2018 2Q

Wireless Communication Engineering

#5 Demodulation and Matched Filter

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June 25, 2018

Course Schedule (1)

	Date	Text	Contents
#1	June 11	1, 7	Introduction to wireless communication systems
#2	June 14	2, 5, etc	Link budget design of wireless access
#3	June 18		Up/down conversion and equivalent baseband system
#4	June 21	3.3, 3.4	Digital modulation and pulse shaping
#5	June 25	3.5	Demodulation and matched filter
#6	June 28		Collaborative exercise for better understanding 1
#7	July 2	3.5	Detection and error due to noise
#8	July 5	4.4	Channel fading and diversity combining

From Previous Lecture

- Digital modulation

$$s_{\mathrm{D}}(t) = s_{\mathrm{DI}}(t) + js_{\mathrm{DQ}}(t) = f\big(m(t)\big)$$

- Amplitude
- Phase
- Frequency

Data rate, power efficiency, complexity, error rate

- Pulse shaping (band limitation)

$$s_{\mathrm{B}}(t) = \int g(\tau)s_{\mathrm{D}}(t-\tau)\,\mathrm{d}\tau$$

- Rectangular
- Nyquist
- Gaussian

Bandwidth, error rate

- IQ analog modulation

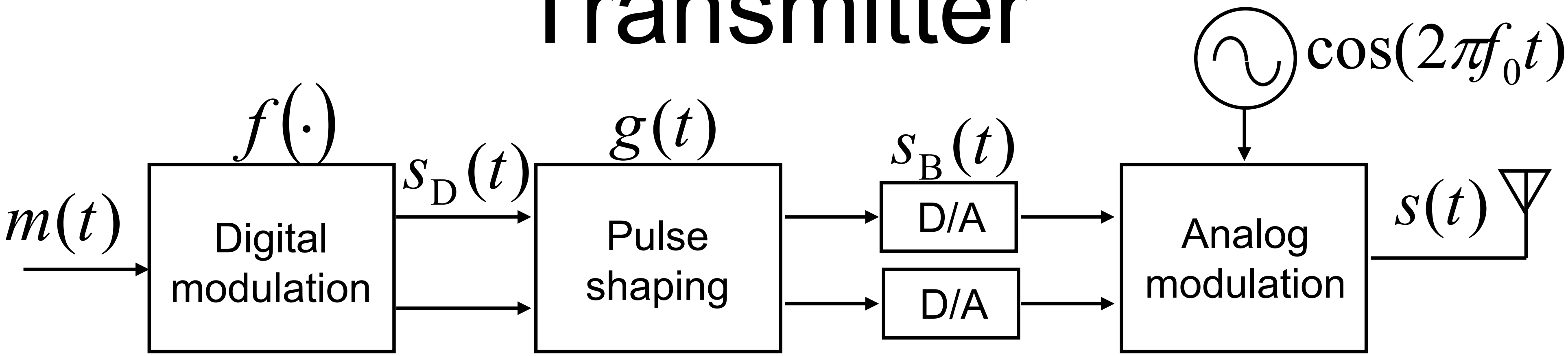
$$s(t) = s_{\mathrm{BI}}(t)\cos(2\pi f_{\mathrm{c}}t) - s_{\mathrm{BQ}}(t)\sin(2\pi f_{\mathrm{c}}t)$$

Carrier frequency

Contents

- Structure of receiver
- Analog demodulation
- Noise through analog demodulator
- Receive sampling
- Matched filter
- Output SNR

