

Endogenous Growth Theory (2): The Role of Public Policies

IEE.B402. Advanced Macroeconomics

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Plan

- ▶ On the inefficiency of competitive equilibrium in the model of Romer (1986)
 - ▶ Social planner's problem → Derivation of social optimum
 - ▶ Comparison btw market equilibrium and social optimum
- ▶ Endogenous growth with productive government spending
 - ▶ Households' and firms' behavior
 - ▶ Characterization of equilibrium
 - ▶ Growth-maximizing tax rate

Brief Review of Romer (1986)

- ▶ Key assumptions (LBD externalities)
 1. The technology level $A(t)$ = the aggregate stock of capital $K(t)$
 2. Each agent takes $A(t)$ as given.

Although each firm takes $A(t)$ as given, this technology advances **endogenously** for the economy **as a whole**.

- ▶ We have shown the unique existence of competitive equilibrium path, where the economy grows at a positive constant rate from the initial date.

Implications

- ▶ Not only from the model by itself, the importance of Paul Romer's paper stems from its emphasis on the externalities.
- ▶ Although such a nonrival nature makes a sustained growth possible, at the same time it makes the equilibrium allocation inferior to the first-best allocation.
- ▶ Recall that the dynamic system in the market equilibrium is given by:

$$\dot{K}(t) = (\bar{A} - \delta)K(t) - C(t) \quad (1)$$

$$\dot{C}(t)/C(t) = \gamma \equiv \frac{1}{\theta}(\alpha\bar{A} - \delta - \rho) \quad (2)$$

$$\lim_{t \rightarrow \infty} K(t) \exp[-(\alpha\bar{A} - \delta)t] = 0 \quad (3)$$

where $C(t) = c(t)\bar{L}$ and $\bar{A} = (z\bar{L})^{1-\alpha}$.

Social Planner' Problem

- Social planner's problem:

$$\begin{aligned} \max_{\{c(t), K(t)\}} \quad & U = \int_0^{\infty} e^{-\rho t} \frac{c(t)^{1-\theta} - 1}{1-\theta} dt \\ \text{s.t.} \quad & \dot{K}_t = (\bar{A} - \delta)K(t) - c(t)\bar{L} \\ & K(0) > 0 \text{ given.} \end{aligned}$$

↓

- Conditions for social optimum:

$$\dot{K}(t) = (\bar{A} - \delta)K(t) - C(t) \tag{4}$$

$$\dot{C}(t)/C(t) = \gamma^{sp} \equiv \frac{1}{\theta}(\bar{A} - \delta - \rho) \tag{5}$$

$$\lim_{t \rightarrow \infty} K(t) \exp[-(\bar{A} - \delta)t] = 0 \tag{6}$$

Inefficiency of Market Equilibrium in a LBD model

- ▶ From (2), (5), and the fact that $\alpha\bar{A} < \bar{A}$, we arrive at the following proposition:

Proposition 1

In the endogenous growth model of learning-by-doing externalities,

- 1. The allocation in the competitive equilibrium is inferior to the first best allocation,*
- 2. The growth rate in the competitive equilibrium, γ , is lower than that in the socially optimal allocation, γ^{sp} .*

A Remedy: The Policy to Achieve the First-Best Allocation

- ▶ Back to the market economy, and introduce the government's activity.
- ▶ Suppose that the profit maximization problem of firm i is now given by

$$\max_{k_i(t), L_i(t)} \left[k_i(t)^\alpha A(t)^{1-\alpha} - (1-s)R(t)k_i(t) - w(t) \right] L_i(t)$$

where $s \in (0, 1)$: the subsidy rate.

