2018 2Q

Wireless Communication Engineering

# #7 Detection and Error due to Noise

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# Course Schedule (1)

| | Date | Text | Contents |
|---|---|---|---|
| #1 | June 11 | 1, 7 | Introduction to wireless communication systems |
| #2 | June 14 | 2, 5, etc | Link budget design of wireless access |
| #3 | June 18 | | Up/down conversion and equivalent baseband system |
| #4 | June 21 | 3.3, 3.4 | Digital modulation and pulse shaping |
| #5 | June 25 | 3.5 | Demodulation and matched filter |
| #6 | June 28 | | Collaborative exercise for better understanding 1 |
| #7 | July 2 | 3.5 | Detection and error due to noise |
| #8 | July 5 | 4.4 | Channel fading and diversity combining |

# From Previous Lecture

- Analog demodulation

$$y(t) = y_{\mathrm{BI}}(t)\cos 2\pi f_0 t - y_{\mathrm{BQ}}(t)\sin 2\pi f_0 t$$

$$y_{\mathrm{BI}}(t) = \mathrm{LPF}\left[2y(t)\cos 2\pi f_0 t\right] \quad y_{\mathrm{BQ}}(t) = \mathrm{LPF}\left[-2y(t)\sin 2\pi f_0 t\right]$$

- Matched filter

$$P_{\mathrm{s}} = \left|\int G_{\mathrm{r}}(f)G_{\mathrm{s}}(f)\,\mathrm{d}f\right|^2 \le \int \left|G_{\mathrm{r}}(f)\right|^2 \mathrm{d}f \int \left|G_{\mathrm{s}}(f)\right|^2 \mathrm{d}f$$

$$G_{\mathrm{r}}(f) = \left(G_{\mathrm{s}}(f)\right)^* \quad \Leftrightarrow \quad g_{\mathrm{r}}(t) = g_{\mathrm{s}}(-t)$$

- Output SNR

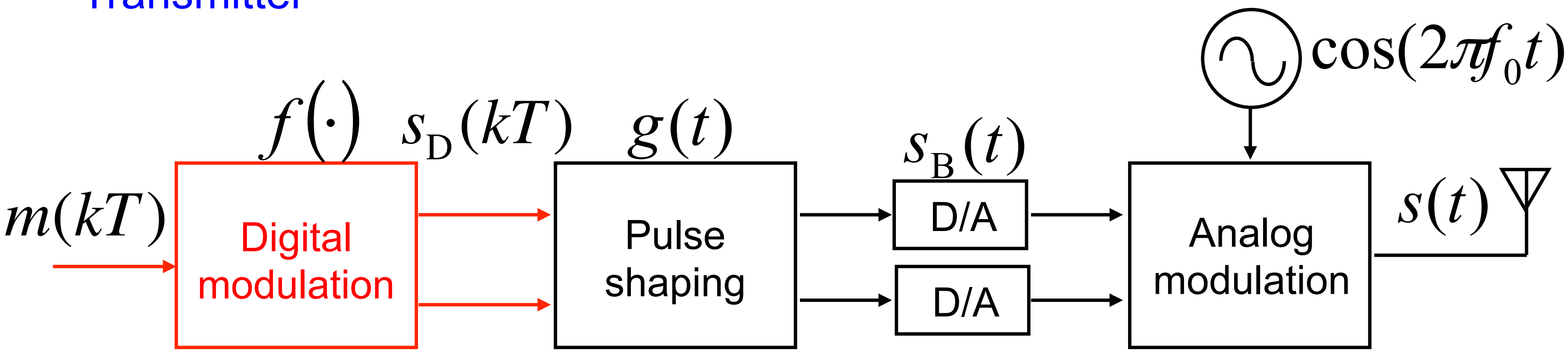
$$\gamma_{\max} = \frac{1}{N_0}\int \left|G_{\mathrm{s}}(f)\right|^2 \mathrm{d}f = \frac{E_{\mathrm{s}}}{N_0} = \frac{P_{\mathrm{s}}}{\sigma^2}$$

# Contents

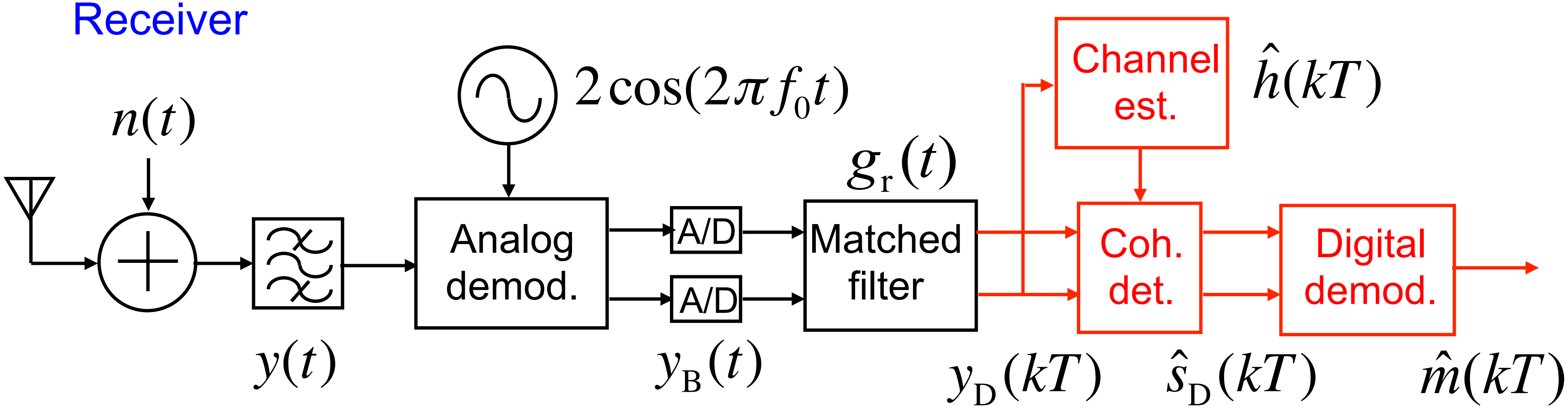
- QAM modulation
- Coherent detection
- Error rate of BPSK signal
- Error rate of QPSK signal
- Error rate of QAM signal
- Demonstration