Transmitter



Carrier freq. f_0

Coverage $d_0 = \sqrt{\frac{A_t P_t}{4\pi\alpha N_0 f_0 \gamma_0}}$

Symbol length T_s

Bandwidth $B \propto 1/T_s$

Modulation order M

Data rate $\frac{\log_2 M}{T_s}$

Modulation $f(\cdot)$

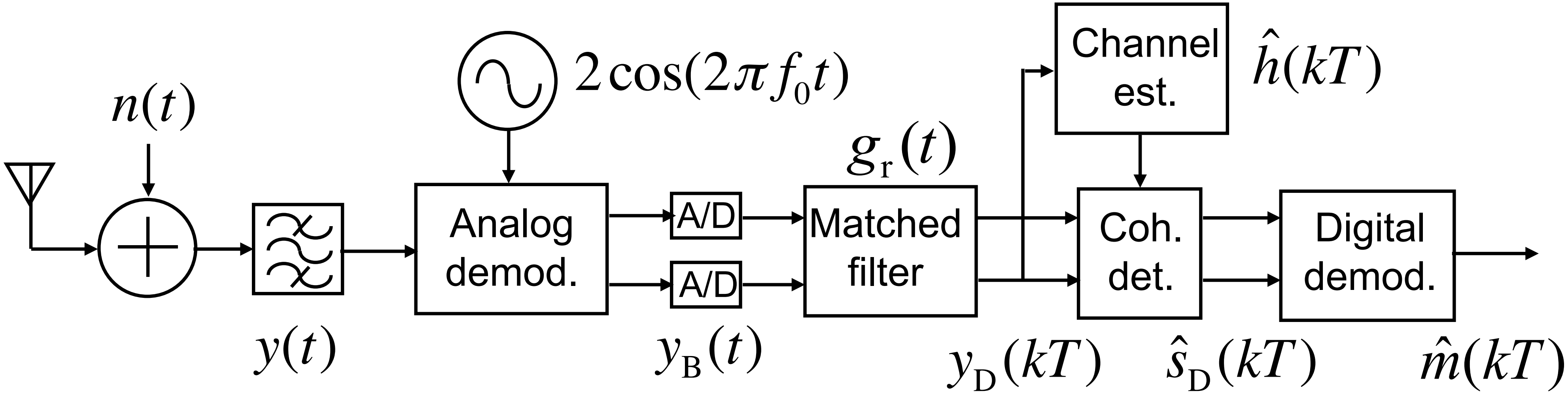
Energy efficiency,
Complexity of circuit,
Bit error rate p_e

