2018 2Q
Wireless Communication Engineering

#3 Up/Down Conversion and Equivalent Baseband System

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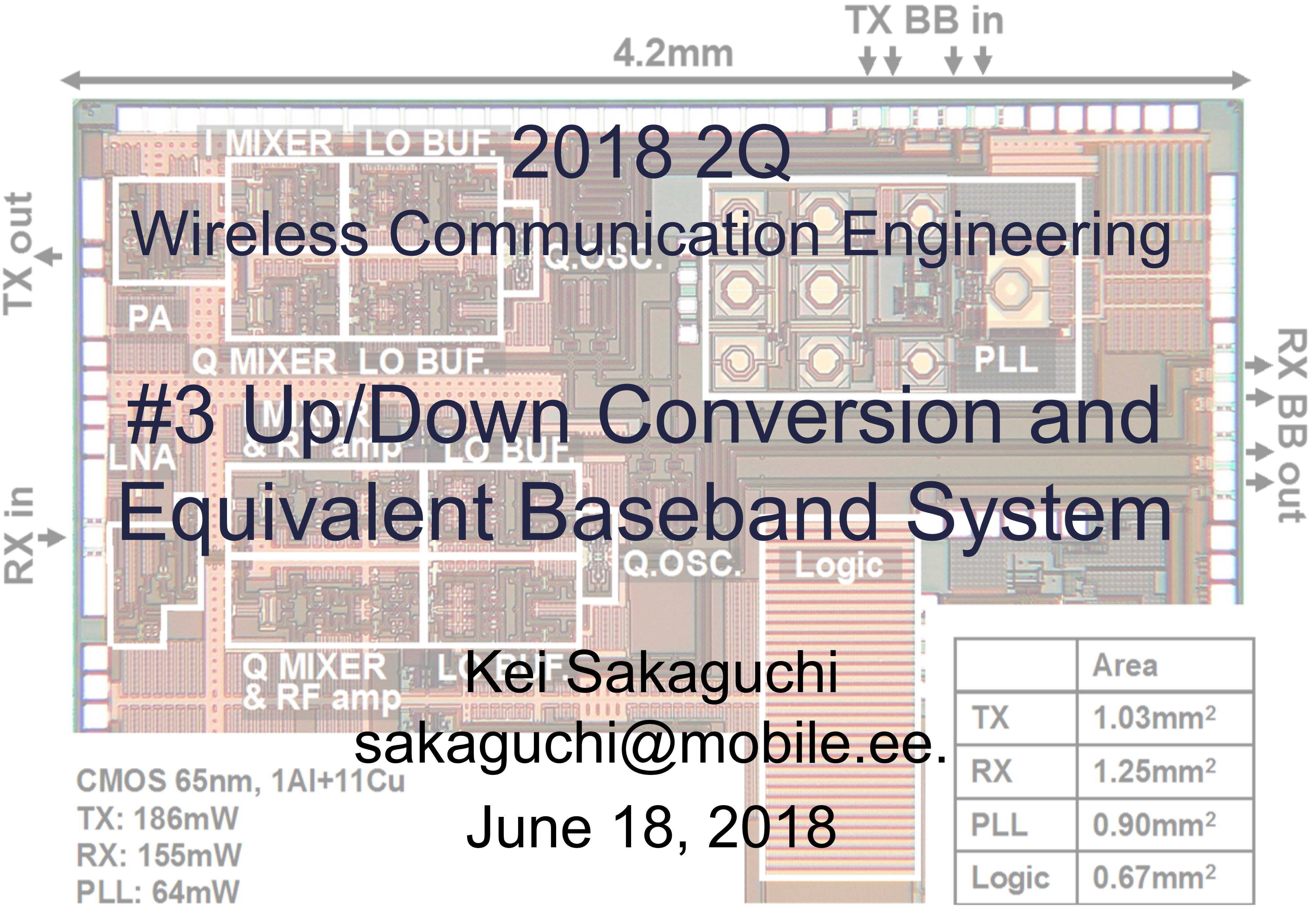


Course Schedule (1)

