s_{\mathrm{D}}(t) = \exp\left(j\pi m(t)\right)$$
$$= \cos(\pi m(t)) + j\sin(\pi m(t))$$
$$= \sum_n a_{2n}\delta\left(t - nT_{\mathrm{s}}\right) \quad a_{2n} = \begin{cases} 1, & \text{if } a_n = 0 \\ -1, & \text{if } a_n = 1 \end{cases}$$

NRZ signal

Message sequence



RF signal



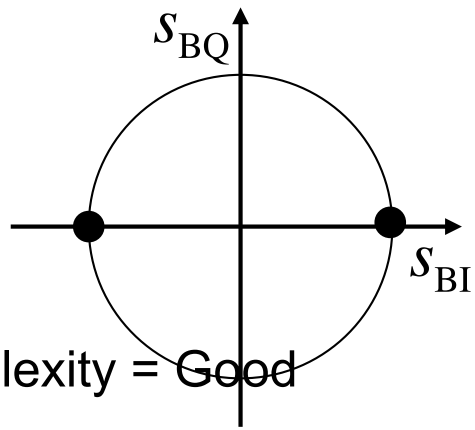
Pulse shaping (rectangular)

$$s_{\mathrm{B}}(t) = \sum_n a_{2n} g_{\mathrm{rect}}(t - nT_{\mathrm{s}})$$

Analog modulation

$$s(t) = s_{\mathrm{B}}(t)\cos(2\pi f_0 t) = \begin{cases} \cos(2\pi f_0 t), & \text{if } a_n = 0 \\ -\cos(2\pi f_0 t), & \text{if } a_n = 1 \end{cases}$$

Constellation



Main features

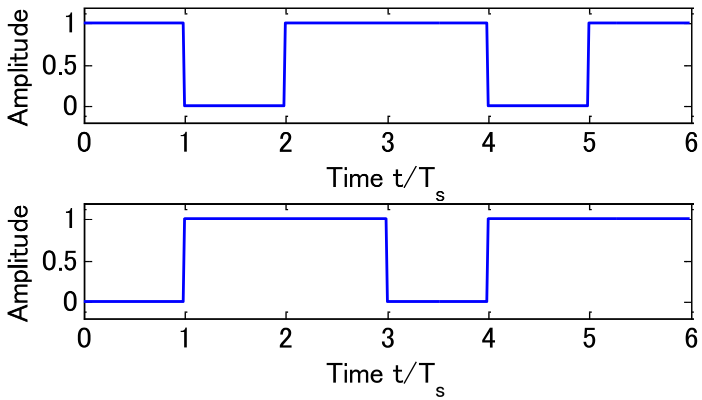
Error rate = Excellent, Power efficiency = Good, Complexity = Good

Quadrature Phase Shift Keying (QPSK)

Message signal

$M = 4 \longrightarrow$ Two binary sequences $m_{\mathrm{I}}(t), m_{\mathrm{Q}}(t)$

Message sequence



Digital modulation & pulse shaping

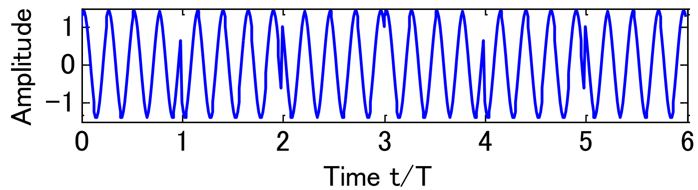
$$s_{\mathrm{D}}(t) = \exp\left(j\pi m_{\mathrm{I}}(t)\right) + \exp\left(j(\pi m_{\mathrm{Q}}(t) + \pi/2)\right)$$

$$= \cos\left(\pi m_{\mathrm{I}}(t)\right) + j\cos\left(\pi m_{\mathrm{Q}}(t)\right)$$

$$= \sum_n a_{2\mathrm{I}n}\delta(t - nT_{\mathrm{s}}) + j\sum_n a_{2\mathrm{Q}n}\delta(t - nT_{\mathrm{s}})$$

$$s_{\mathrm{B}}(t) = \sum_n a_{2\mathrm{I}n} g_{\mathrm{rect}}(t - nT_{\mathrm{s}}) + j\sum_n a_{2\mathrm{Q}n} g_{\mathrm{rect}}(t - nT_{\mathrm{s}})$$

RF signal

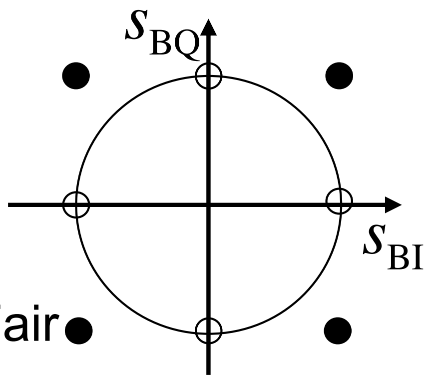


Analog modulation

$$s(t) = s_{\mathrm{BI}}(t)\cos(2\pi f_0 t) - s_{\mathrm{BQ}}(t)\sin(2\pi f_0 t)$$

$$= \sqrt{2}\cos(2\pi f_0 t + \mathrm{atan}(s_{\mathrm{BI}}(t), s_{\mathrm{BQ}}(t)))$$