Pulse $g(t)$

Power leackage

$$S_B^s(t) = |g(f)|^2 S_D^s(f)$$

Receiver



Additive white noise

Thermal noise generated in receiver

Bandpass fileter

Inter system interference cancellation

Analog demodulation

Convert signal from RF to BB

Matched filter

Maximization of SNR

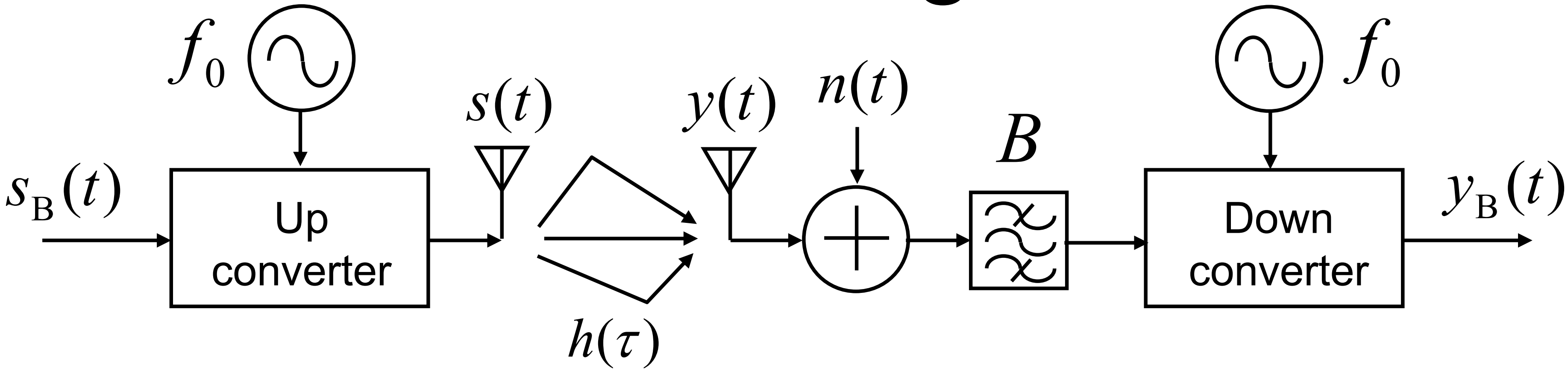
Coherent detection

Compensation of channel response

Digital demodulation

Convert complex signal to message

RF & BB Signals



Transmitter

$$s(t) = \mathrm{Re}\left[s_A(t)\right]$$

$$s_A(t) = s_B(t)e^{j2\pi f_0 t}$$

$$s_B(t)$$

Receiver

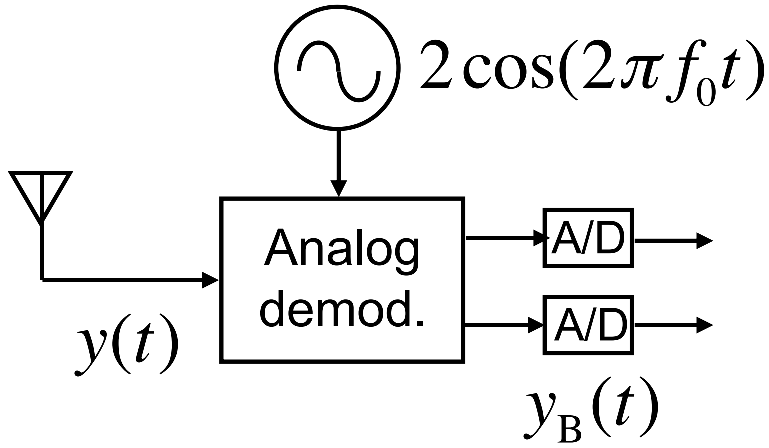
$$y(t) = \int h(\tau)s(t-\tau)\mathrm{d}\tau$$

$$y_A(t) = y(t) + j\,\mathrm{hilb}\left(y(t)\right)$$

$$y_B(t) = y_A(t)e^{-j2\pi f_0 t}$$

Analog Demodulation

Inputs & outputs of analog demodulator



$$y_B(t) = y_{BI}(t) + j y_{BQ}(t)$$

$$y_A(t) = y_B(t) e^{j2\pi f_0 t}$$

$$y(t) = \mathrm{Re}\left[y_A(t) \right]$$

RF input

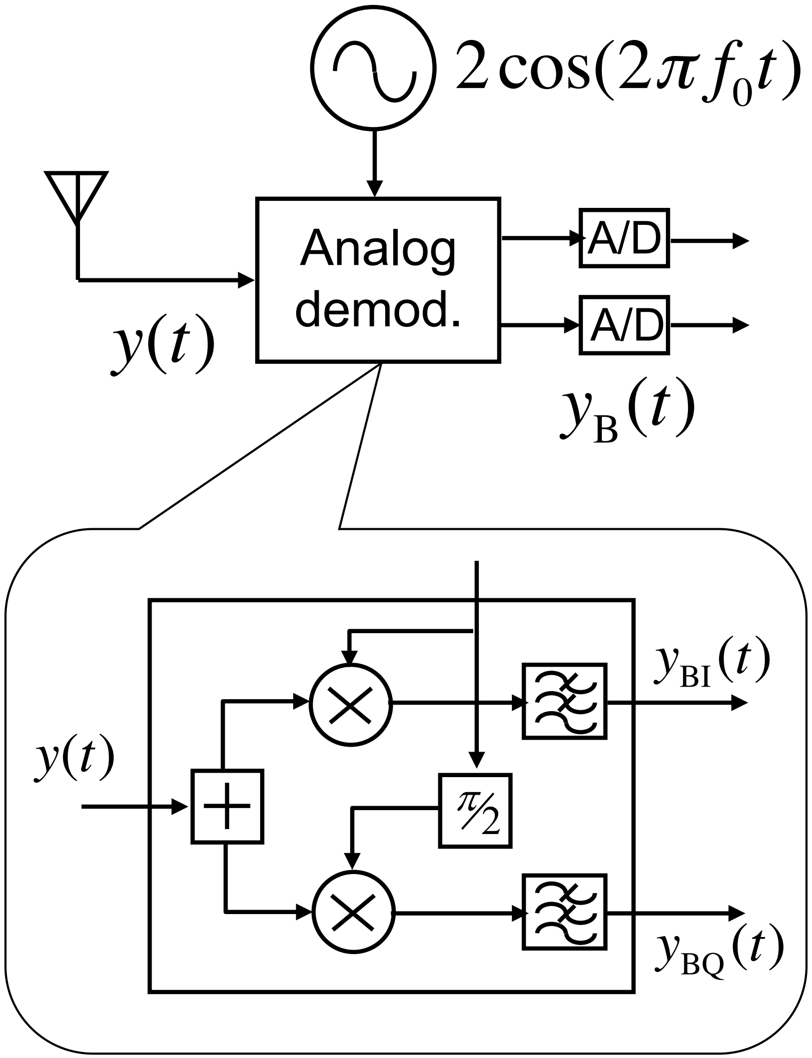
$$= y_{BI}(t) \cos 2\pi f_0 t - y_{BQ}(t) \sin 2\pi f_0 t$$