# Transmitter & Receiver

Transmitter

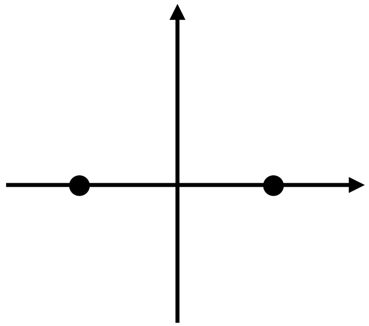



Receiver



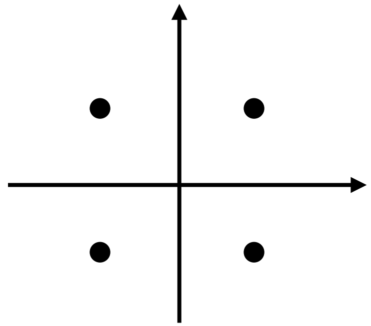
# QAM Modulation

BPSK



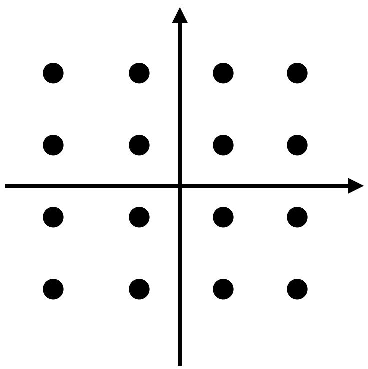
$$\log_2 M = 1$$

QPSK



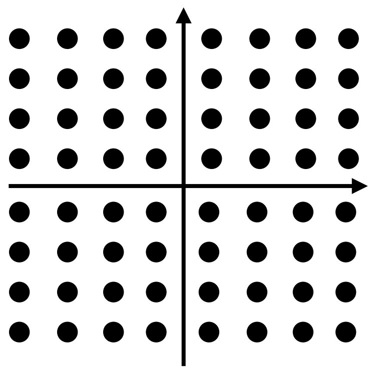
$$\log_2 M = 2$$

16QAM



$$\log_2 M = 4$$

64QAM

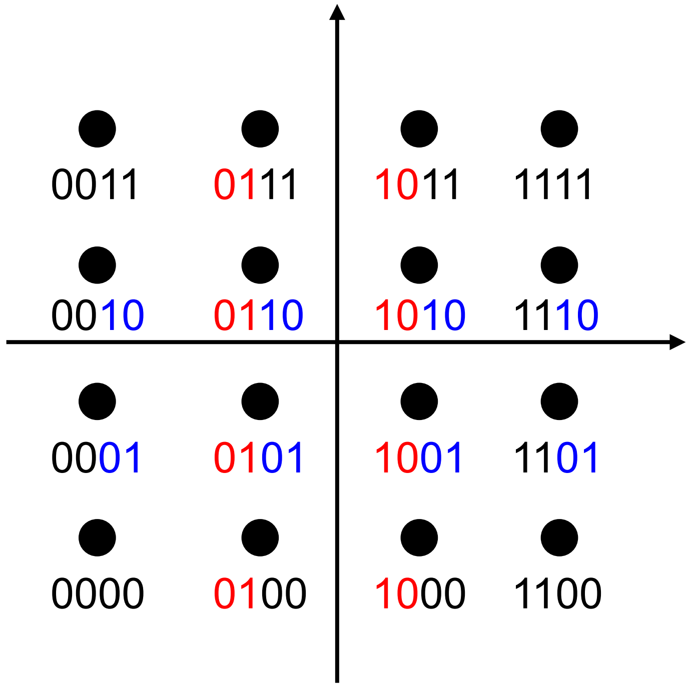


$$\log_2 M = 6$$

# Gray Coding (Mapping)

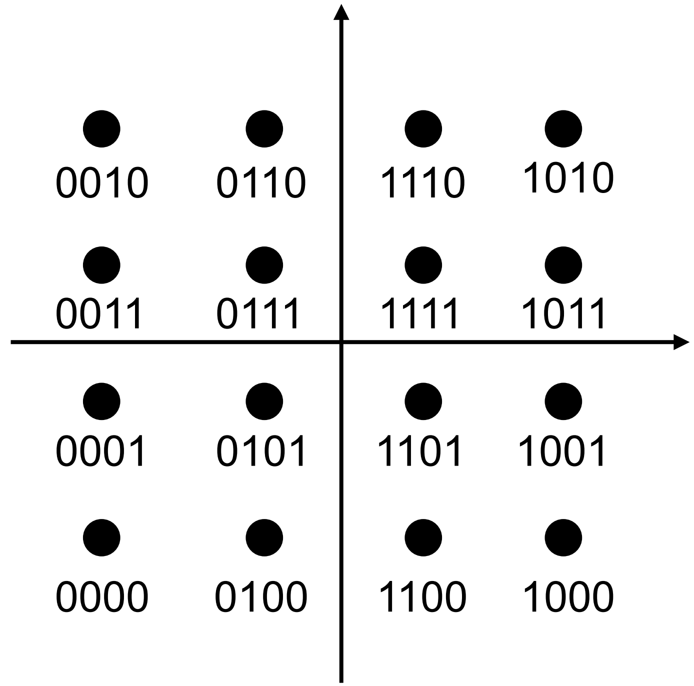
Binary Mapping

00, 01, 10, 11




Gray Mapping

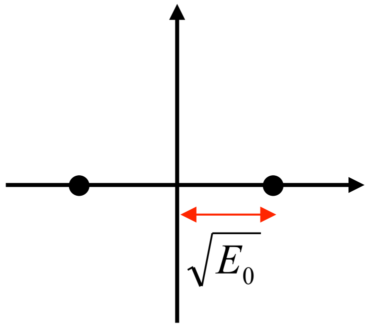
00, 01, 11, 10