Constellation



Main features

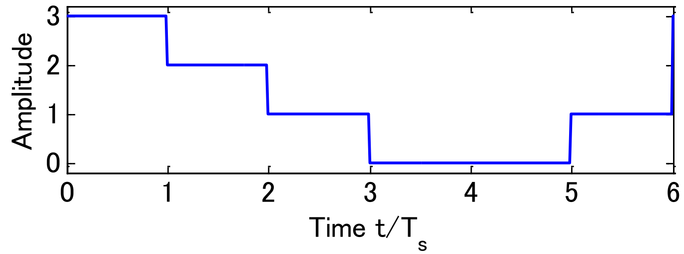
Rate = Good, Error = Excellent, Power = Good, Complexity = Fair

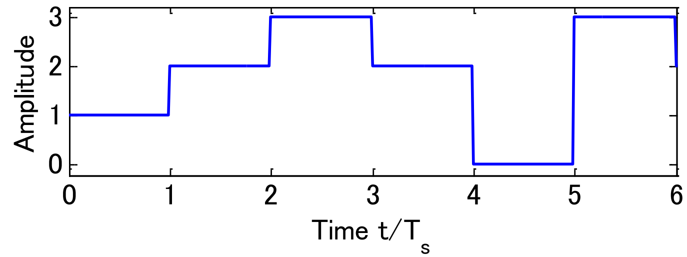
Quadrature Amplitude Modulation (QAM)

Message signal

$M = 16 \longrightarrow$ Two 4-level sequences $m_{\mathrm{I}}(t), m_{\mathrm{Q}}(t)$

Message sequence





Digital modulation & pulse shaping

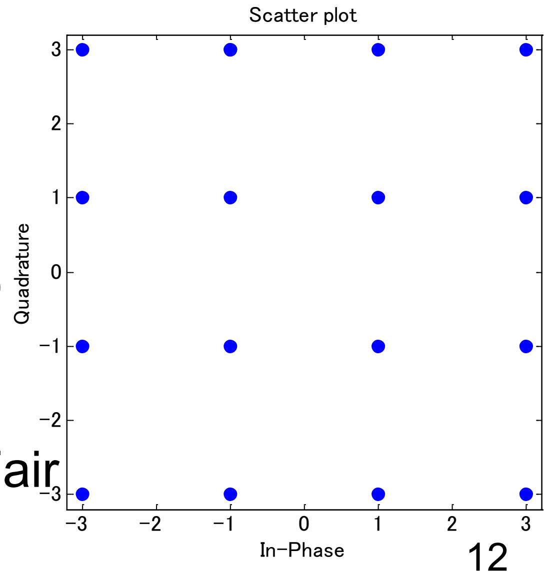
$$s_{\mathrm{D}}(t) = s_{\mathrm{DI}}(t) + j s_{\mathrm{DQ}}(t)$$

$$s_{\mathrm{DI}}(t) = 2m_{\mathrm{I}}(t) - \sqrt{M} + 1 = \sum_n a_{4\mathrm{I}n} \delta(t - nT_{\mathrm{s}})$$

$$s_{\mathrm{DQ}}(t) = 2m_{\mathrm{Q}}(t) - \sqrt{M} + 1 = \sum_n a_{4\mathrm{Q}n} \delta(t - nT_{\mathrm{s}})$$

$$s_{\mathrm{B}}(t) = \sum_n a_{4\mathrm{I}n} g_{\mathrm{rect}}(t - nT_{\mathrm{s}}) + j \sum_n a_{4\mathrm{Q}n} g_{\mathrm{rect}}(t - nT_{\mathrm{s}})$$

Constellation $M = 16$



Analog modulation

$$s(t) = s_{\mathrm{BI}}(t)\cos(2\pi f_0 t) - s_{\mathrm{BQ}}(t)\sin(2\pi f_0 t)$$

$$= \sqrt{s_{\mathrm{BI}}^2(t) + s_{\mathrm{BQ}}^2(t)}\,\cos(2\pi f_0 t + \mathrm{atan}(s_{\mathrm{BI}}(t), s_{\mathrm{BQ}}(t)))$$

Main features

Rate = Excellent, Error = Fair, Power, Fair, Complexity = Fair

Binary Freq. Shift Keying (BFSK)

BPSK modulation

$$\tilde{s}_{\mathrm{D}}(t) = \sum_n a_{2n} g_{\mathrm{rect}}(t - nT_{\mathrm{s}})$$