BB output

BB output

Analog Demodulation

$$y(t) = y_{BI}(t)\cos 2\pi f_0 t - y_{BQ}(t)\sin 2\pi f_0 t$$



In-phase BB receive signal

$$y_{BI}(t) = \mathrm{LPF}\left[2y(t)\cos 2\pi f_0 t\right]$$

$$= \mathrm{LPF}\left[y_{BI}(t)\left(1 + \cos 4\pi f_0 t\right) - y_{BQ}(t)\sin 4\pi f_0 t\right]$$

Quadrature BB receive signal

$$y_{BQ}(t) = \mathrm{LPF}\left[-2y(t)\sin 2\pi f_0 t\right]$$

$$= \mathrm{LPF}\left[-y_{BI}(t)\sin 4\pi f_0 t + y_{BQ}(t)\left(1 - \cos 4\pi f_0 t\right)\right]$$

Noise Through Analog Demodulator

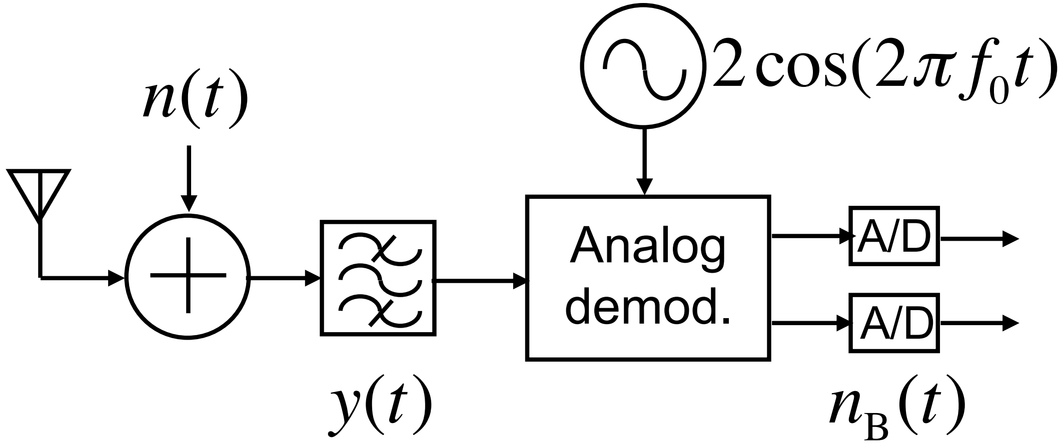
BB noise

$$n_{\mathrm{B}}(t) = n_{\mathrm{BI}}(t) + jn_{\mathrm{BQ}}(t)$$