	Date	Text	Contents
#1	June 11	1, 7	Introduction to wireless communication systems
#2	June 14	2, 5, etc	Link budget design of wireless access
#3	June 18		Up/down conversion and equivalent baseband system
#4	June 21	3.3, 3.4	Digital modulation and pulse shaping
#5	June 25	3.5	Demodulation and matched fileter
#6	June 28		Collaborative exercise for better understanding 1
#7	July 2	3.5	Detection and error due to noise
#8	July 5	4.4	Channel fading and diversity combining

From Previous Lecture

- Channel capacity

$$C = B\log_2(1+\gamma) = \alpha \times f_0 \times R \ \text{[bps]}$$

- Friis propagation model

$$P_\mathrm{r} = \left(\frac{\lambda_0}{4\pi d}\right)^2 G_\mathrm{r} G_\mathrm{t} P_\mathrm{t} \qquad \gamma = \left(\frac{\lambda_0}{4\pi \mathrm{d}}\right)^2 \cdot \frac{G_\mathrm{r} G_\mathrm{t} P_\mathrm{t}}{P_\mathrm{n}}$$

- User rate and multiple access

$$C_\mathrm{UE} = \frac{B\log_2(1+\gamma)}{N_\mathrm{UE}} = \frac{B\log_2(1+\gamma)}{\pi d_0^2 \eta}$$

- Design of wireless access systems

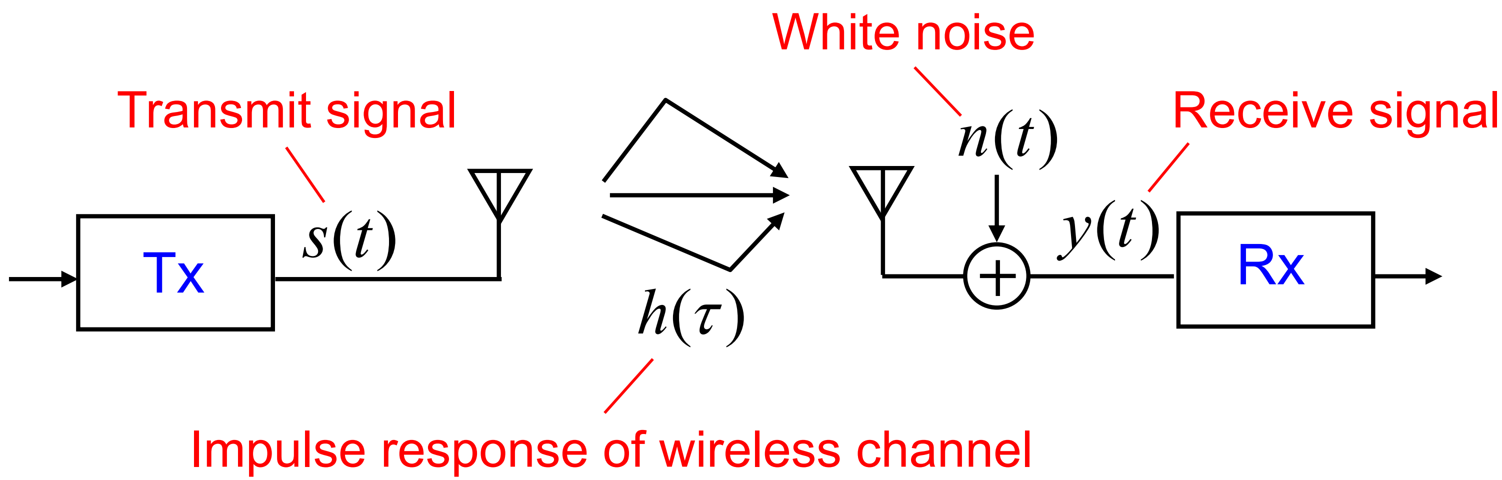
$$C_\mathrm{UE0}^\mathrm{req} \longrightarrow N_\mathrm{UE}, C \longrightarrow d_0, B, R \longrightarrow f_0, P_\mathrm{t}, G_\mathrm{t}$$