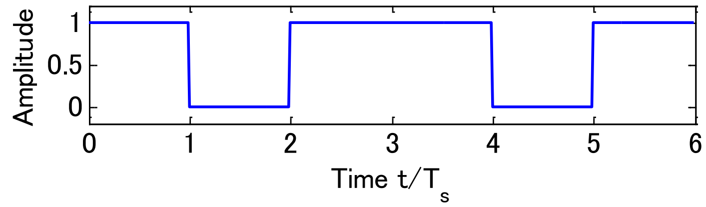
BFSK modulation

$$s_{\mathrm{B}}(t) = \cos(\theta(t)) + j\sin(\theta(t))$$

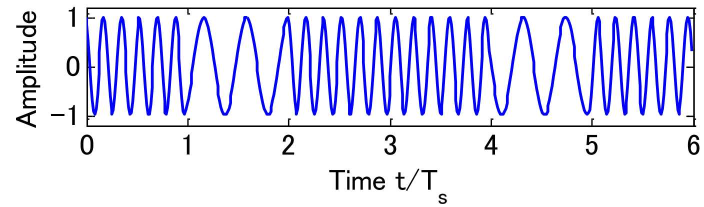
$$\theta(t) = \pi \Delta f \int_{-\infty}^{t} \tilde{s}_{\mathrm{D}}(\tau)\mathrm{d}\tau$$

$$f_{\mathrm{B}}(t) = \frac{1}{2\pi}\frac{\mathrm{d}\theta(t)}{\mathrm{d}t} = \frac{\Delta f}{2}\tilde{s}_{\mathrm{D}}(t)$$

Message sequence



RF signal

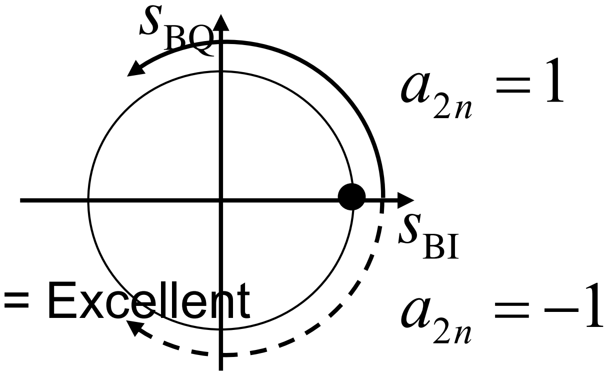


Analog modulation

$$s(t) = \cos(\theta(t))\cos(2\pi f_0 t) - \sin(\theta(t))\sin(2\pi f_0 t)$$

$$= \cos(2\pi f_0 t + \theta(t)) = \cos(2\pi(f_0 + {}^{\Delta f}\!/_{2}\, \tilde{s}_{\mathrm{D}}(t))t)$$

Constellation



Main features

Rate = Fair, Error = Fair, Power = Excellent, Complexity = Excellent

Minimum Shift Keying (MSK)

Differential representation of phase

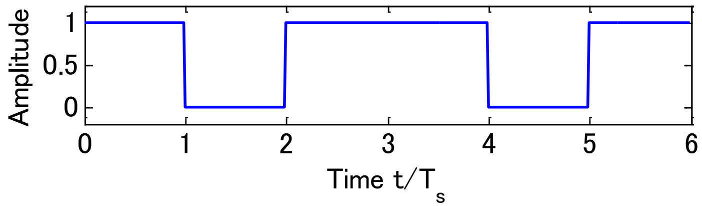
$$\theta(nT_s) = \pi\Delta f \int_{-\infty}^{nT_s} \tilde{s}_D(\tau)\mathrm{d}\tau$$

$$= \pi\Delta f \int_{(n-1)T_s}^{nT_s} \tilde{s}_D(\tau)\mathrm{d}\tau + \theta((n-1)T_s)$$

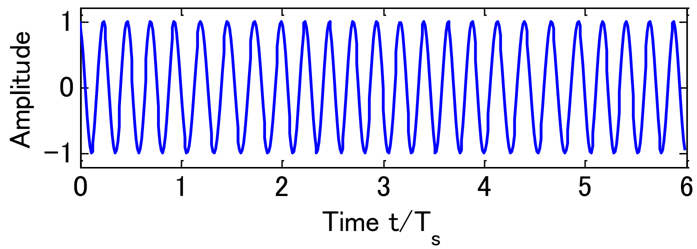
$$= \underline{\pi\Delta f T_s} a_{2n} + \theta((n-1)T_s)$$

Modulation order (BT factor)

Message sequence



RF signal

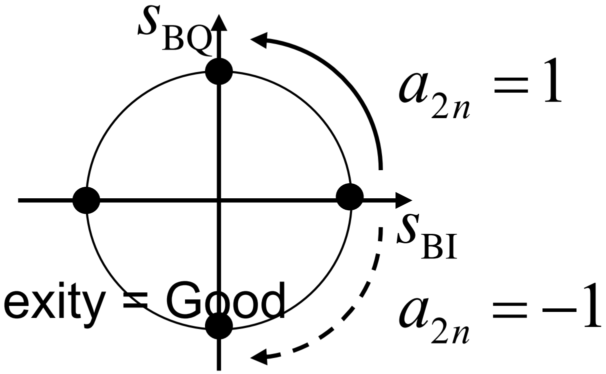


Condition on constant phase difference

$$\pi\Delta f T a_{2n} = k\frac{\pi}{2} \longrightarrow \Delta f T = \frac{k}{2}$$