$$n_{\mathrm{A}}(t) = n_{\mathrm{B}}(t)e^{j2\pi f_0 t}$$

$$n(t) = \mathrm{Re}\left[n_{\mathrm{A}}(t)\right]$$

$$= n_{\mathrm{BI}}(t)\cos 2\pi f_0 t - n_{\mathrm{BQ}}(t)\sin 2\pi f_0 t$$



Analog demodulation

$$n_{\mathrm{BI}}(t) = \mathrm{LPF}\left[2n(t)\cos 2\pi f_0 t\right]$$

$$n_{\mathrm{BQ}}(t) = \mathrm{LPF}\left[-2n(t)\sin 2\pi f_0 t\right]$$

Property of Noise

Noise power

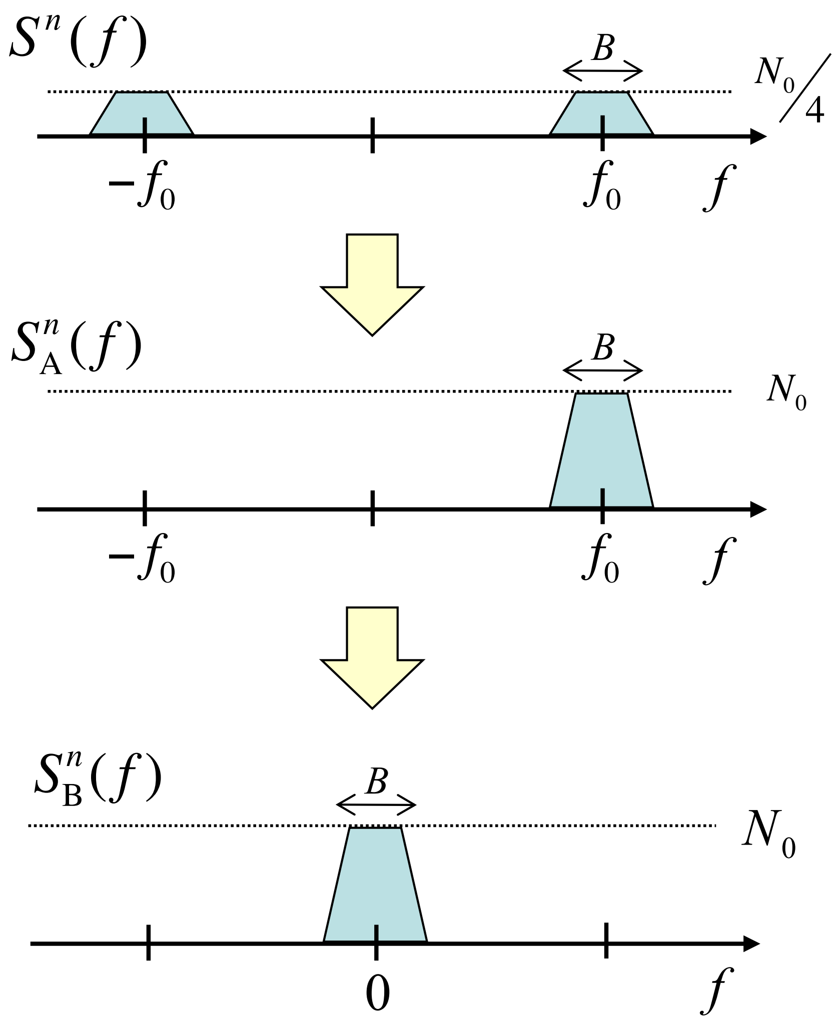
$$P_n = P_{nI} + P_{nQ} = N_0 B = \sigma^2$$

PDF of noise

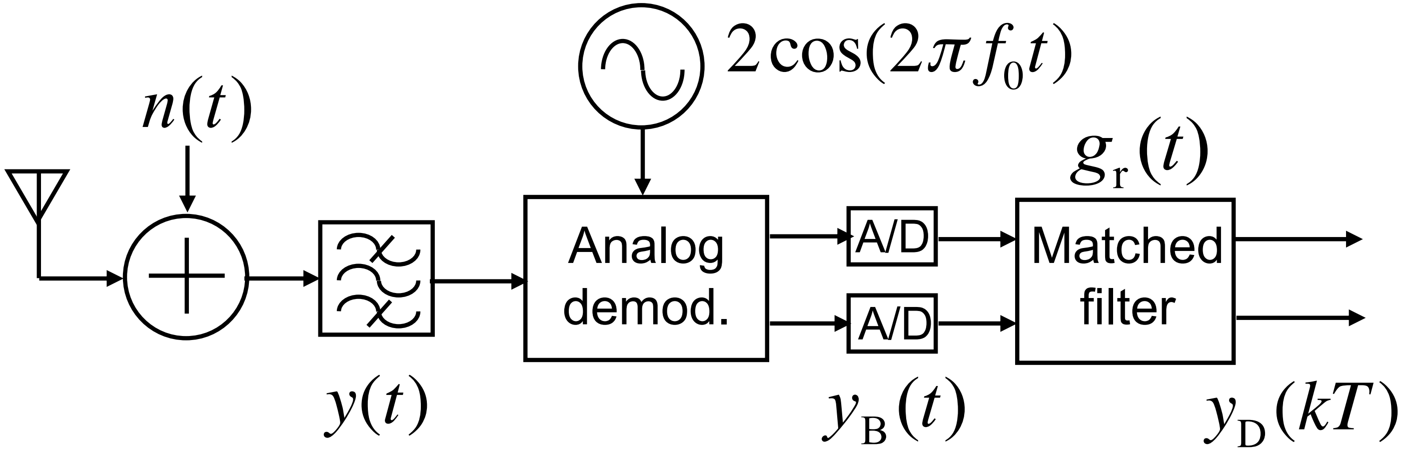
$$p(n_I) = p(n_Q) = \frac{1}{\sqrt{\pi\sigma^2}} e^{-\frac{n_x^2}{\sigma^2}}$$