# Average Power with Fixed $E_0$

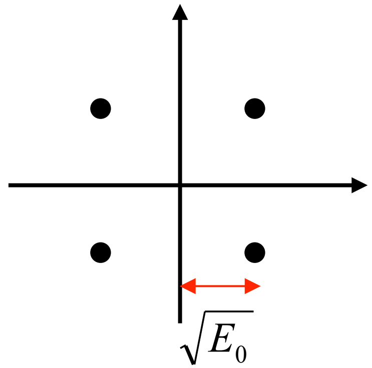
BPSK

$$P_2 = E_0$$



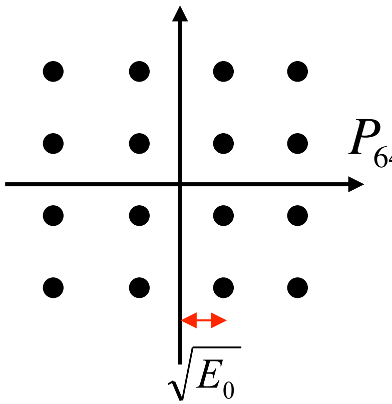

QPSK

$$P_4 = 2E_0$$



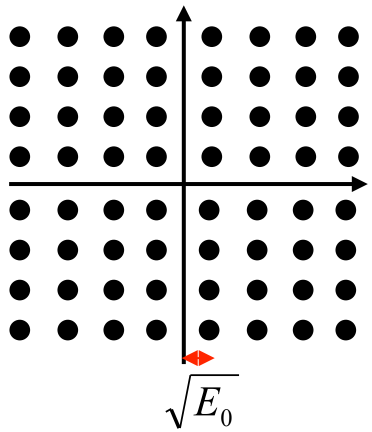

16QAM

$$P_{16} = 2 \times \frac{1}{2}\left(E_0 + 3^2 E_0\right) = 10E_0$$




64QAM

$$P_{64} = 2 \times \frac{1}{4}\sum_{i=1}^{4}\left(2i-1\right)^2 E_0 = 42E_0$$

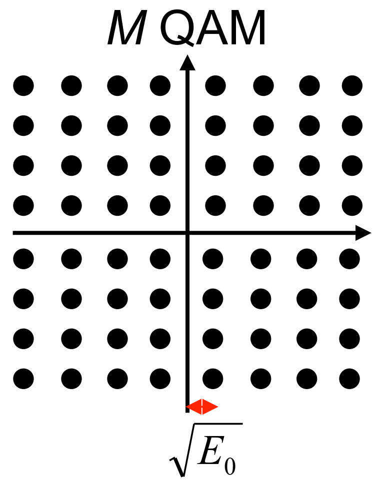


# Power Normalization

Average power with fixed $E_0$

$$\tilde{P}_M = 2\left(\frac{2}{\sqrt{M}}\sum_{i=1}^{\sqrt{M}/2}(2i-1)^2 E_0\right)$$

