Endogenous Growth Theory (2): The Role of Public Policies

IEE.B402. Advanced Macroeconomics

Ryoji Ohdoi
Department of Industrial Engineering and Economics,
Tokyo Institute of Technology

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Plan

- On the inefficiency of competitive equilibrium in the model of Romer (1986)
  - Social planner's problem $\rightarrow$ Derivation of social optimum
  - Comparison btw market equilibrium and social optimum
- Endogenous growth with productive government spending
  - Households' and firms' behavior
  - Characterization of equilibrium
  - Growth-maximizing tax rate

Brief Review of Romer (1986)

- Key assumptions (LBD externalities)
  1. The technology level $A(t) =$ the aggregate stock of capital $K(t)$
  2. Each agent takes $A(t)$ as given.

  Although each firm takes $A(t)$ as given, this technology advances endogenously for the economy as a whole.

- We have shown the unique existence of competitive equilibrium path, where the economy grows at a positive constant rate from the initial date.

Implications

- Not only from the model by itself, the importance of Paul Romer's paper stems from its emphasis on the externalities.

- Although such a nonrival nature makes a sustained growth possible, at the same time it makes the equilibrium allocation inferior to the first-best allocation.

- Recall that the dynamic system in the market equilibrium is given by:

$$\dot{K}(t) = (\bar{A} - \delta)K(t) - C(t) \tag{1}$$

$$\dot{C}(t)/C(t) = \gamma \equiv \frac{1}{\theta}(\alpha\bar{A} - \delta - \rho) \tag{2}$$

$$\lim_{t\to\infty} K(t)\exp[-(\alpha\bar{A} - \delta)t] = 0 \tag{3}$$

where $C(t) = c(t)\bar{L}$ and $\bar{A} = (z\bar{L})^{1-\alpha}$.

Social Planner' Problem

- Social planner's problem:

$$\max_{\{c(t),K(t)\}} \quad U = \int_0^\infty e^{-\rho t}\frac{c(t)^{1-\theta}-1}{1-\theta}dt$$

$$\text{s.t.} \quad \dot{K}(t) = (\bar{A}-\delta)K(t) - c(t)\bar{L}$$

$$K(0) > 0 \text{ given.}$$

$\downarrow$

- Conditions for social optimum:

$$\dot{K}(t) = (\bar{A}-\delta)K(t) - C(t) \tag{4}$$

$$\dot{C}(t)/C(t) = \gamma^{sp} \equiv \tfrac{1}{\theta}(\bar{A}-\delta-\rho) \tag{5}$$

$$\lim_{t\to\infty} K(t)\exp[-(\bar{A}-\delta)t] = 0 \tag{6}$$

Inefficiency of Market Equilibrium in a LBD model

- From (2), (5), and the fact that $\alpha\bar{A} < \bar{A}$, we arrive at the following proposition:

Proposition 1

In the endogenous growth model of learning-by-doing externalities,

1. *The allocation in the competitive equilibrium is inferior to the first best allocation,*
2. *The growth rate in the competitive equilibrium, γ, is lower than that in the socially optimal allocation, γ^{sp}.*

A Remedy: The Policy to Achieve the First-Best Allocation

- Back to the market economy, and introduce the government's activity.
- Suppose that the profit maximization problem of firm i is now given by

$$\max_{k_i(t), L_i(t)} \left[k_i(t)^\alpha A(t)^{1-\alpha} - (1-s)R(t)k_i(t) - w(t) \right] L_i(t)$$

  where $s \in (0,1)$: the subsidy rate.
- First-order-conditions (F.O.Cs):

$$(1-s)R(t) = \alpha \left(\frac{k_i(t)}{A(t)} \right)^{-(1-\alpha)} \tag{7}$$

$$w(t) = (1-\alpha) k_i(t)^\alpha A(t)^{1-\alpha} \tag{8}$$

A Remedy: The Policy to Achieve the First-Best Allocation

- The subsidy is financed by the lump-sum tax:
  1. Household's budget constraint:

$$\dot{a}(t) = r(t)a(t) + w(t) - c(t) - T(t)/\bar{L}$$
$$\Leftrightarrow \dot{K}(t) = (R(t) - \delta)K(t) + w(t)\bar{L} - C(t) - T(t) \quad (C(t) \equiv c(t)\bar{L})$$

  2. Euler equation:

$$\begin{aligned} \dot{C}(t)/C(t) &= \frac{1}{\theta}(r(t) - \rho) \\ &= \frac{1}{\theta}(R(t) - \delta - \rho) \end{aligned}$$

  3. Government's budget constraint:

$$sR(t)K(t) = T(t)$$

- The market-clearing conditions are the same as those in the baseline model on July 6.

A Remedy: The Policy to Achieve the First-Best Allocation

Then, we can show the following proposition:

Proposition 2

The market economy achieves the first-best allocation if $s = 1 - \alpha$.

Endogenous growth with productive government spending

Introduction

- In this section, we show that productive government spending (e.g., public services) is another possible source of sustained growth.
- This type of model is proposed by Barro (1990).

($*$) The notations used here is almost the same as the model with LBD externalities.

Firms' Behavior: Production Function

- Suppose that there are M firms, indexed by $i \in [0, M]$.
- Firm i's production function:

$$Y_i(t) = \bar{Z} K_i(t)^\alpha (G(t) L_i(t))^{1-\alpha}$$
$$= \bar{Z} k_i(t)^\alpha G(t)^{1-\alpha} L_i(t), \quad \bar{Z} > 0, \alpha \in (0,1) \tag{9}$$

$G(t)$: public services provided by the government

$k_i(t)$: firm i's capital-labor ratio

- It is assumed that the public services are public goods, i.e., the goods that are both non-excludable and non-rivalrous.