$$\mathrm{E}[n_I] = \mathrm{E}[n_Q] = 0$$

$$\mathrm{E}\left[|n_I|^2\right] = \mathrm{E}\left[|n_Q|^2\right] = \frac{\sigma^2}{2}$$



BB Receive Filter



BB received signal

$$y_B(t) = h_B s_B(t) + n_B(t)$$

BB transmit signal

$$s_B(t) = g_s(t) \otimes s_D(t) = \sum_n a_n g_s(t - nT)$$

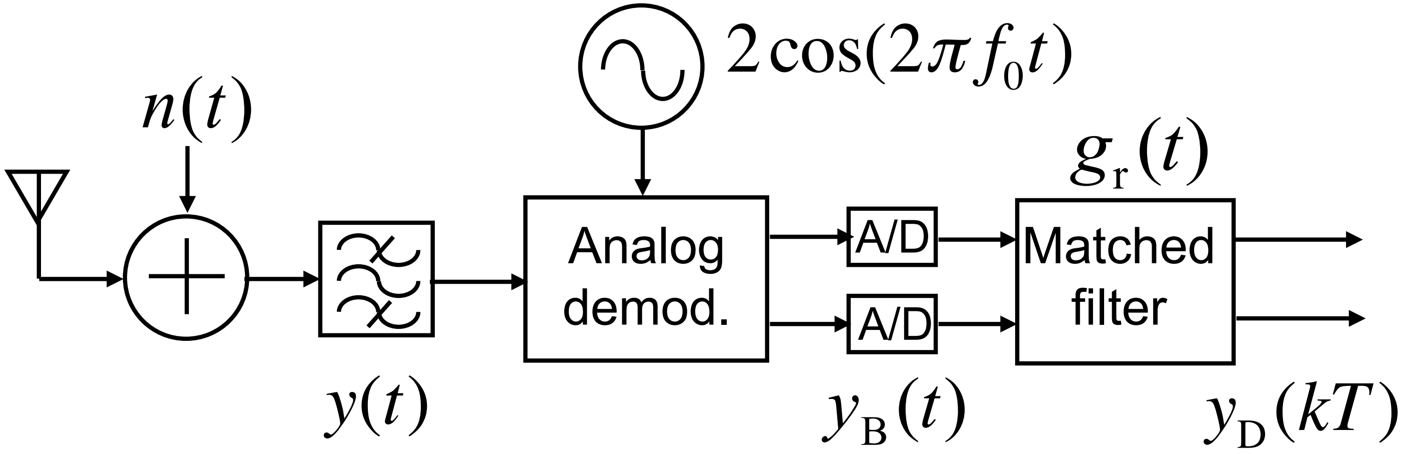
Transmit pulse (g_s)

Output of receiver filter

$$y_D(t) = g_r(t) \otimes y_B(t) = \underline{g_r(t) \otimes g_s(t)} \otimes s_D(t) + g_r(t) \otimes n_B(t)$$

Receive pulse (g_r) Combined pulse ($g_r(t) \otimes g_s(t)$)