$$= \frac{2(M-1)E_0}{3} \qquad (M \geq 4)$$




Power normalization

Constant average power

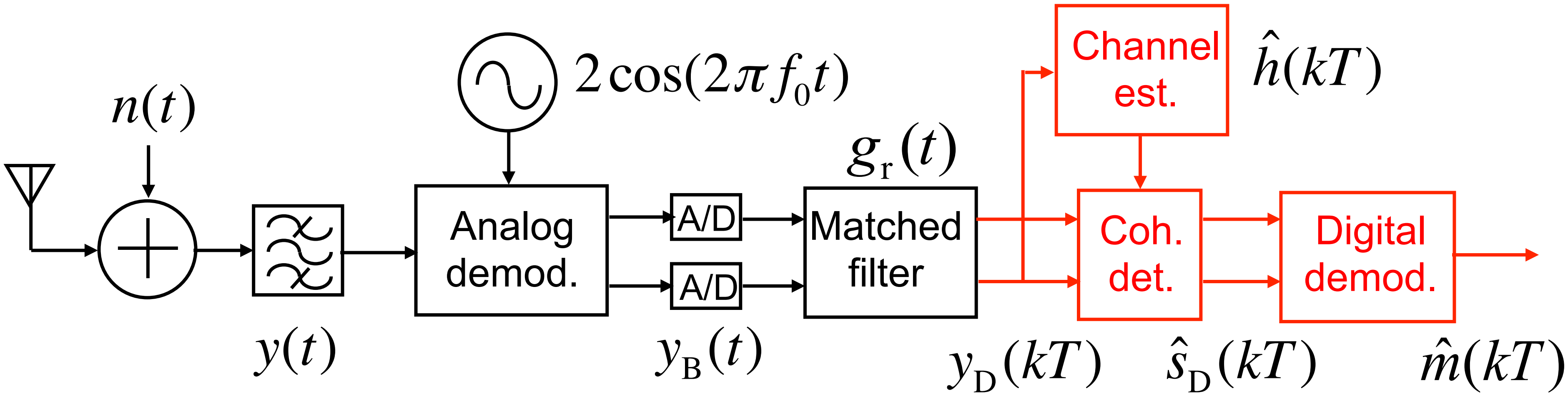
$$\tilde{P}_M = P_s \longrightarrow$$

$E_0$ is inversely proportional to $M$

$$E_0 = \frac{3}{2(M-1)}P_s \qquad (M \geq 4)$$

$$E_0 = P_s \qquad (M = 2)$$

# Output of Matched Filter

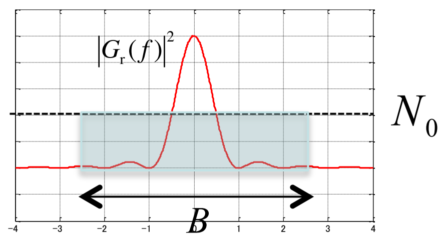



Output of matched filter

$$y_D(t) = g_r(t) \otimes g_s(t) h_B s_D(t) + g_r(t) \otimes n_B(t) = g(t) \otimes h_B s_D(t) + n_D(t)$$

Normalized matched filter

$$g(0) = \int G(f) df = \int G_r(f) G_s(f) df = \int |G_r(f)|^2 \, d f = 1$$



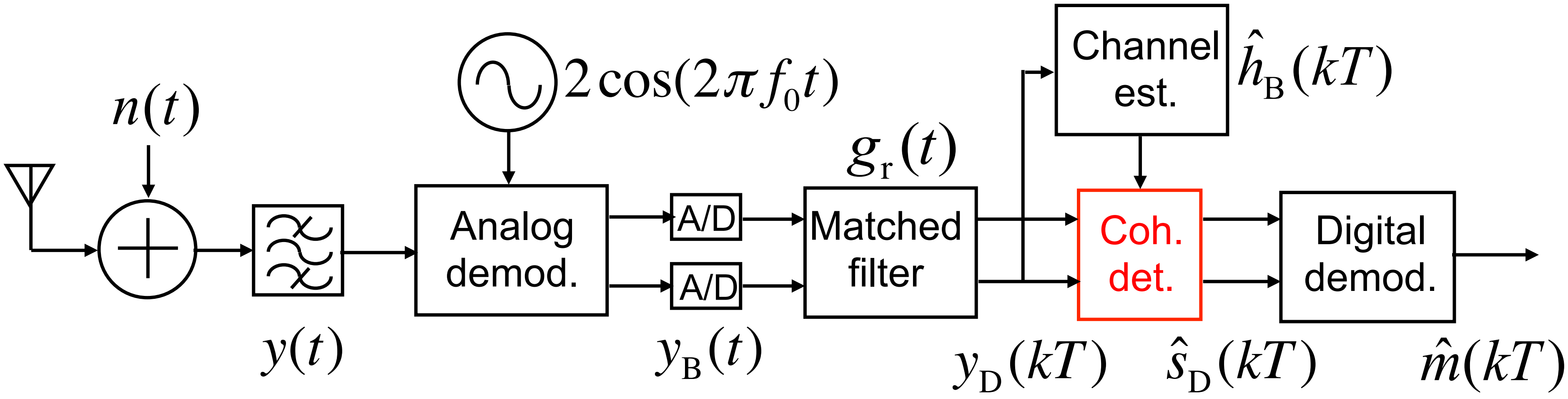

If bandwidth of matched filter < *B*

$$y_D(k) = h_B s_D(k) + n_D(k), \quad \mathrm{E}[|n_D(k)|^2] = \mathrm{E}[|n_B(k)|^2] = N_0 B = \sigma^2$$

# Detection Schemes

| Detection | Modulation | Demodulation |
| --- | --- | --- |
| Envelope | ASK | $\hat{s}_{\mathrm{D}}(k) = \left\lvert y_{\mathrm{D}}(k) \right\rvert \big/ \left\lvert h_{\mathrm{B}} \right\rvert$ |
| Correlation | FSK | $\left\lvert \int y_{\mathrm{B}}(t)\exp(j\pi\Delta f t)\mathrm{d}t \right\rvert \gtrless \left\lvert \int y_{\mathrm{B}}(t)\exp(-j\pi\Delta f t)\mathrm{d}t \right\rvert$ |
| Coherent | PSK, QAM, MSK | $\hat{s}_{\mathrm{D}}(t) = y_{\mathrm{D}}(t) \big/ h_{\mathrm{B}}$ |

