Technological Progress and Economic Growth: An Expanding Variety Model

IEE.B402. Advanced Macroeconomics

Ryoji Ohdoi
Department of Industrial Engineering and Economics,
Tokyo Institute of Technology

Introduction

- We consider the model of economic growth with endogenous technological progress.

- The model presented here is the simplified version of the model developed by Paul Romer in the following paper:

  Romer, P. M. (1990) "Endogenous Technological Change," *Journal of Political Economy* 98, S71–S102.

- Basic idea:
  1. R&D leads to the creation of new varieties of intermediate inputs (e.g., machines).
  2. A greater variety of inputs raises the productivity of the final-good firms.

Plan

- Setup
- Households' behavior
- Firms' behavior
  - Final-good firms
  - Intermediate-good firms
- R&D activities by potential entrants and "knowledge externalities"
- Characterization of the equilibrium path
  - Market-clearing conditions
  - Derivation of dynamic system
  - Proof of unique BGP
- On the inefficiency of the equilibrium path

Setup

- Time is continuous, denoted by $t \in [0, \infty)$
- Two types of economic agents: households and firms
- Population of the households is fixed at $\bar{L} > 0$ over time, that is, there is no population growth (i.e., $n = 0$)
- Firms are in turn classified into three types:
  - Final good firms
  - Intermediate good firms
  - R&D firms (or potential entrants)
- There is a single final good used for consumption. This type of good is supplied under perfect competition.
- There is a continuum of intermediate goods, used for the inputs of final good production. This type of goods is supplied under monopolistic competition.

Households' Behavior: Utility Maximization Problem

- Households are homogeneous.
- A representative household's dynamic utility maximization problem:

$$\max_{\{c(t),a(t)\}} \quad U = \int_0^\infty e^{-\rho t} u(c(t))dt$$

$$\text{s.t.} \quad \dot{a}(t) = r(t)a(t) + w(t) - c(t) \qquad \forall t \in [0, \infty) \tag{1}$$

$$\lim_{t \to \infty} a(t) \exp\left(- \int_0^t r(s)ds \right) \geq 0 \tag{2}$$

$$a(0) \text{ given}$$

- Specification of $u(c)$ as the CRRA form:

$$u(c) = \begin{cases} \dfrac{c^{1-\theta} - 1}{1 - \theta} & \text{if } \theta > 0, \theta \neq 1, \\ \ln c & \text{if } \theta = 1. \end{cases}$$

Households' Behavior: Euler equation and TVC

- Current-value Hamiltonian: $H(a, c, \lambda) = u(c) + \lambda[ra + w - c]$
- First-order-conditions (F.O.Cs):

$$\frac{\partial H}{\partial c(t)} = 0 : \quad c(t)^{-\theta} = \lambda(t) \tag{3}$$

$$\dot{\lambda}(t) = \rho\lambda(t) - \frac{\partial H}{\partial a(t)} : \quad \dot{\lambda}(t) = (\rho - r(t))\lambda(t) \tag{4}$$

Eqs. (3) and (4) lead the Euler equation:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\theta}(r(t) - \rho) \tag{5}$$

- Transversality condition (TVC) $\rightarrow$ the binding condition of (2):

$$\lim_{t\to\infty} a(t) \exp\left(-\int_0^t r(s)ds\right) = 0 \tag{6}$$

Firms' Behavior

There are three types of firms:

1. Producers of a single final good: They hire labor and a continuum of differentiated intermediate inputs under perfect competition, and combine them to produce the final good used for consumption.

2. Producers of intermediate inputs: Each firm of this type holds a patent for its own brand, and therefore supplies its input to the final good firms with monopoly pricing.

3. R&D firms (potential entrants): Each firm of this type engages in R&D activities.
   If it succeeds, it becomes an intermediate good supplier of new brand.

Final-Good Firms: Production Function

- The production technology of a representative firm is

$$Y(t) = \frac{1}{\alpha} \left[L_Y(t)\right]^{1-\alpha} \left(\int_0^{N(t)} \left[x(j,t)\right]^{\alpha} dj \right), \quad \alpha \in (0,1) \tag{7}$$

  where

  - $L_Y(t)$ is demand for labor by the final good firm.
  - $x(j,t)$ is demand for variety $j \in [0, N(t)]$.
  - $N(t)$ is the measure of varieties available at time t.

- Note that final good firms take $N(t)$ as given. So the production technology exhibits constant returns to scale.

Final-Good Firms: Profit Maximization

($*$) Hereafter, we omit "(t)" unless to do so would cause confusions.