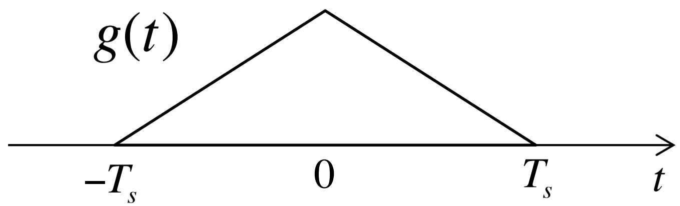
BB Receive Sampling



Output of receive filter (wo noise)

$$y_D(t) = g_r(t) \otimes g_s(t) \otimes s_D(t)$$
$$= g(t) \otimes s_D(t)$$

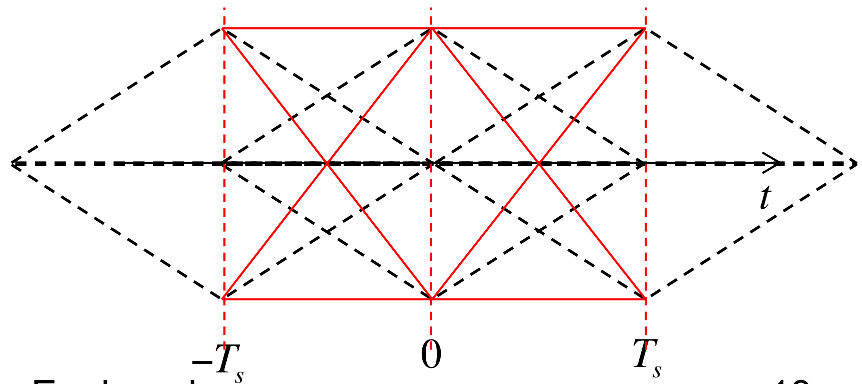
Combined pulse



Digital modulated signal (BPSK)

$$s_D(t) = \sum_k a_{2k}\delta(t - kT_s)$$

Eye pattern



Receive sampling

$$y_D(kT_s) = g(0)a_{2k}$$

BB Receive Power

Signal power

$$g(t) = g_r(t) \otimes g_s(t) \quad \Leftrightarrow \quad G(f) = G_r(f) G_s(f)$$

$$g(0) = \int G_r(f) G_s(f) e^{j2\pi ft} \, \mathrm{d}f$$

$$P_s = |g(0)|^2 = \left| \int G_r(f) G_s(f) \, \mathrm{d}f \right|^2$$

Noise power

$$R_B^n(\tau) = \mathrm{E}\left[n_B^*(t) n_B(t+\tau) \right] = N_0 \delta(0) \quad \Leftrightarrow \quad S_B^n(f) = N_0$$

$$n_D(t) = g_r(t) \otimes n_B(t) \quad \Leftrightarrow \quad S_D^n(f) = |G_r(f)|^2 S_B^n(f) = N_0 |G_r(f)|^2$$

$$P_n = \mathrm{E}\left[|n_D(t)|^2 \right] = N_0 \int |G_r(f)|^2 e^{j2\pi f\tau} \, \mathrm{d}f = N_0 \int |G_r(f)|^2 \, \mathrm{d}f$$

Matched Filter

SNR maximization

$$\gamma_{\max} = \max \frac{P_\mathrm{s}}{P_\mathrm{n}} = \max_{G_\mathrm{r}(f)} \frac{\left| \int G_\mathrm{r}(f) G_\mathrm{s}(f) \,\mathrm{d}f \right|^2}{N_0 \int \left| G_\mathrm{r}(f) \right|^2 \mathrm{d}f}$$

Schwartz inequality

$$\left| \int G_\mathrm{r}(f) G_\mathrm{s}(f) \,\mathrm{d}f \right|^2 \leq \int \left| G_\mathrm{r}(f) \right|^2 \mathrm{d}f \int \left| G_\mathrm{s}(f) \right|^2 \mathrm{d}f$$

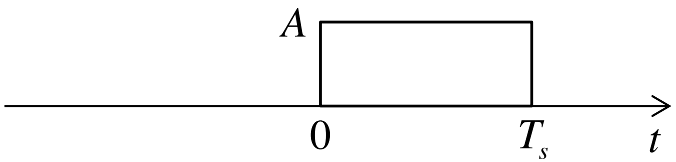
Matched filter

$$G_\mathrm{r}(f) = \left(G_\mathrm{s}(f) \right)^* \quad \Leftrightarrow \quad g_\mathrm{r}(t) = g_\mathrm{s}(-t)$$