# Coherent Detection

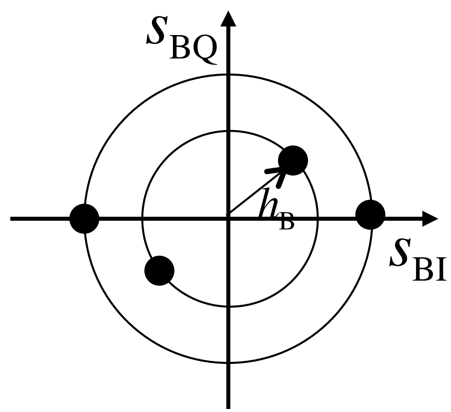



## Output of matched filter

$$y_{\mathrm{D}}(k) = h_{\mathrm{B}} s_{\mathrm{D}}(k) + n_{\mathrm{D}}(k)$$

$$\Rightarrow \quad y(k) = h_{\mathrm{B}} s(k) + n(k)$$

Constellation




## Coherent detection

$$\hat{s}(k) = y(k) / \hat{h}_{\mathrm{B}}$$

Compensation of channel response

## Digital demodulation

$$\hat{m}(k) = f^{-1}\left(\hat{s}(k)\right)$$

for each modulation method of PSK, QAM, MSK

# Channel Estimation

Output of matched filter

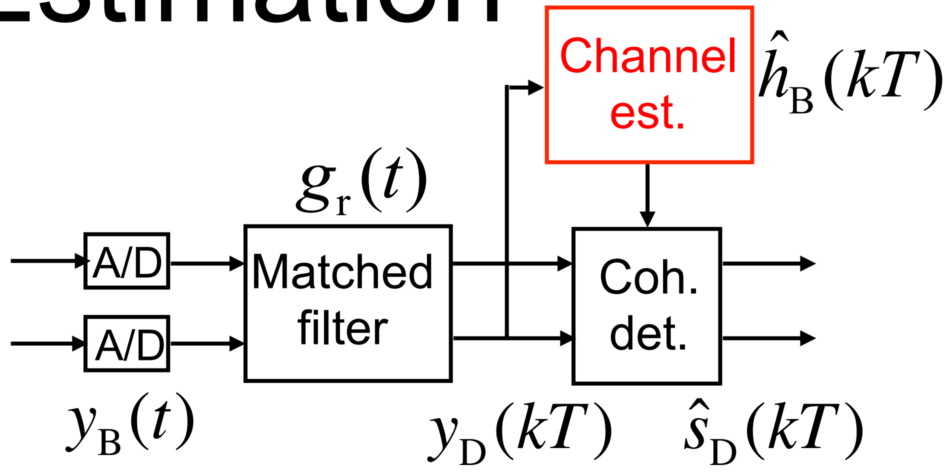
$$y(k) = h_B s_{TR}(k) + n(k)$$

Training signal

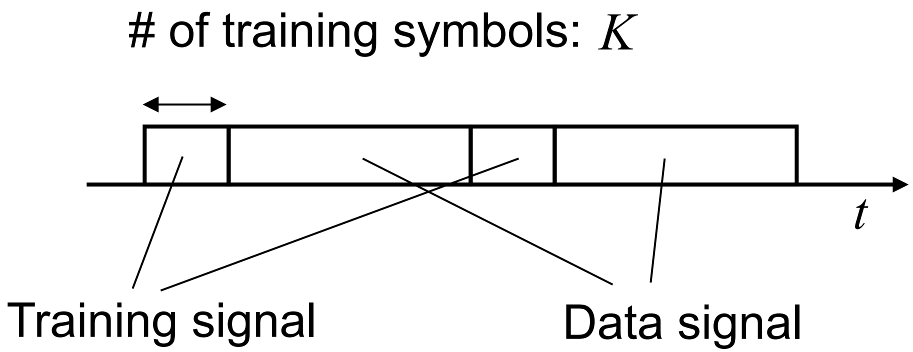
Channel estimation

$$\tilde{h}_B = y(k) \big/ s_{TR}(k)$$

$$\hat{h}_B = \frac{1}{K} \sum_{k=1}^{K} \tilde{h}_B(k)$$




Frame structure of transmit signal

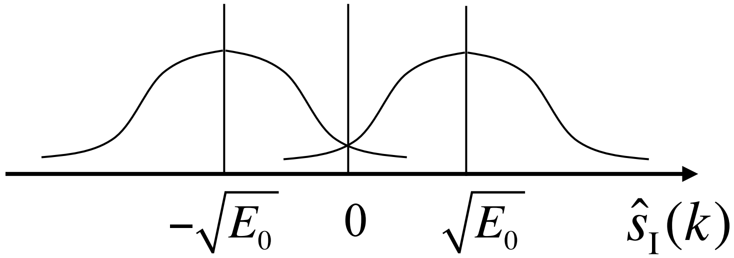


# Output of Coherent Detection

Output of coherent detection

$$\hat{s}(k) = y(k)/h_B = s(k) + n(k)/h_B$$

Complex Gaussian with variance $\sigma^2/|h_B|^2$



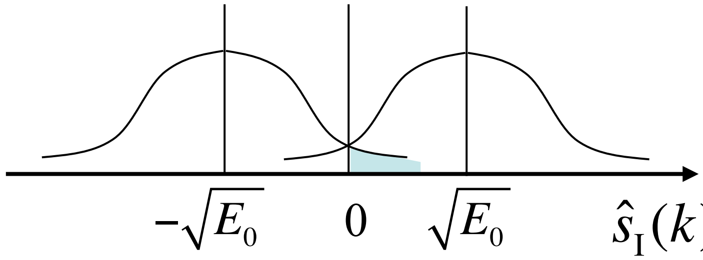

Probability Density Function (PDF)

$$p(\hat{s}_I) = \begin{cases} \dfrac{1}{\sqrt{\pi\sigma^2/|h_B|^2}} \exp\left(-\dfrac{(\hat{s}_I - \sqrt{E_0})^2}{\sigma^2/|h_B|^2}\right), & m = 0 \\ \dfrac{1}{\sqrt{\pi\sigma^2/|h_B|^2}} \exp\left(-\dfrac{(\hat{s}_I + \sqrt{E_0})^2}{\sigma^2/|h_B|^2}\right), & m = 1 \end{cases}$$