$$k = 1 \longrightarrow \text{MSK}$$

Constellation

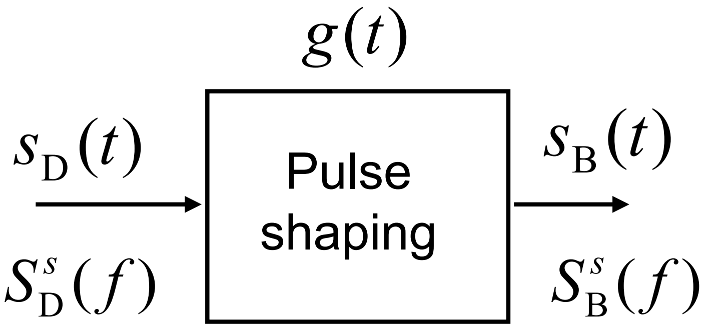


Main features

Rate = Fair, Error = Excellent, Power = Excellent, Complexity = Good

Pulse Shaping Filter

Pulse shaping filter

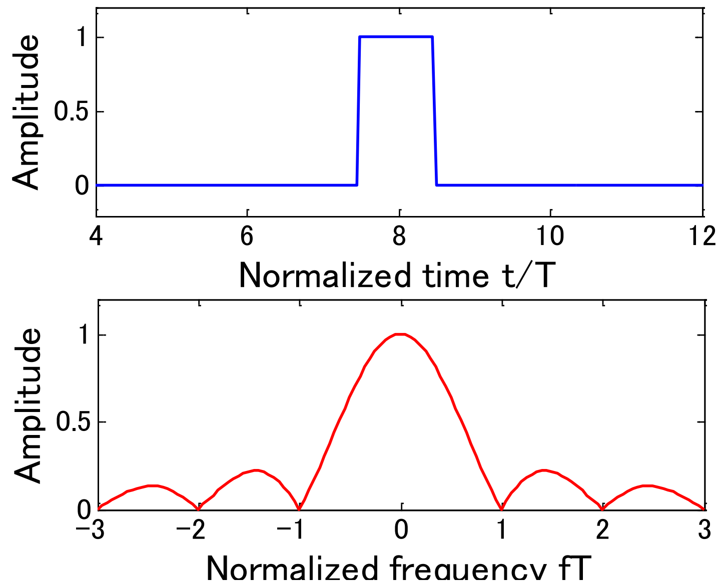


$$s_B(t) = \int g(\tau) s_D(t-\tau)\, d\tau$$

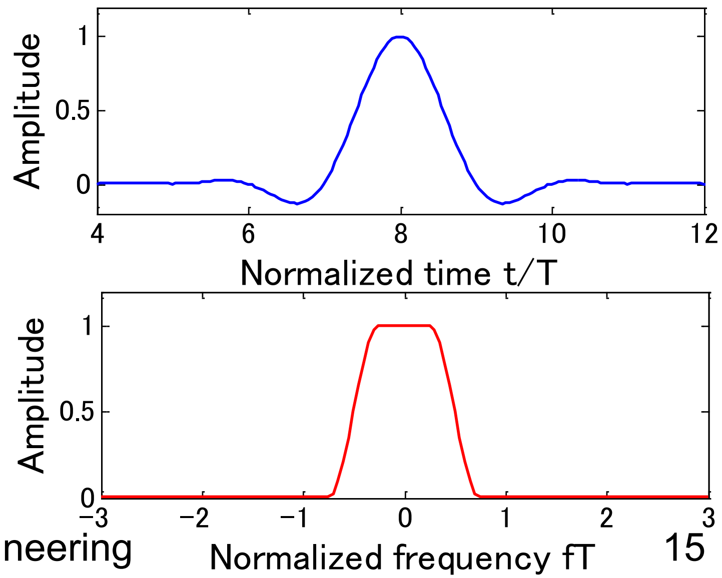
$$S_B^s(f) = |G(f)|^2 S_D^s(f)$$

Trade-off between
bandwidth & inter-symbol interference

Rectangular pulse



Nyquist pulse



Nyquist Pulse

Frequency response

$$G(f)=$$

$$=\begin{cases} 1 & 0 \le |fT_s| < \dfrac{1-\alpha}{2} \\ \dfrac{1}{2}\left(1-\sin\left(\dfrac{\pi}{2\alpha}(2fT_s-1)\right)\right) & \dfrac{1-\alpha}{2} \le |fT_s| < \dfrac{1+\alpha}{2} \\ 0 & \dfrac{1+\alpha}{2} \le |fT_s| \le 1 \end{cases}$$