Contents

- Transmit signal (up conversion)
- Receive signal (down conversion)
- Equivalent baseband signal
- Auto-correlation & power spectrum
- Frequency domain analysis

System Model

- System model



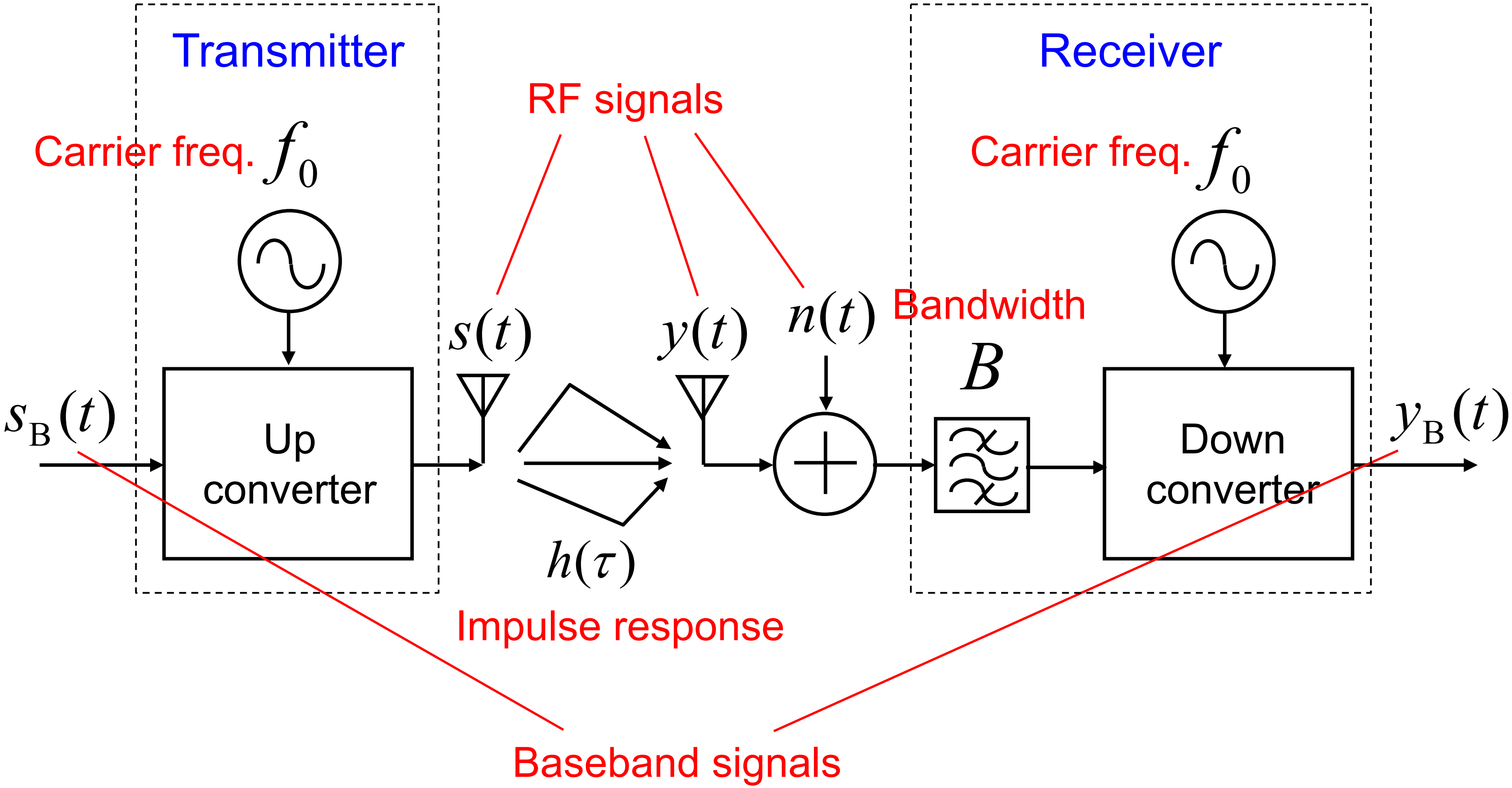
- Receive signal model

$$y(t) = \int h(\tau) s(t-\tau) \mathrm{d}\tau + n(t)$$

Assuming delay spread
(frequency selective fading)