# Error Rate of BPSK Signal

Pairwise error rate

$$p_{\rm pw} = \frac{1}{\sqrt{\pi\sigma^2/|h_{\rm B}|^2}} \int_0^\infty \exp\left(-\frac{(\hat{s}_{\rm I}+\sqrt{E_0})^2}{\sigma^2/|h_{\rm B}|^2}\right) d\hat{s}_{\rm I}$$




$$= \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_0|h_{\rm B}|^2}{\sigma^2}}\right)$$

Complementary error function

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty \exp(-z^2)\,dz$$

Error rate of BPSK signal

$$E_0 = P_s$$

$$p_{\rm eb} = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{P_s|h_{\rm B}|^2}{\sigma^2}}\right) = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\gamma}\right)$$

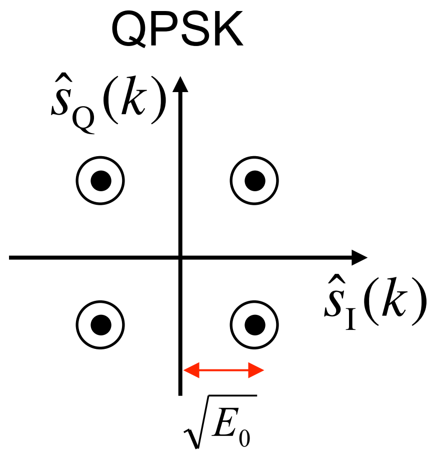
Receive SNR

$$\gamma = \frac{P_s|h_{\rm B}|^2}{\sigma^2}$$

# Error Rate of QPSK Signal

Output of coherent detection

$$\hat{s}(k) = s_{\mathrm{I}}(k) + js_{\mathrm{Q}}(k) + n(k)/h_{\mathrm{B}}$$




Bit error rate

$$p_{\mathrm{pw}} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{E_0 |h_{\mathrm{B}}|^2}{\sigma^2}}\right)$$

$$E_0 = P_s/2$$

$$p_{\mathrm{eb}} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{P_{\mathrm{s}} |h_{\mathrm{B}}|^2}{2\sigma^2}}\right) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{\gamma}{2}}\right)$$

Proportional to SNR per bit

Symbol error rate

$$p_{\mathrm{es}} = 1 - \underbrace{(1 - p_{\mathrm{eb}})}_{\mathrm{I}}\underbrace{(1 - p_{\mathrm{eb}})}_{\mathrm{Q}} = 2p_{\mathrm{eb}} - p_{\mathrm{eb}}^2 \cong 2p_{\mathrm{eb}}$$

# Error Rate of QAM Signal

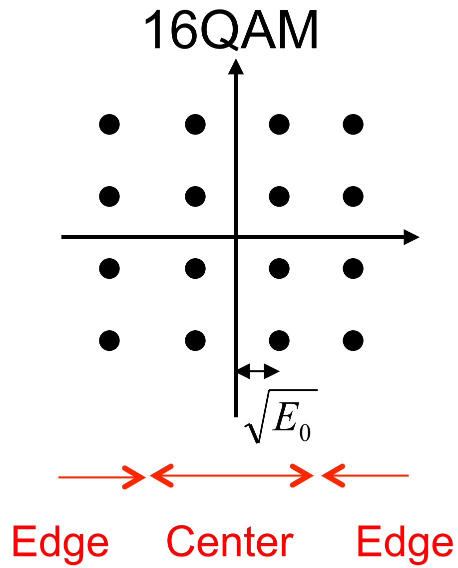
Symbol error rate

$$p_{\mathrm{es}} = 1 - \left(1 - p_{\mathrm{esI}}\right)\left(1 - p_{\mathrm{esQ}}\right) \cong 2p_{\mathrm{esI}} = 2p_{\mathrm{esQ}}$$

$$p_{\mathrm{esI}} = \left(\frac{\sqrt{M} - 2}{\sqrt{M}}\right) \times 2p_{\mathrm{pw}} + \left(\frac{2}{\sqrt{M}}\right) \times p_{\mathrm{pw}}$$