Role-off factor

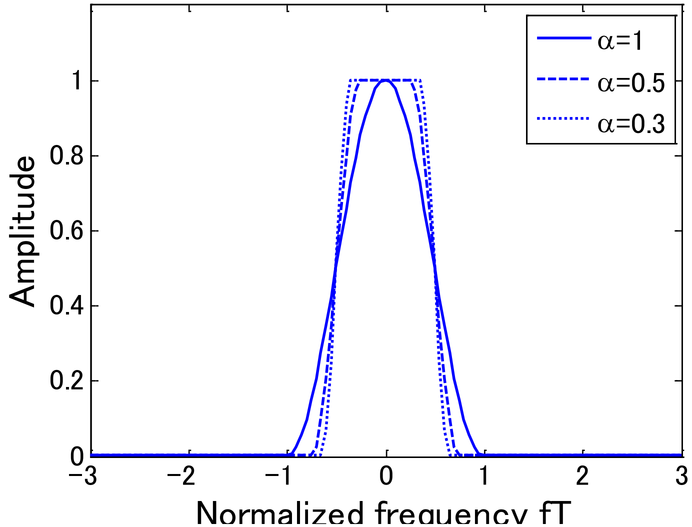
Impulse response

$$g(t)=\frac{\sin\left(\dfrac{\pi t}{T_s}\right)}{\dfrac{\pi t}{T_s}}\,\frac{\cos\left(\dfrac{\alpha \pi t}{T_s}\right)}{1-\left(\dfrac{2\alpha t}{T_s}\right)^2}$$

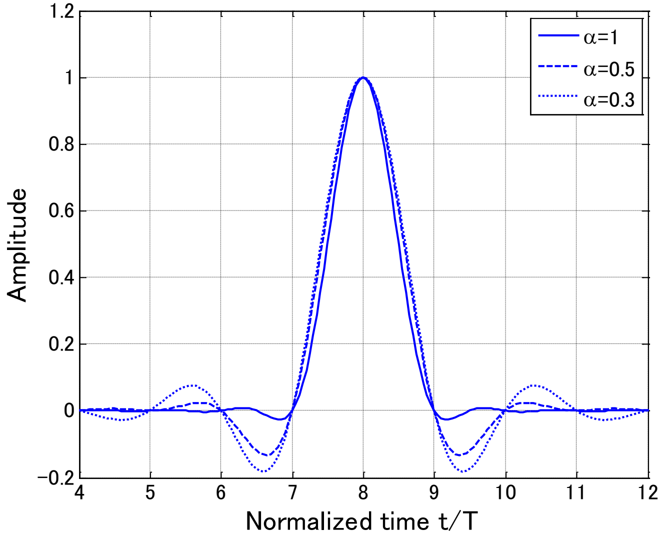
Zero cross every symbol period

No inter-symbol interference

Frequency response



Impulse response



Gaussian Pulse

Frequency response

$$G(f) = \exp\left(-\left(\frac{f}{f_c}\right)^2\right)$$

3dB bandwidth

$$f = \frac{B}{2},\ G(f) = \frac{1}{\sqrt{2}} \longrightarrow f_c = \frac{B}{\sqrt{2\ln 2}}$$

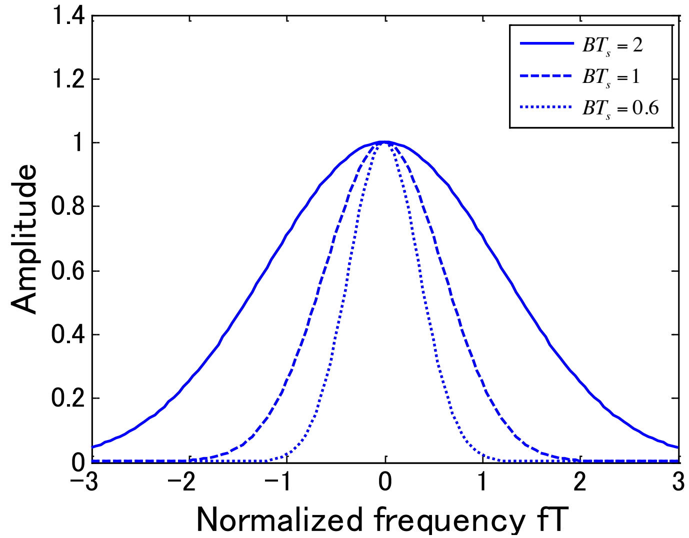
$$G(f) = \exp\left(-2\ln 2\left(\frac{f}{B}\right)^2\right)$$

Impulse response

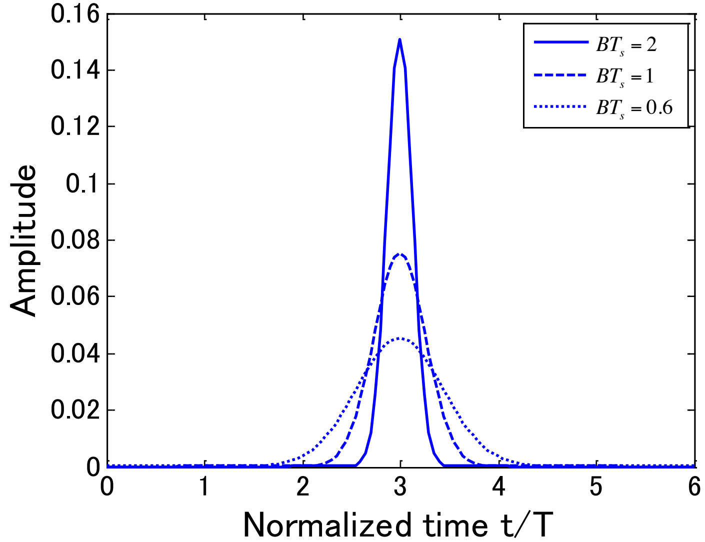
$$g(t) = \sqrt{\frac{\pi}{2\ln 2}}B\exp\left(-\frac{\pi^2}{2\ln 2}(Bt)^2\right)$$