Carrier freq. f_0 & bandwidth B

Carrier Frequency and Modem



Transmit Signal (Time Domain)

- Baseband & RF analytic signal

Baseband transmit signal: $s_{\mathrm{B}}(t) = s_{\mathrm{BR}}(t) + j s_{\mathrm{BI}}(t)$

In-phase Quadrature

RF analytic signal: $s_{\mathrm{A}}(t) = s_{\mathrm{B}}(t) e^{j2\pi f_0 t}$

Up conversion (BB → RF)

- Transmit signal

$$
\begin{aligned}
s(t) &= \mathrm{Re}\left[s_A(t) \right] \\
&= s_{\mathrm{BR}}(t) \cos 2\pi f_0 t - s_{\mathrm{BI}}(t) \sin 2\pi f_0 t \\
&= \sqrt{s_{\mathrm{BR}}^2(t) + s_{\mathrm{BI}}^2(t)} \cos\left(2\pi f_0 t + \angle s_{\mathrm{BI}} / s_{\mathrm{BR}} \right)
\end{aligned}
$$

Receive Signal (Time Domain)

- Receive signal

$$y(t) = \int h(\tau) s(t-\tau) \mathrm{d}\tau$$

$$= \mathrm{Re}\left[\int h(\tau) s_{\mathrm{R}}(t-\tau) \mathrm{d}\tau \right]$$

$$= \mathrm{Re}\left[\int h(\tau) s_{\mathrm{B}}(t-\tau) e^{j2\pi f_0 (t-\tau)} \mathrm{d}\tau \right]$$

- RF analytic & baseband (BB) receive signal

Analytic receive signal: $y_{\mathrm{A}}(t) = y(t) + j\,\mathrm{hilb}\left(y(t) \right)$

Hilbert transformation

Baseband receive signal: $y_{\mathrm{B}}(t) = y_{\mathrm{A}}(t) e^{-j2\pi f_0 t}$

Down conversion (RF → BB)

Equivalent Baseband Signal

- RF and baseband (BB) receive signal

$$y(t) = \mathrm{Re}\left[\int h(\tau) s_{\mathrm{B}}(t-\tau) e^{j2\pi f_0 (t-\tau)} \mathrm{d}\tau\right]$$

$$y_{\mathrm{B}}(t) = y_{\mathrm{A}}(t) e^{-j2\pi f_0 t} = \left(y(t) + j\,\mathrm{hilb}\left(y(t)\right)\right) e^{-j2\pi f_0 t}$$