- ▶ First-order-conditions (F.O.Cs):

$$(1-s)R(t) = \alpha \left(\frac{k_i(t)}{A(t)} \right)^{-(1-\alpha)} \quad (7)$$

$$w(t) = (1-\alpha)k_i(t)^\alpha A(t)^{1-\alpha} \quad (8)$$

A Remedy: The Policy to Achieve the First-Best Allocation

- ▶ The subsidy is financed by the lump-sum tax:

1. Household's budget constraint:

$$\begin{aligned}\dot{a}(t) &= r(t)a(t) + w(t) - c(t) - T(t)/\bar{L} \\ \Leftrightarrow \dot{K}(t) &= (R(t) - \delta)K(t) + w(t)\bar{L} - C(t) - T(t) \quad (C(t) \equiv c(t)\bar{L})\end{aligned}$$

2. Euler equation:

$$\begin{aligned}\dot{C}(t)/C(t) &= \frac{1}{\theta}(r(t) - \rho) \\ &= \frac{1}{\theta}(R(t) - \delta - \rho)\end{aligned}$$

3. Government's budget constraint:

$$sR(t)K(t) = T(t)$$

- ▶ The market-clearing conditions are the same as those in the baseline model on July 6.

A Remedy: The Policy to Achieve the First-Best Allocation

Then, we can show the following proposition:

Proposition 2

The market economy achieves the first-best allocation if $s = 1 - \alpha$.

Endogenous growth with productive government spending

Introduction

- ▶ In this section, we show that productive government spending (e.g., public services) is another possible source of sustained growth.
- ▶ This type of model is proposed by Barro (1990).

(*) The notations used here is almost the same as the model with LBD externalities.

Firms' Behavior: Production Function

- ▶ Suppose that there are M firms, indexed by $i \in [0, M]$.
- ▶ Firm i 's production function:

$$\begin{aligned} Y_i(t) &= \bar{Z} K_i(t)^\alpha (G(t) L_i(t))^{1-\alpha} \\ &= \bar{Z} k_i(t)^\alpha G(t)^{1-\alpha} L_i(t), \quad \bar{Z} > 0, \alpha \in (0, 1) \end{aligned} \quad (9)$$

$G(t)$: public services provided by the government

$k_i(t)$: firm i 's capital-labor ratio

- ▶ It is assumed that the public services are public goods, i.e., the goods that are both non-excludable and non-rivalrous.

Firm's Behavior: Profit Maximization

- Profit maximization problem of firm i :

$$\max_{k_i(t), L_i(t)} \left[(1 - \tau) \bar{Z} k_i(t)^\alpha G(t)^{1-\alpha} - R(t) k_i(t) - w(t) \right] L_i(t)$$

with $G(t)$ given.

- $R(t)$: the rental price of capital, satisfying $R(t) - \delta = r(t)$
- $\tau \in (0, 1)$: the tax rate levied on the sales of each firm Y_i
- F.O.Cs:

$$R(t) = (1 - \tau) \alpha \bar{Z} \left(\frac{k_i(t)}{G(t)} \right)^{-(1-\alpha)} \quad (10)$$

$$w(t) = (1 - \tau)(1 - \alpha) \bar{Z} k_i(t)^\alpha G(t)^{1-\alpha} \quad (11)$$

Thus, all firms determine the same capital-labor ratio.

Households' Behavior

- ▶ There are homogeneous households of size $\bar{L} > 0$.
- ▶ The utility maximization problem of a household is essentially the same as that in the LBD model.

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- ▶ Euler equation and TVC:

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}(R(t) - \delta - \rho) \quad (12)$$

$$\lim_{t \rightarrow \infty} K(t) \exp\left(-\int_0^t (R(s) - \delta) ds\right) = 0 \quad (13)$$

Government

- ▶ The government's budget constraint:

$$\underbrace{G(t)}_{\text{expenditure}} = \tau \underbrace{\int_0^M Y_i(t) di}_{\text{tax revenue}} \quad (14)$$