Sharpest response in both freq. & time

Frequency response



Impulse response



Summary

- **Digital modulation**

$$s_D(t) = s_{DI}(t) + j s_{DQ}(t) = f\left(m(t)\right)$$

- Amplitude
- Phase
- Frequency

⇓

Data rate, power efficiency, complexity, error rate

- **Pulse shaping (band limitation)**

$$s_B(t) = \int g(\tau) s_D(t-\tau)\,\mathrm{d}\tau$$

- Rectangular
- Nyquist
- Gaussian

⇓

Bandwidth, error rate

- **IQ analog modulation**

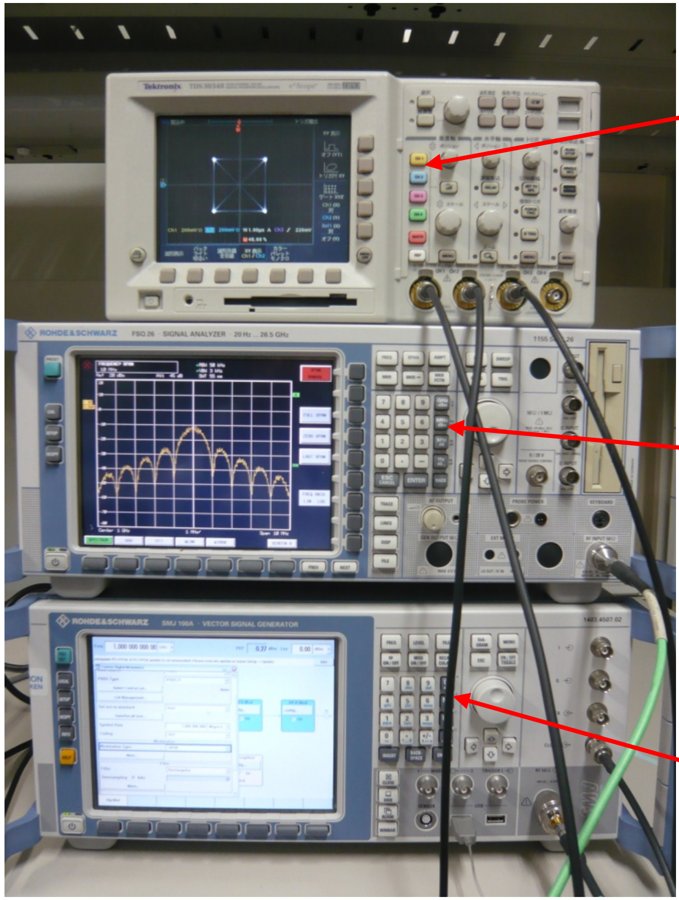
$$s(t) = s_{BI}(t)\cos(2\pi f_0 t) - s_{BQ}(t)\sin(2\pi f_0 t)$$

⇓

Carrier frequency

Experiment

Set up



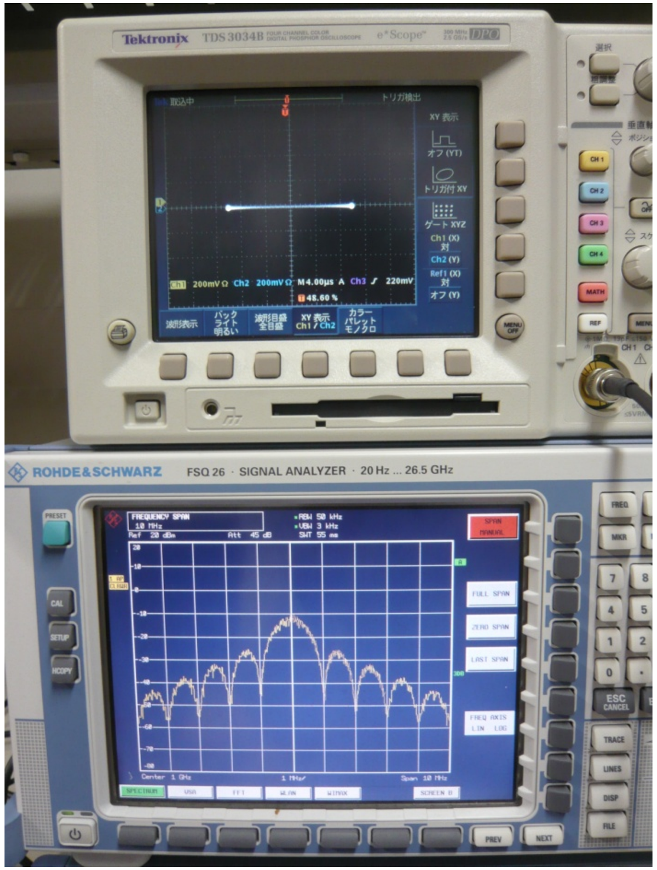
Osilloscope
- TDS 3034B (Tektoronix)
- Time domain analysis
- Constellation (X-Y) analysis