Center Two edges

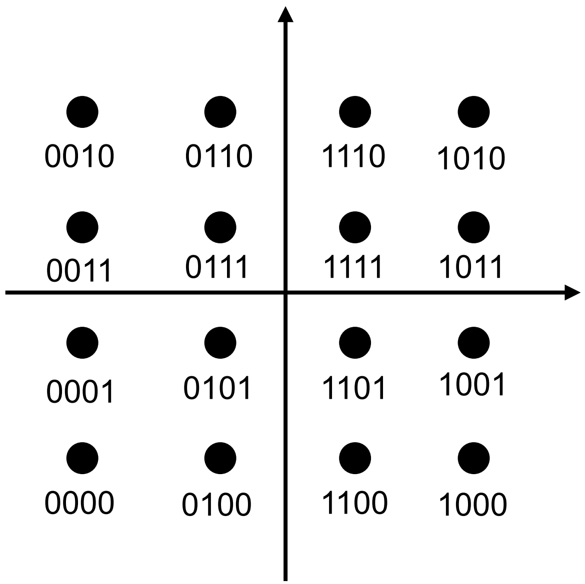
$$= \left(1 - \frac{1}{\sqrt{M}}\right) \mathrm{erfc}\left(\sqrt{\frac{E_0 \left|h_{\mathrm{B}}\right|^2}{\sigma^2}}\right)$$



# Error Rate of QAM Signal

Symbol error rate

$$p_{\mathrm{es}} = 2\left(1 - \frac{1}{\sqrt{M}}\right)\mathrm{erfc}\left(\sqrt{\frac{E_0 \left|h_{\mathrm{B}}\right|^2}{\sigma^2}}\right)$$




Bit error rate

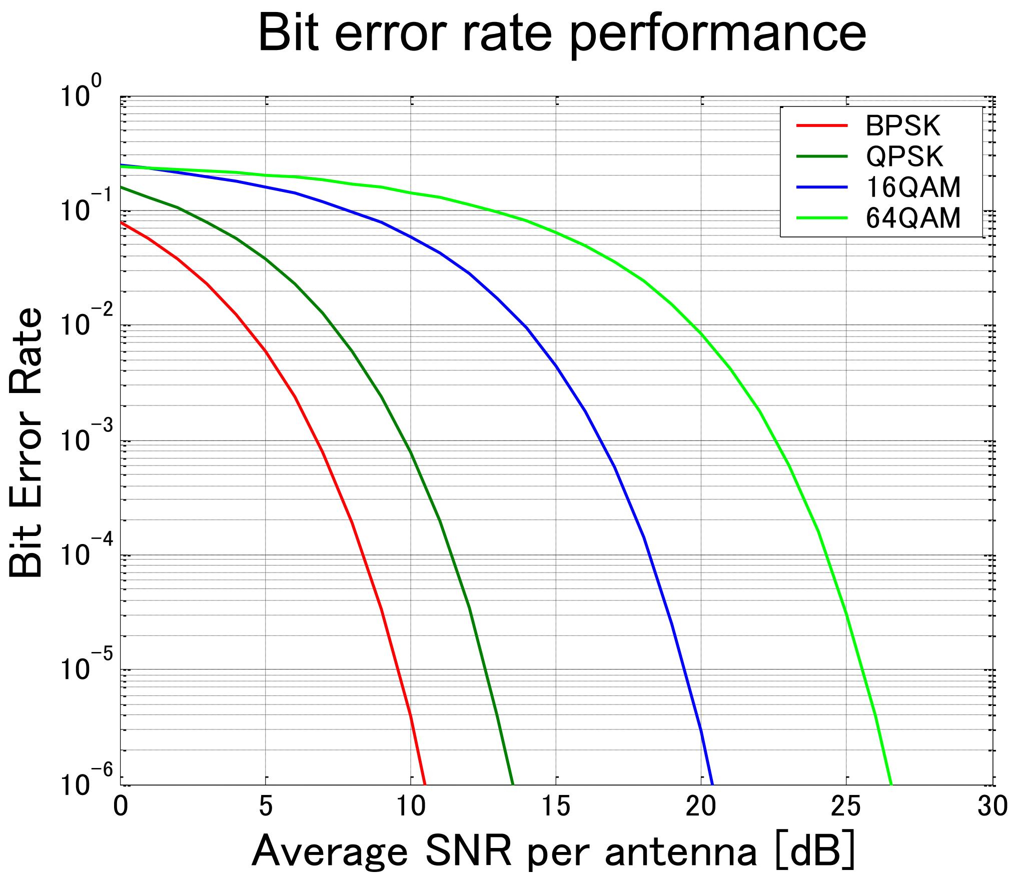
$$E_0 = \frac{3}{2(M-1)} P_s$$

$$p_{\mathrm{eb}} \cong \frac{1}{\log_2 M} p_{\mathrm{es}}$$

Symbol error corresponds to one bit error owing to Gray coding

$$= \frac{2}{\log_2 M}\left(1 - \frac{1}{\sqrt{M}}\right)\mathrm{erfc}\left(\sqrt{\frac{3\gamma}{2(M-1)}}\right)$$

# Error Rate Performance



# Summary

- Channel estimation & coherent detection

$$\hat{h}_{\mathrm{B}} = \frac{1}{K}\sum_{k=1}^{K} y(k) \Big/ s_{\mathrm{TR}}(k) \quad \longrightarrow \quad \hat{s}(k) = y(k)/h_{\mathrm{B}} = s(k) + n(k)/h_{\mathrm{B}}$$