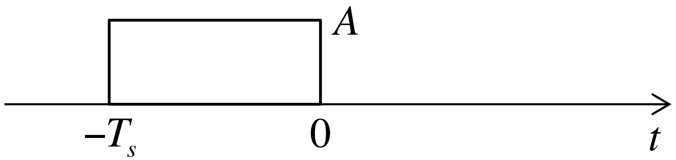
$$\gamma_{\max} = \frac{1}{N_0} \int \left| G_\mathrm{s}(f) \right|^2 \mathrm{d}f$$

Example of Matched Filter

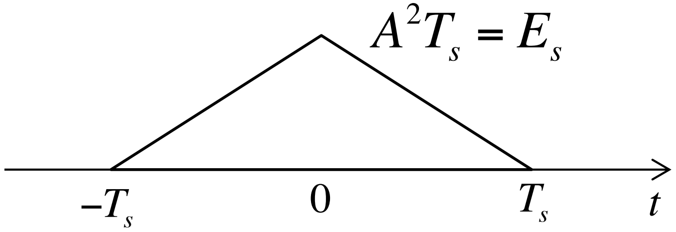
$g_s(t)$



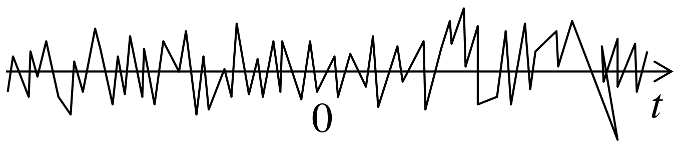
$g_r(t) = g_s(-t)$



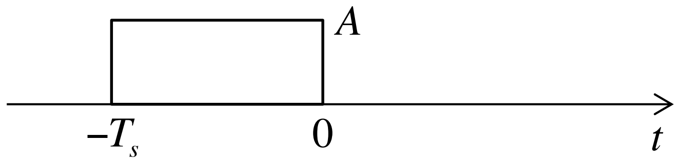
$$g(t) = \int g_r(\tau) g_s(t-\tau)\,\mathrm{d}t$$



$n_B(t)$

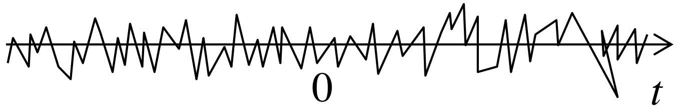


$g_r(t) = g_s(-t)$



$$n_D(t) = \int g_r(\tau) n_B(t-\tau)\,\mathrm{d}t$$

$$\mathrm{E}\left[\left|n_D(t)\right|^2\right] = N_0 \int \left|G_r(f)\right|^2 \mathrm{d}f = A^2 T_s N_0$$



Output SNR

Matched filter output

$$\gamma_{\max} = \frac{1}{N_0} \int \left| G_s(f) \right|^2 \mathrm{d} f$$

Parseval's theorem

$$\int \left| G_s(f) \right|^2 \mathrm{d} f = \int \left| g_s(t) \right|^2 \mathrm{d} t \quad = E_s$$

Energy per symbol

Output SNR

$$\gamma_{\max} = \frac{E_s}{N_0} \quad = \frac{P_s T_s}{N_0} \quad = \frac{P_s}{N_0 B} = \frac{P_s}{\sigma^2}$$

Summary

- Analog demodulation

$$y(t) = y_{BI}(t)\cos 2\pi f_0 t - y_{BQ}(t)\sin 2\pi f_0 t$$

$$y_{BI}(t) = \mathrm{LPF}\left[2y(t)\cos 2\pi f_0 t\right] \quad y_{BQ}(t) = \mathrm{LPF}\left[-2y(t)\sin 2\pi f_0 t\right]$$

- Matched filter

$$P_s = \left|\int G_r(f) G_s(f)\,\mathrm{d}f\right|^2 \le \int \left|G_r(f)\right|^2 \mathrm{d}f \int \left|G_s(f)\right|^2 \mathrm{d}f$$

$$G_r(f) = \left(G_s(f)\right)^* \quad \Leftrightarrow \quad g_r(t) = g_s(-t)$$

- Output SNR

$$\gamma_{max} = \frac{1}{N_0}\int \left|G_s(f)\right|^2 \mathrm{d}f = \frac{E_s}{N_0} = \frac{P_s}{\sigma^2}$$