Spectrum analyzer
- FSQ 26（R&S）
- Spectrum analyss

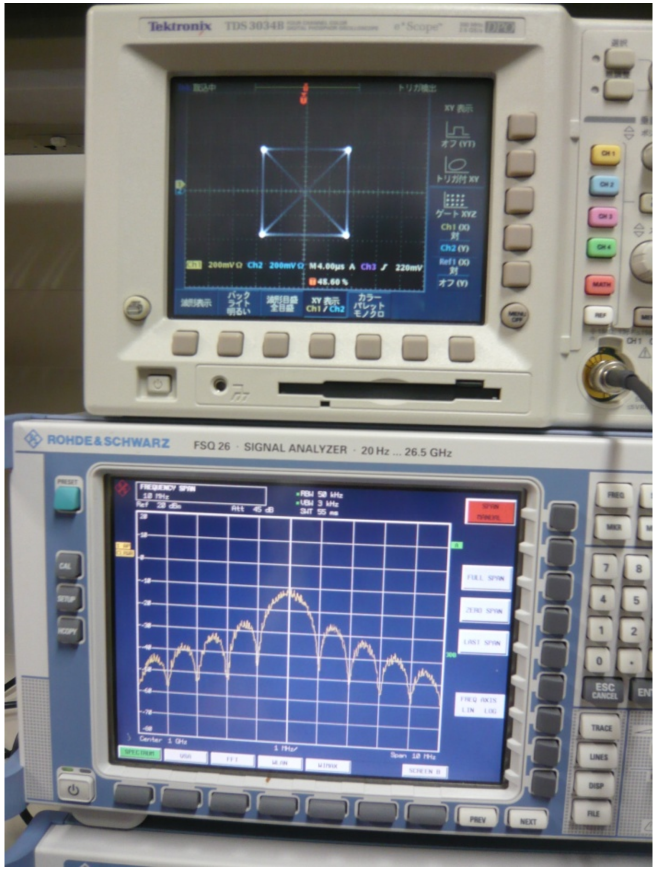
Signal generator
- SMJ 100A（R&S）
- Modulation, pulse shaping