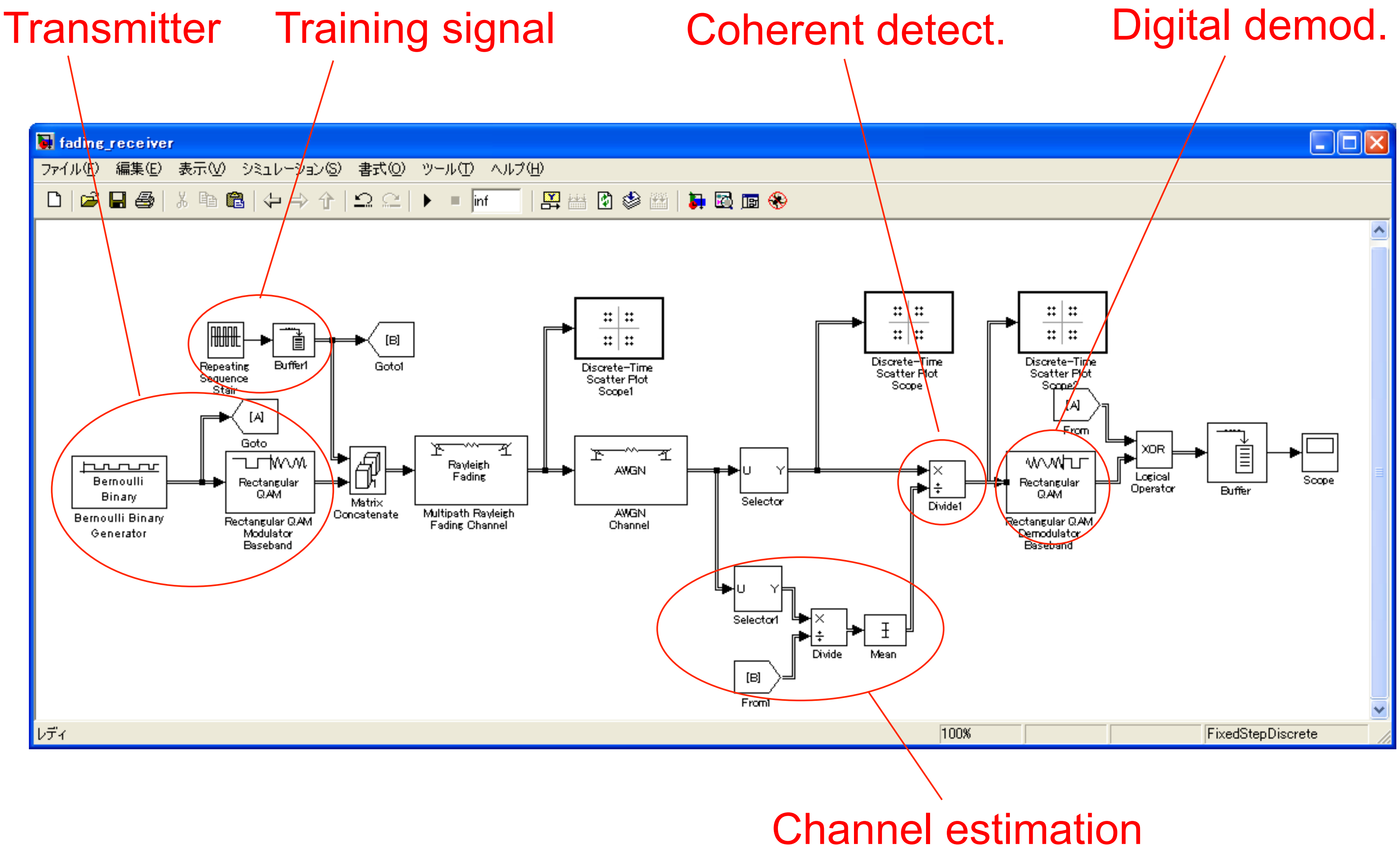
- Error rate of BPSK signal

$$p_{\mathrm{eb}} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{P_{\mathrm{s}}\left|h_{\mathrm{B}}\right|^2}{\sigma^2}}\right) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\gamma}\right)$$

- Error rate of QAM signal

$$p_{\mathrm{eb}} = \frac{2}{\log_2 M}\left(1 - \frac{1}{\sqrt{M}}\right)\mathrm{erfc}\left(\sqrt{\frac{3\gamma}{2(M-1)}}\right)$$

# Demo


Transmitter
Training signal
Coherent detect.
Digital demod.
Channel estimation
fading_receiver
ファイル(F) 編集(E) 表示(V) シミュレーション(S) 書式(O) ツール(T) ヘルプ(H)
inf
Repeating Sequence Stair
Buffer1
[B]
Goto1
[A]
Goto
Bernoulli Binary
Bernoulli Binary Generator
Rectangular QAM
Rectangular QAM Modulator Baseband
Matrix Concatenate
Rayleigh Fading
Multipath Rayleigh Fading Channel
Discrete-Time Scatter Plot Scope1
AWGN
AWGN Channel
U Y
Selector
Discrete-Time Scatter Plot Scope
Divide1
Selector1
Divide
Mean
[B]
From1
Discrete-Time Scatter Plot Scope2
[A]
From
Rectangular QAM Demodulator Baseband
XOR
Logical Operator
Buffer
Scope
レディ
100%
FixedStepDiscrete

# Error Rate of MSK

Output of coherent detection

$$\hat{s}(k) = s(k) + n(k)/h_{\mathrm{B}}$$

MSK modulation

$$s(k) = \exp(j\theta(k))$$

$$\theta(k) = \pi a_k/2 + \theta(k-1)$$

BER of MSK signal

$$p_{\mathrm{eb}} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{P_s|h_{\mathrm{B}}|^2}{\sigma^2}}\right) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\gamma}\right)$$