- The final good is taken as a numeraire.
- Let $p(j,t)$ is the price of the intermediate good of variety j.
- Profit maximization problem:

$$\max_{L_Y, [x(j)]_{j\in[0,N]}} \frac{1}{\alpha} L_Y^{1-\alpha} \left(\int_0^N x(j)^\alpha dj \right) - wL_Y - \int_0^N p(j)x(j)dj$$

- First-order-conditions (F.O.Cs) for L_Y and $x(j')$ are

$$w = \frac{1-\alpha}{\alpha} L_Y^{-\alpha} \left(\int_0^N x(j)^\alpha dj \right) \tag{8}$$

$$p(j') = x(j')^{\alpha-1} L_Y^{1-\alpha} \; \forall j' \in [0,N] \tag{9}$$

Int.-Good Firms: Assumption of Monopolistic Competition

- Key assumption by Romer (1990):
  - We assume that the intermediate-good firms operate under a *monopolistic competition*, developed by Spence (1976) and Dixit and Stiglitz (1977).
  - That is, a firm owns a monopoly right to its own variety, but has no market power to the other firms' varieties.

  This means that the each firm's product is patent-protected.

- Hereafter, let us call the firm producing variety i, "firm j." At each point in time, this firm faces the inverse demand function given by (9):

$$p(j,t) = \left(\frac{x(j,t)}{L_Y(t)}\right)^{-(1-\alpha)} \tag{10}$$

Int.- Good Firms: Profit maximization

- To produce one unit of the good, $\psi(>0)$ units of the final good is required.
- Let $\pi(j,t)$ denote the firm j's profit flow. Then, the firm j's profit maximization problem is

$$\max_{x(j,t)} \pi(j,t) \equiv p(j,t)x(j,t) - \psi x(j,t)$$
$$\text{s.t. } p(j,t) = \left(\frac{x(j,t)}{L_Y(t)}\right)^{-(1-\alpha)}$$
$$L_Y(t) \text{ given}$$

It is shown that the profit-maximizing price is given by

$$p(j,t) = \frac{1}{\alpha}\psi \quad \forall j,t \tag{11}$$

- Note that the charging price is higher than the marginal cost ψ.

Int.-Good Firms: The Stock Value

- Notice that all intermediate-good firms charge the same price, ψ/α from (11).
  $\rightarrow$ Using this result and (10), the output x accordingly becomes the same across the firms.
- So hereafter we can omit the index of varieties (j or i):

$$x(j,t) = x(t), \quad \pi(j,t) = \pi(t)$$

- Let us define $v(t)$ as the stock value of the intermediate-good firm at date t. That is,

$$v(t) = \int_t^{\infty} \pi(\tau) \exp\left(-\int_t^{\tau} r(s)ds\right) d\tau$$

or equivalently,

$$\dot{v}(t) = r(t)v(t) - \pi(t) \tag{12}$$

R&D Activities by "Potential Entrants"

- R&D firms can be interpreted as "potential entrants" to the intermediate goods market.
- To obtain one unit of patent of brand-new variety, each potential firm must employ $B(t)$ units of workers for R&D.

Knowledge Spillovers (Romer, 1990)

Romer (1990) specifies $B(t)$ as

$$B(t) = \frac{1}{\eta N(t)} \tag{13}$$

where $\eta > 0$.

Free Entry Condition of R & D Activities

- Let $L_R(t)$ denote the aggregate employment for R&D activities. This is an endogenous variable.
- Then, the flow of newly-born varieties $\dot{N}(t)$ is

$$\begin{aligned}\dot{N}(t) &= L_R(t)/B(t) \\ &= \underbrace{\eta N(t)}_{\text{externality}} L_R(t)\end{aligned} \tag{14}$$

- Free entry condition of R & D activities is given by

$$v(t) \leq \frac{w(t)}{\eta N(t)}, \quad \dot{N}(t) \geq 0, \quad \dot{N}(t)\left(v(t) - \frac{w(t)}{\eta N(t)}\right) = 0 \tag{15}$$

This means that if the R & D is conducted at time t (that is, if $\dot{N}(t) \geq 0$), then $v(t) = w(t)/(\eta N(t))$.

Characterization of Equilibrium Path

Closing the Model

Market-clearing conditions:

- Labor market equilibrium:

$$\bar{L} = L_Y(t) + L_R(t) \tag{16}$$

- Assets:

$$a(t)\bar{L} = v(t)N(t) \tag{17}$$

- Final-good market :

$$Y(t) = c(t)\bar{L} + \psi x(t)N(t) \tag{18}$$

Normalization

- Hereafter, for simplicity, we normalize the marginal cost of machine production, ψ, to

$$\psi = \alpha$$

$\downarrow$

- From the above normalization and the monopoly pricing (11),

$$p(t) = 1 \quad \forall t$$

- Then, the F.O.Cs of the final-good firm (8) and (9) show

$$x(t) = L_Y(t) \tag{19}$$

$$w(t) = \frac{1-\alpha}{\alpha} N(t) \tag{20}$$

Derivation of Dynamic System

- Using (20), the free-entry condition of R&D (15) is rewritten as

$$(15): \; v(t) = \frac{w(t)}{\eta N(t)} \Leftrightarrow v(t) = \frac{1-\alpha}{\eta\alpha} \tag{21}$$

Namely, in equilibrium, $v(t)$ is constant over time.