Closing the Model

- ▶ Let $L(t)$ and $K(t)$ respectively denote the aggregate demand for labor and capital:

$$L(t) \equiv \int_0^M L_i(t) di, \quad K(t) \equiv \int_0^M K_i(t) di$$

- ▶ Market-clearing conditions:

$$\text{Asset market:} \quad K(t) = a(t)\bar{L} \quad (15)$$

$$\text{Labor market:} \quad L(t) = \bar{L} \quad (16)$$

Equilibrium Supply of Public Services

- ▶ As also verified in the LBD model, we can show that

$$k_i(t) = K(t)/\bar{L} \quad (17)$$

- ▶ The government budget constraint (14) is rewritten as

$$\begin{aligned} G(t) &= \tau \int_0^M \bar{Z} k_i(t)^\alpha G(t)^{1-\alpha} L_i(t) di \\ &= \tau \bar{Z} \left(\frac{K(t)}{L(t)} \right)^\alpha G(t)^{1-\alpha} \underbrace{\int_0^M L_i(t) di}_{=L(t)} \\ &= \tau \bar{Z} K(t)^\alpha G(t)^{1-\alpha} \bar{L}^{1-\alpha} \quad (\because \text{Eq.(16)}) \end{aligned}$$

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- ▶ Equilibrium supply of public services:

$$G(t) = [\tau \bar{Z} \bar{L}^{1-\alpha}]^{1/\alpha} K(t) \quad (18)$$

Equilibrium

- ▶ Hereafter, let $\bar{A} \equiv (\bar{Z}\bar{L}^{1-\alpha})^{1/\alpha}$.
- ▶ Substituting (16)–(18) into (10) and (11),

$$R(t) = (1 - \tau)\tau^{\frac{1-\alpha}{\alpha}}\alpha\bar{A} \quad (19)$$

$$w(t) = (1 - \tau)\tau^{\frac{1-\alpha}{\alpha}}(1 - \alpha)\bar{A}K(t)/\bar{L} \quad (20)$$

- ▶ Aggregate output:

$$Y_t \equiv \int_0^M Y_i(t)di = \tau^{\frac{1-\alpha}{\alpha}}\bar{A}K(t)$$

Dynamic System

- ▶ From (19) and (20), we can obtain the dynamics of $K(t)$:

$$\begin{aligned}\dot{K}(t) &= R(t)K(t) + w(t)\bar{L} - C(t) \\ &= (1 - \tau)\tau^{\frac{1-\alpha}{\alpha}}\bar{A}K(t) - \delta K(t) - C(t)\end{aligned}\quad (21)$$

- ▶ Substituting (19) into (12), the dynamics of $C(t)$ is given by

$$\frac{\dot{C}(t)}{C(t)} = \gamma \equiv \frac{1}{\theta} \left[(1 - \tau)\tau^{\frac{1-\alpha}{\alpha}}\alpha\bar{A} - \delta - \rho \right] \quad (22)$$

We can show the unique existence of balanced growth path in the same way as the LBD model.

Effect of Tax on the Growth Rate

- By following the same procedure as the LBD model, we can show that

$$\dot{C}(t)/C(t) = \dot{K}(t)/K(t) = \gamma.$$

- How does a change in τ affect γ ?

$$\begin{aligned}\frac{d\gamma}{d\tau} &= \frac{\alpha \bar{A}}{\theta} \left(-\tau^{\frac{1-\alpha}{\alpha}} + (1-\tau) \frac{1-\alpha}{\alpha} \tau^{\frac{1-2\alpha}{\alpha}} \right) \\ &= \frac{\alpha \bar{A}}{\theta} \tau^{\frac{1-\alpha}{\alpha}} \left(-1 + \frac{1-\tau}{\tau} \frac{1-\alpha}{\alpha} \right) \gtrless 0 \Leftrightarrow \tau \gtrless 1-\alpha\end{aligned}\quad (23)$$

Proposition 3

The growth rate is maximized at $\tau = 1 - \alpha$.

References

- ▶ Barro, R. J. (1990) "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy* 98, S103–S125.
- ▶ Barro, R. J. and X. Sala-i-Martin (2004) *Economic Growth*, Second Edition, Cambridge, MIT Press.