Firm's Behavior: Profit Maximization

- Profit maximization problem of firm i:

$$\max_{k_i(t), L_i(t)} \quad \Big[(1-\tau)\bar{Z}k_i(t)^{\alpha}G(t)^{1-\alpha} - R(t)k_i(t) - w(t)\Big]L_i(t)$$

with $G(t)$ given.

  - $R(t)$: the rental price of capital, satisfying $R(t) - \delta = r(t)$
  - $\tau \in (0,1)$: the tax rate levied on the sales of each firm Y_i

- F.O.Cs:

$$R(t) = (1-\tau)\alpha\bar{Z}\left(\frac{k_i(t)}{G(t)}\right)^{-(1-\alpha)} \tag{10}$$

$$w(t) = (1-\tau)(1-\alpha)\bar{Z}k_i(t)^{\alpha}G(t)^{1-\alpha} \tag{11}$$

Thus, all firms determine the same capital-labor ratio.

Households' Behavior

- There are homogeneous households of size $\bar{L} > 0$.
- The utility maximization problem of a household is essentially the same as that in the LBD model.

  $\downarrow$
- Euler equation and TVC:

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}(R(t) - \delta - \rho) \tag{12}$$

$$\lim_{t \to \infty} K(t) \exp(-\int_0^t (R(s) - \delta) ds) = 0 \tag{13}$$

Government

- The government's budget constraint:

$$\underbrace{G(t)}_{\text{expenditure}} = \underbrace{\tau \int_0^M Y_i(t)di}_{\text{tax revenue}} \tag{14}$$

Closing the Model

- Let $L(t)$ and $K(t)$ respectively denote the aggregate demand for labor and capital:

$$L(t) \equiv \int_0^M L_i(t)di, \quad K(t) \equiv \int_0^M K_i(t)di$$

- Market-clearing conditions:

$$\text{Asset market:} \quad K(t) = a(t)\bar{L} \tag{15}$$

$$\text{Labor market:} \quad L(t) = \bar{L} \tag{16}$$

Equilibrium Supply of Public Services

- As also verified in the LBD model, we can show that

$$k_i(t) = K(t)/\bar{L} \tag{17}$$

- The government budget constraint (14) is rewritten as

$$\begin{aligned} G(t) &= \tau \int_0^M \bar{Z} k_i(t)^\alpha G(t)^{1-\alpha} L_i(t) di \\ &= \tau \bar{Z} \left(\frac{K(t)}{L(t)} \right)^\alpha G(t)^{1-\alpha} \underbrace{\int_0^M L_i(t) di}_{=L(t)} \\ &= \tau \bar{Z} K(t)^\alpha G(t)^{1-\alpha} \bar{L}^{1-\alpha} \quad (\because \text{Eq.}(16)) \end{aligned}$$

$\downarrow$

- Equilibrium supply of public services:

$$G(t) = \left[\tau \bar{Z} \bar{L}^{1-\alpha} \right]^{1/\alpha} K(t) \tag{18}$$

Equilibrium

- Hereafter, let $\bar{A} \equiv (\bar{Z}\bar{L}^{1-\alpha})^{1/\alpha}$.
- Substituting (16)–(18) into (10) and (11),

$$R(t) = (1-\tau)\tau^{\frac{1-\alpha}{\alpha}}\alpha\bar{A} \tag{19}$$

$$w(t) = (1-\tau)\tau^{\frac{1-\alpha}{\alpha}}(1-\alpha)\bar{A}K(t)/\bar{L} \tag{20}$$

- Aggregate output:

$$Y_t \equiv \int_0^M Y_i(t)di = \tau^{\frac{1-\alpha}{\alpha}}\bar{A}K(t)$$

Dynamic System

- From (19) and (20), we can obtain the dynamics of $K(t)$:

$$\begin{aligned}\dot{K}(t) &= (R(t)-\delta)K(t) + w(t)\bar{L} - C(t) \\ &= (1-\tau)\tau^{\frac{1-\alpha}{\alpha}}\bar{A}K(t) - \delta K(t) - C(t)\end{aligned} \tag{21}$$

- Substituting (19) into (12), the dynamics of $C(t)$ is given by

$$\frac{\dot{C}(t)}{C(t)} = \gamma \equiv \frac{1}{\theta}\left[(1-\tau)\tau^{\frac{1-\alpha}{\alpha}}\alpha\bar{A} - \delta - \rho\right] \tag{22}$$

We can show the unique existence of balanced growth path in the same way as the LBD model.

Effect of Tax on the Growth Rate

- By following the same procedure as the LBD model, we can show that
$$\dot{C}(t)/C(t) = \dot{K}(t)/K(t) = \gamma.$$
- How does a change in τ affect γ?

$$\begin{aligned}\frac{d\gamma}{d\tau} &= \frac{\alpha\bar{A}}{\theta}\left(-\tau^{\frac{1-\alpha}{\alpha}} + (1-\tau)\frac{1-\alpha}{\alpha}\tau^{\frac{1-2\alpha}{\alpha}}\right)\\ &= \frac{\alpha\bar{A}}{\theta}\tau^{\frac{1-\alpha}{\alpha}}\left(-1 + \frac{1-\tau}{\tau}\frac{1-\alpha}{\alpha}\right) \gtreqless 0 \;\Leftrightarrow\; \tau \lesseqgtr 1-\alpha \qquad (23)\end{aligned}$$

Proposition 3

The growth rate is maximized at $\tau = 1-\alpha$.

References

- Barro, R. J. (1990) "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy* 98, S103–S125.
- Barro, R. J. and X. Sala-i-Martin (2004) *Economic Growth*, Second Edition, Cambridge, MIT Press.