- Then, imposing $\dot{v} = 0$ in (12) and using (21), $r(t)$ is expressed as

$$\begin{aligned} r(t) &= \frac{\pi(t)}{v(t)} \quad \left(\text{where} \quad \pi = (1-\alpha)x\right) \\ &= \eta\alpha L_Y(t)\left(\because (19)\right) \end{aligned} \tag{22}$$

Dynamic System

- Then, substituting (22) into the Euler equation, the dynamics of $c(t)$ is given by
$$\dot{c}(t)/c(t) = \frac{\eta\alpha L_Y(t) - \rho}{\theta}. \tag{23}$$
- On the other hand, Using the labor-market equilibrium (16), the dynamics of $N(t)$, (14) is rewritten as
$$\dot{N}(t)/N(t) = \eta(\bar{L} - L_Y(t)). \tag{24}$$

That is, once $L_Y(t)$ is obtained, (23) and (24) govern the dynamic system.

Dynamic System

- Now derive L_Y
- Using (19) and (20), the market equilibrium for the final good (18) is rewritten as

$$
\begin{aligned}
Y = c\bar{L} + \psi N x \Leftrightarrow & \; wL_Y + Nx = c\bar{L} + \psi N x \\
\Leftrightarrow & \; \frac{1-\alpha}{\alpha} N L_Y + N L_Y = c\bar{L} + \alpha N L_Y \\
\Leftrightarrow & \; \frac{1-\alpha^2}{\alpha} L_Y = Z
\end{aligned}
$$

where

$$Z(t) \equiv \frac{c(t)\bar{L}}{N(t)}$$

Dynamic System

- Therefore, L_Y is given by

$$L_Y = \frac{\alpha}{1-\alpha^2} Z \tag{25}$$

- From (23)–(25), the dynamics of $Z(t)$ is described as

$$\begin{aligned} \dot{Z}(t)/Z(t) &= \dot{c}(t)/c(t) - \dot{N}(t)/N(t) \\ &= \frac{1}{\theta}\left[\eta(\alpha+\theta)L_Y(t) - (\rho + \theta\eta\bar{L})\right] \end{aligned} \tag{26}$$

That is, (25) and (26) jointly constitute the dynamic system of the economy.

Unique Existence of BGP

- From these two equations, we easily have

$$\dot{Z}(t)/Z(t) = \frac{1}{\theta}\left[\frac{\eta\alpha(\alpha+\theta)}{1-\alpha^2}Z(t) - (\rho + \theta\eta\bar{L})\right]$$

- $Z(0)$ is an endogenous variable. Then, for the same reasoning as in the AK model, it holds that

$$\dot{Z}(t) = 0 \forall t \geq 0 \;\Leftrightarrow\; Z(t) = Z^* \equiv \frac{1-\alpha^2}{\alpha}\frac{\rho+\theta\eta\bar{L}}{\eta(\theta+\alpha)} \tag{27}$$

Namely, there is no transition and the economy is on the balanced growth path from the initial date.

The Growth Rate

- The growth rate of the economy, denoted by g^*, is given by

$$g^* = \frac{\eta\alpha L_Y^* - \rho}{\theta} \tag{28}$$

where

$$L_Y^* = \frac{\rho + \theta\eta\bar{L}}{\eta(\alpha + \theta)}$$

Implications

- The mathematical structure, especially, the "no transition" result is similar to the AK model, discussed in the previous chapter. As in the AK model, the economy always grows at a constant rate.
- However, even though the mathematical structure of these two models are similar, the economic implications of these models are very different, in the sense that the equilibrium in this expanding variety exhibits the endogenous technological progress.
  - In particular, R&D firms spend resources to invent new intermediate goods.
  - They do so because they can profitably sell these goods to the final-good firms.

Thus, in this model, profit incentives drive R&D activities, and this in turn drives economic growth.

On the Inefficiency of the Equilibrium

Inefficiency of Equilibrium

- Is the market equilibrium in this model Pareto optimal? $\rightarrow$ No.
- The sources of inefficiency

1. Intermediate goods firms' monopoly pricing: There is a mark up over the marginal cost.
2. R&D firms' behavior (1): They decide whether or not to engage in R&D *without* taking into account that $N(t)$ improves the productivity of the final good firms.

$$Y(t) = \frac{1}{\alpha} X(t)^{\alpha} (N(t) L(t))^{1-\alpha}$$

   where $X = Nx$.

3. R&D firms' behavior (2): They decide whether or not to engage in R&D *without* taking into account that $N(t)$ improves the productivity of "future" R&D activities.