- Equivalent baseband system

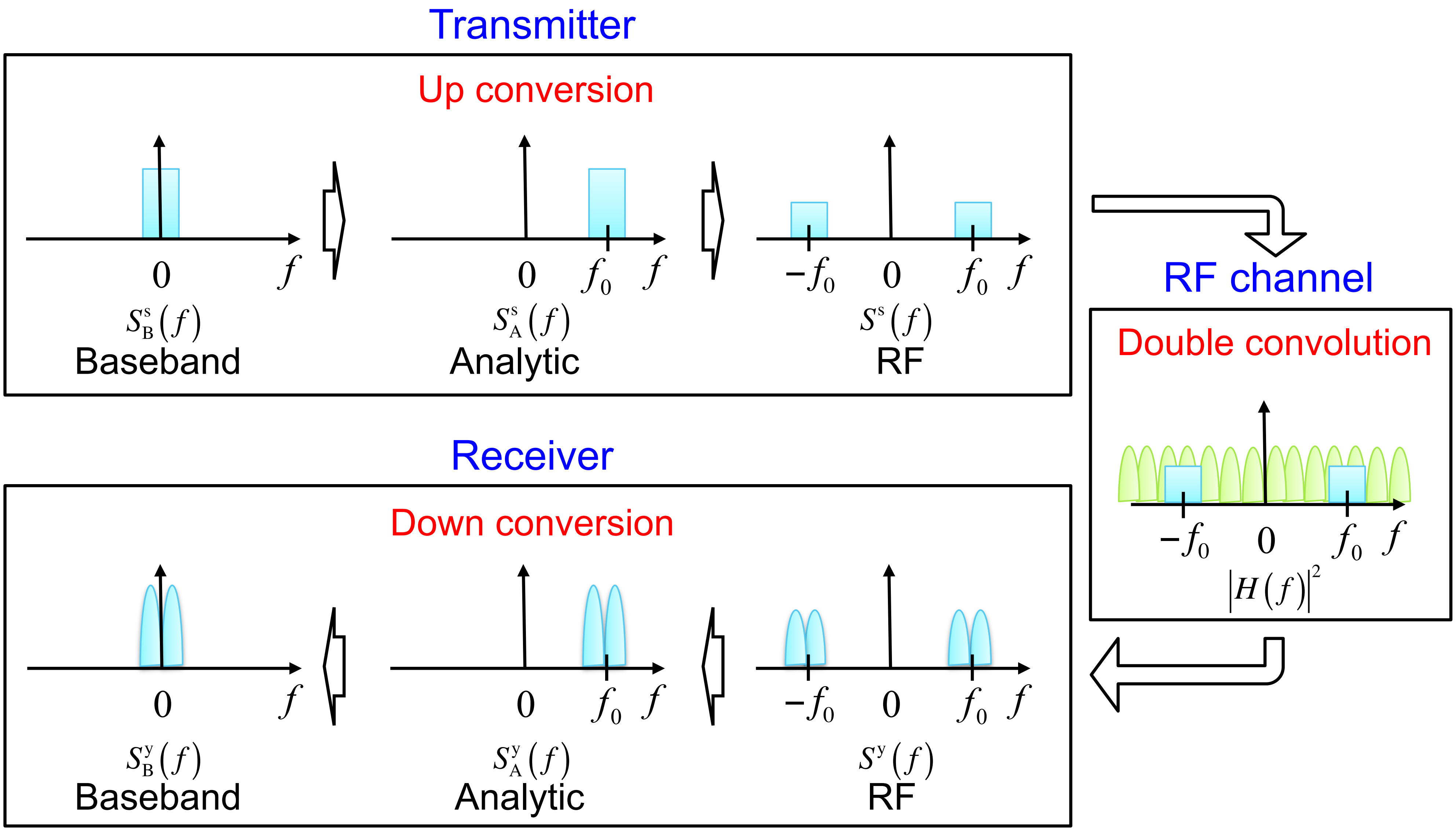
$$y_{\mathrm{B}}(t) = \int h(\tau) s_{\mathrm{B}}(t-\tau) e^{-j2\pi f_0 \tau} \mathrm{d}\tau = \int h_{\mathrm{B}}(\tau) s_{\mathrm{B}}(t-\tau) \mathrm{d}\tau$$

Separation from carrier freq.

Equivalent baseband impulse response: $h_{\mathrm{B}}(\tau) = h(\tau) e^{-j2\pi f_0 \tau}$

Phase rotation depending on carrier freq.