BPSK(QPSK) + Rect. Pulse

BPSK

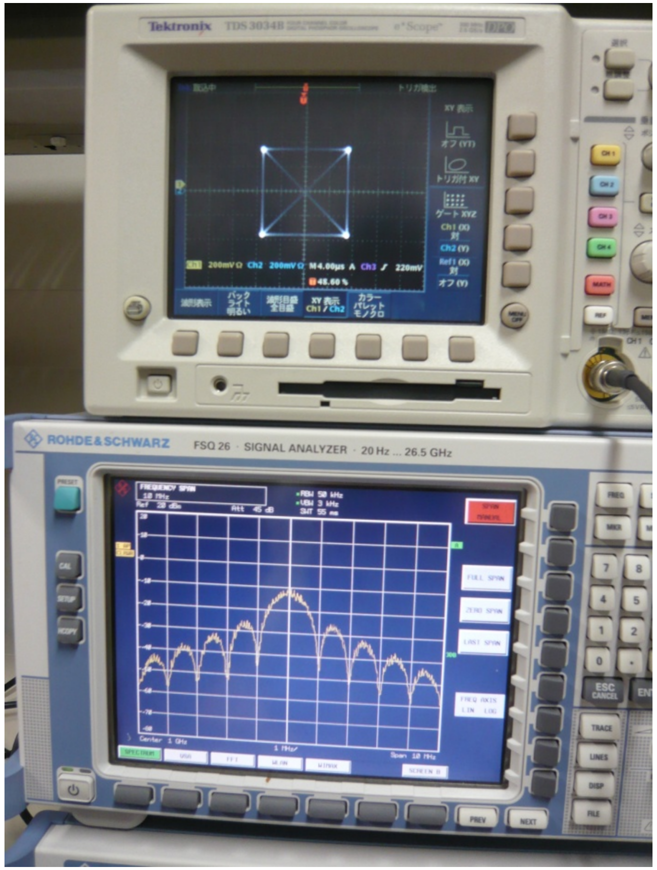


QPSK

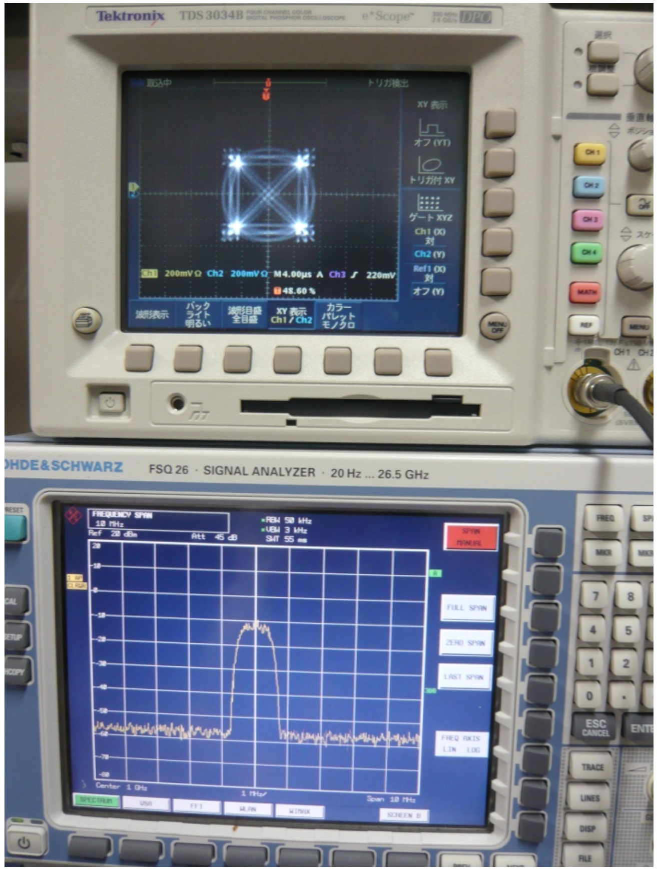


QPSK + Nyquist Pulse

Rectangular pulse

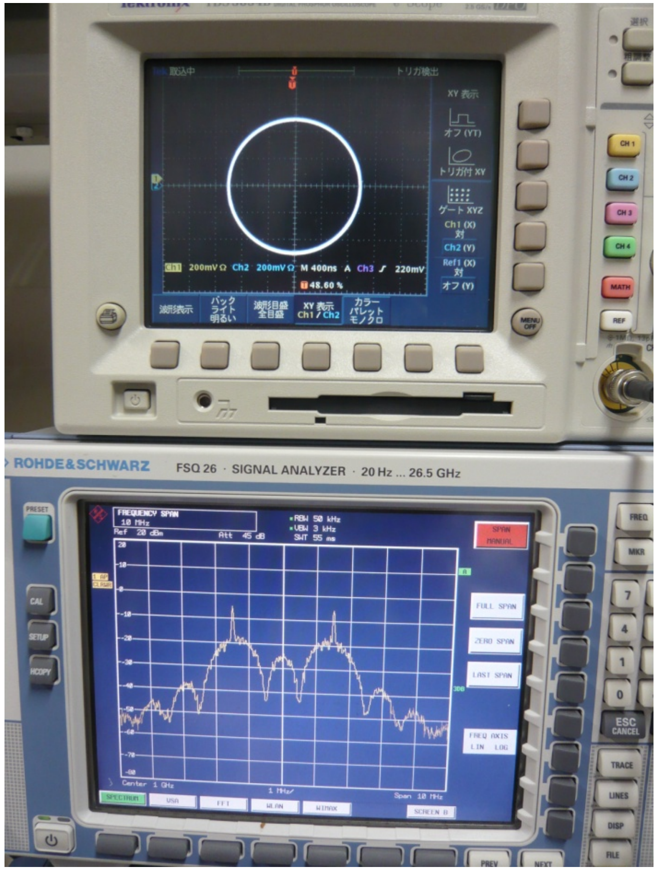


Nyquist pulse

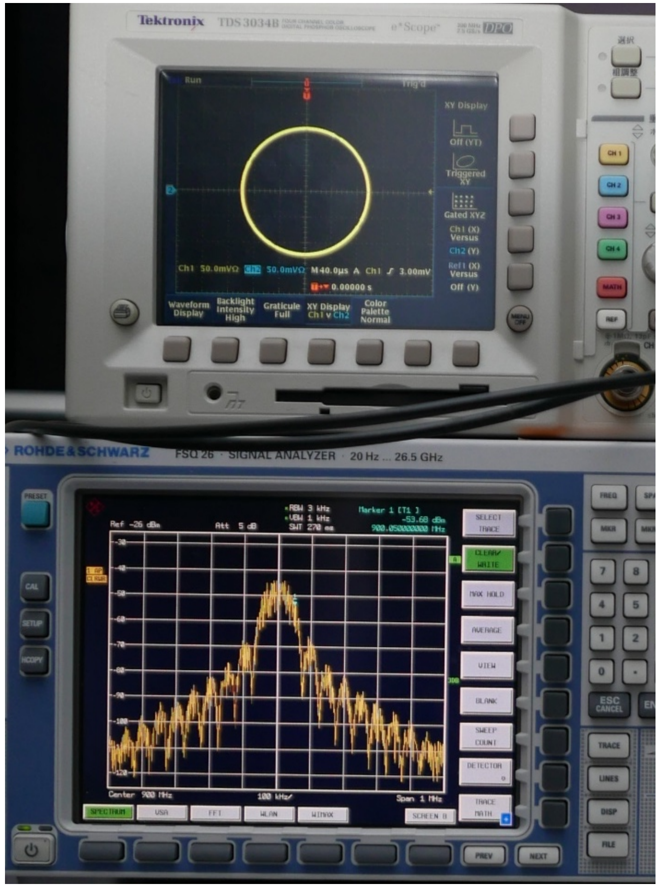


BFSK + Rect. Pulse

BFSK



MSK



MSK + Gaussian Pulse

Nyquist pulse, Gaussian pulse