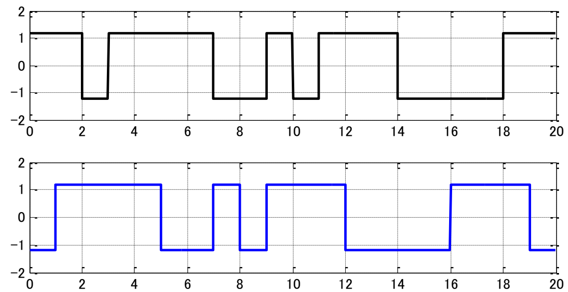
Frequency Domain Analysis



Auto-correlation & Power Spectrum

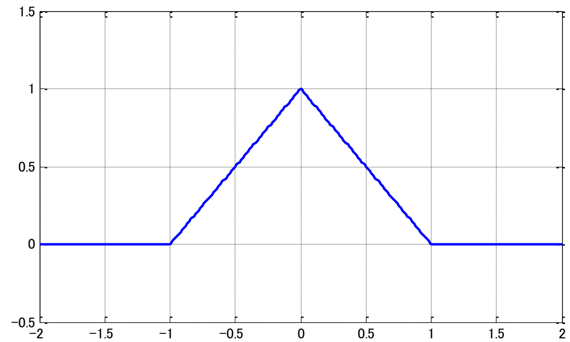
- Random pulse sequence

$$s_{\mathrm{B}}(t)$$



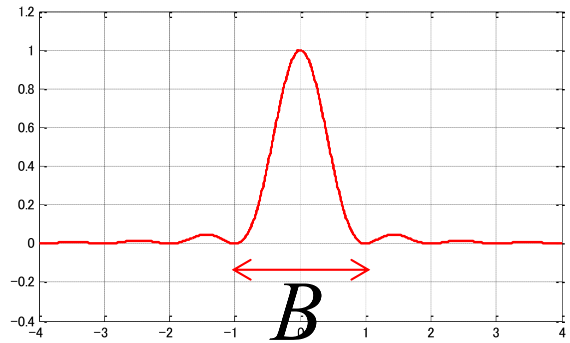
- Auto-correlation

$$R_{\mathrm{B}}^{\mathrm{s}}(\tau) = \mathrm{E}\left[s_{\mathrm{B}}^{*}(t)\, s_{\mathrm{B}}(t+\tau)\right]$$



- Power spectrum

$$S_{\mathrm{B}}^{\mathrm{s}}(f) = \int_{-\infty}^{\infty} R_{\mathrm{B}}^{\mathrm{s}}(\tau)\, e^{-j2\pi f\tau}\, \mathrm{d}\tau$$



Analytic Signal (Freq. Domain)

- Auto-correlation of analytic signal

$$R_{\mathrm{A}}^{\mathrm{s}}\left(\tau\right)=\mathrm{E}\left[s_{\mathrm{A}}^{*}(t)s_{\mathrm{A}}(t+\tau)\right]$$

$$=\mathrm{E}\left[s_{\mathrm{B}}^{*}(t)s_{\mathrm{B}}(t+\tau)\right]e^{j2\pi f_0\tau}=R_{\mathrm{B}}^{\mathrm{s}}\left(\tau\right)e^{j2\pi f_0\tau}$$

Up conversion (BB → RF)

- Power spectrum of analytic signal

$$S_{\mathrm{A}}^{\mathrm{s}}\left(f\right)=\int_{-\infty}^{\infty}R_{\mathrm{A}}^{\mathrm{s}}\left(\tau\right)e^{-j2\pi f\tau}\,\mathrm{d}\tau=S_{\mathrm{B}}^{\mathrm{s}}\left(f-f_0\right)$$

Frequency conversion

Transmit Signal (Freq. Domain)

- Auto-correlation of transmit signal

$$s(t) = \mathrm{Re}\left[s_{\mathrm{A}}(t)\right] = \frac{1}{2}\left(s_{\mathrm{A}}(t) + s_{\mathrm{A}}^{*}(t)\right)$$

$$R^{\mathrm{s}}(\tau) = \mathrm{E}\left[s^{*}(t)s\ (t+\tau)\right]$$

Assuming independency between in-phase & quadrature signals

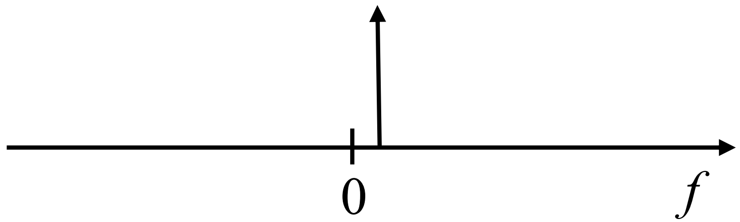
$$= \frac{1}{4}\left(R_{\mathrm{A}}^{\mathrm{s}}(\tau) + R_{\mathrm{A}}^{\mathrm{s}*}(\tau)\right) = \frac{1}{4}\left(R_{\mathrm{B}}^{\mathrm{s}}(\tau)e^{j2\pi f_0\tau} + R_{\mathrm{B}}^{\mathrm{s}*}(\tau)e^{-j2\pi f_0\tau}\right)$$

- Power spectrum of transmit signal

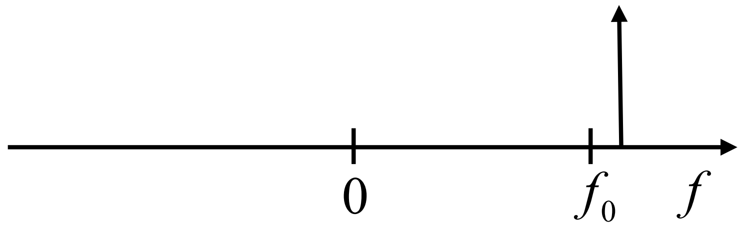
$$S^{\mathrm{s}}(f) = \int_{-\infty}^{\infty} R^{\mathrm{s}}(\tau)e^{-j2\pi f\tau}\,\mathrm{d}\tau = \frac{1}{4}\left(S_{\mathrm{B}}^{\mathrm{s}}(f - f_0) + S_{\mathrm{B}}^{\mathrm{s}}(-f - f_0)\right)$$

Positive freq. Negative freq.

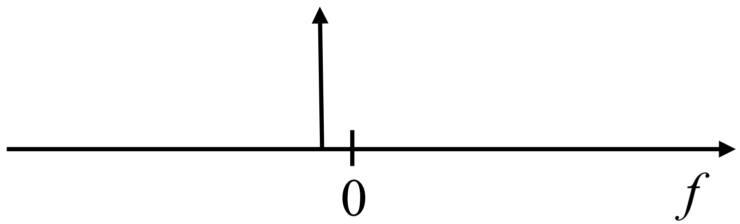
Example of Transmit Spectrum



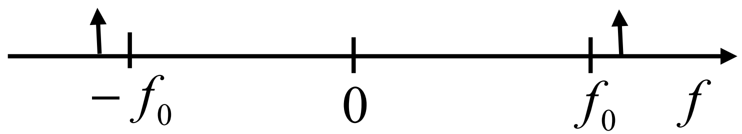
$$S_{\mathrm{B}}^{\mathrm{s}}(f)$$



$$S_{\mathrm{A}}^{\mathrm{s}}(f) = S_{\mathrm{B}}^{\mathrm{s}}(f - f_0)$$



$$S_{\mathrm{B}}^{\mathrm{s}}(-f)$$



$$S^{\mathrm{s}}(f) = \frac{1}{4}\left(S_{\mathrm{B}}^{\mathrm{s}}(f - f_0) + S_{\mathrm{B}}^{\mathrm{s}}(-f - f_0)\right)$$

Receive Signal (Freq. Domain)

- Auto-correlation of receive signal

$$y_{\mathrm{B}}(t) = \int h_{\mathrm{B}}(\tau) s_{\mathrm{B}}(t-\tau)\mathrm{d}\tau$$

$$R_{\mathrm{B}}^{\mathrm{y}}(\tau) = \mathrm{E}\left[y_{\mathrm{B}}^{*}(t) y_{\mathrm{B}}(t+\tau)\right]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_{\mathrm{B}}^{*}(\tau_1) h_{\mathrm{B}}(\tau_2) R_{B}^{s}(\tau - \tau_1 + \tau_2)\mathrm{d}\tau_1\,\mathrm{d}\tau_2$$

Double convolution

- Power spectrum of receive signal

$$S_{\mathrm{B}}^{\mathrm{y}}(f) = \int_{-\infty}^{\infty} R_{\mathrm{B}}^{\mathrm{y}}(\tau) e^{-j2\pi f\tau}\,\mathrm{d}\tau = \left|H_{\mathrm{B}}(f)\right|^{2} S_{\mathrm{B}}^{\mathrm{s}}(f)$$

Feature of double convolution

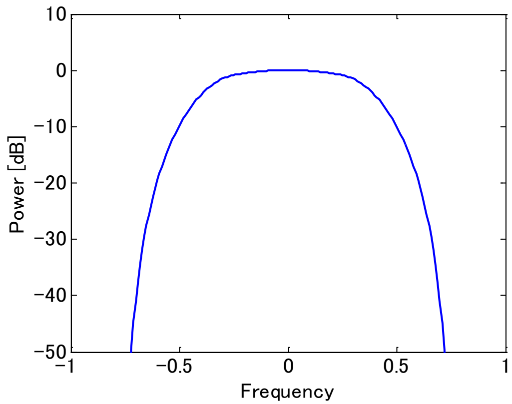
$$H_{\mathrm{B}}(f) = \int_{-\infty}^{\infty} h_{\mathrm{B}}(\tau) e^{-j2\pi f\tau}\,\mathrm{d}\tau$$

Example of Receive Spectrum

$$S_{\mathrm{B}}^{\mathrm{y}}(f) = \left|H_{\mathrm{B}}(f)\right|^2 S_{\mathrm{B}}^{\mathrm{s}}(f)$$

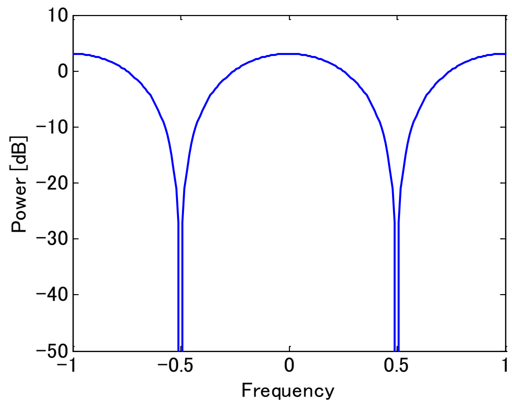
$S_{\mathrm{B}}^{\mathrm{s}}(f)$

Transmit spectrum



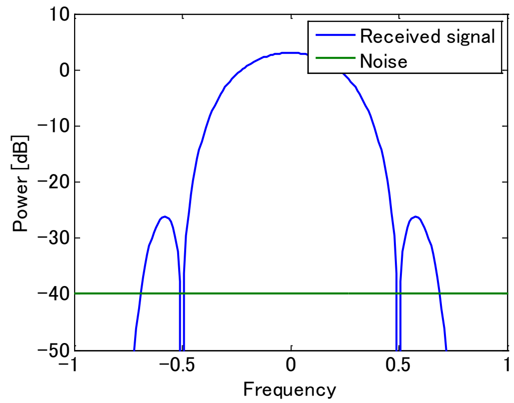
$\left|H_{\mathrm{B}}(f)\right|^2$

Channel response



$S_{\mathrm{B}}^{\mathrm{y}}(f)$

Receive spectrum



Summary

- Equivalent baseband system

$$y_{\mathrm{B}}(t) = \int h_{\mathrm{B}}(\tau) s_{\mathrm{B}}(t-\tau) \mathrm{d}\tau$$

$$y_{\mathrm{B}}(t) = y_{\mathrm{A}}(t) e^{-j2\pi f_0 t} \qquad y_{\mathrm{A}}(t) = y(t) + j\,\mathrm{hilb}\big(y(t)\big)$$

- Power spectrum of transmit signal

$$S^{\mathrm{s}}(f) = \frac{1}{4}\left(S_{\mathrm{B}}^{\mathrm{s}}(f - f_0) + S_{\mathrm{B}}^{\mathrm{s}}(-f - f_0)\right)$$

- Power spectrum of receive signal

$$S_{\mathrm{B}}^{\mathrm{y}}(f) = \left|H_{\mathrm{B}}(f)\right|^2 S_{\mathrm{B}}^{\mathrm{s}}(f)$$

Fourier Transform Pairs

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} \, \mathrm{d} f \quad \Leftrightarrow \quad X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} \, \mathrm{d} t$$

$$x(t) e^{j2\pi f_0 t} \quad \Leftrightarrow \quad X(f - f_0)$$

$$x^*(t) \quad \Leftrightarrow \quad X^*(-f)$$

$$\int_{-\infty}^{\infty} x(\tau) y(t-\tau) \, \mathrm{d} \tau \quad \Leftrightarrow \quad X(f) Y(f)$$