2017 2Q

Wireless Communication Engineering

#5 Demodulation and Detection Error due to Noise

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Course Schedule (1)

	Date	Text	Contents
#1	June 12	1, 7	Introduction to wireless communication systems
#2	June 15	2, 5, etc	Link budget design of wireless access
#3	June 19		Up/down conversion and equivalent baseband system
#4	June 22	3.3, 3.4	Digital modulation and pulse shaping
#5	June 26	3.5	Demodulation and detection error due to noise
#6	June 29		Collaborative exercise for better understanding 1
#7	July 3	4.4	Channel fading and diversity combining
#8	July 6	4.6	Error correction coding

From Previous Lecture

- Digital modulation

$$s_{\mathrm{D}}(t) = s_{\mathrm{DI}}(t) + js_{\mathrm{DQ}}(t) = f\big(m(t)\big)$$

- Amplitude
- Phase
- Frequency

Data rate, power efficiency, complexity, error rate

- Pulse shaping (band limitation)

$$s_{\mathrm{B}}(t) = \int g(\tau)s_{\mathrm{D}}(t-\tau)\,\mathrm{d}\tau$$

- Rectangular
- Nyquist
- Gaussian

Bandwidth, error rate

- IQ analog modulation

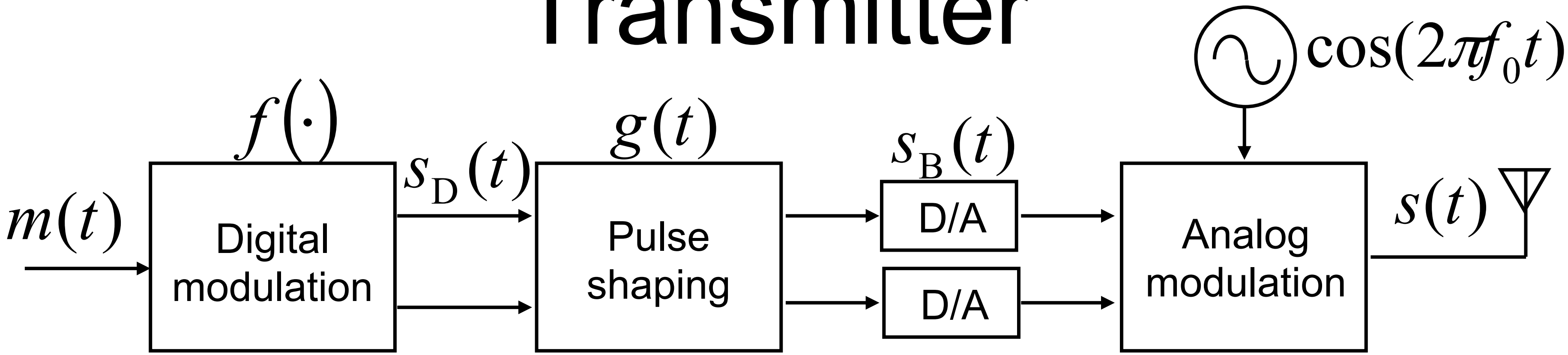
$$s(t) = s_{\mathrm{BI}}(t)\cos(2\pi f_{\mathrm{c}}t) - s_{\mathrm{BQ}}(t)\sin(2\pi f_{\mathrm{c}}t)$$

Carrier frequency

Contents

- Structure of receiver
- Analog demodulation
- Matched filter
- Coherent detection
- Error rate of BPSK signal
- Demonstration

Transmitter



Carrier freq. f_0

Coverage $d_0 = \sqrt{\dfrac{A_t P_t}{4\pi\alpha N_0 f_0 \gamma_0}}$

Symbol length T_s

Bandwidth $B \propto 1/T_s$

Modulation order M

Data rate $\dfrac{\log_2 M}{T_s}$

Modulation $f(\cdot)$

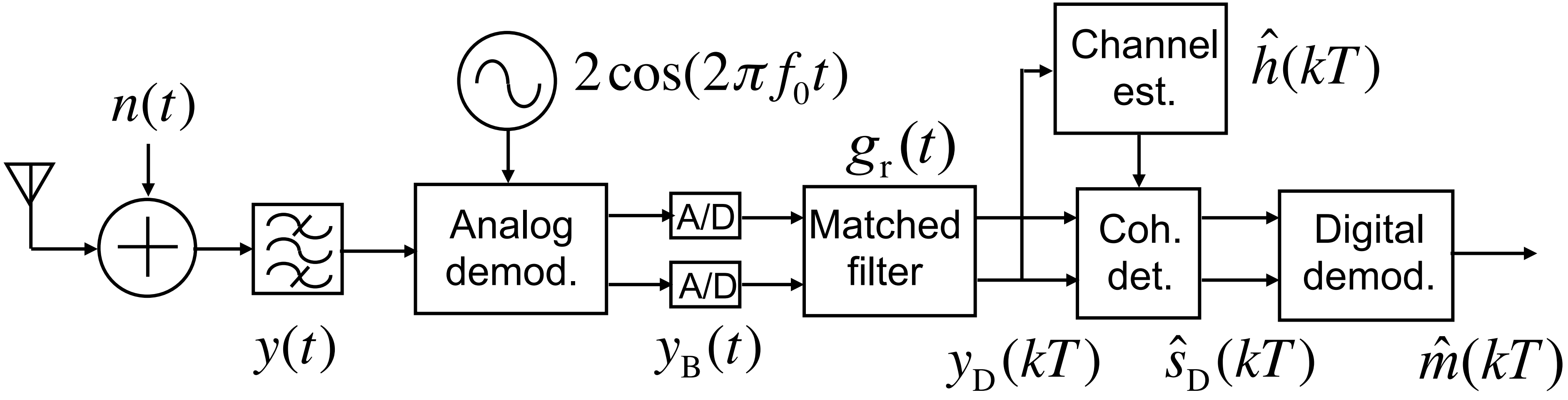
Energy efficiency,
Complexity of circuit,
Bit error rate p_e

Pulse $g(t)$

Power leackage

$$S_B^s(t) = |g(f)|^2 S_D^s(f)$$

Receiver



Additive white noise

Thermal noise generated in receiver

Bandpass fileter

Inter system interference cancellation

Analog demodulation

Convert signal from RF to BB

Matched filter

Maximization of SNR

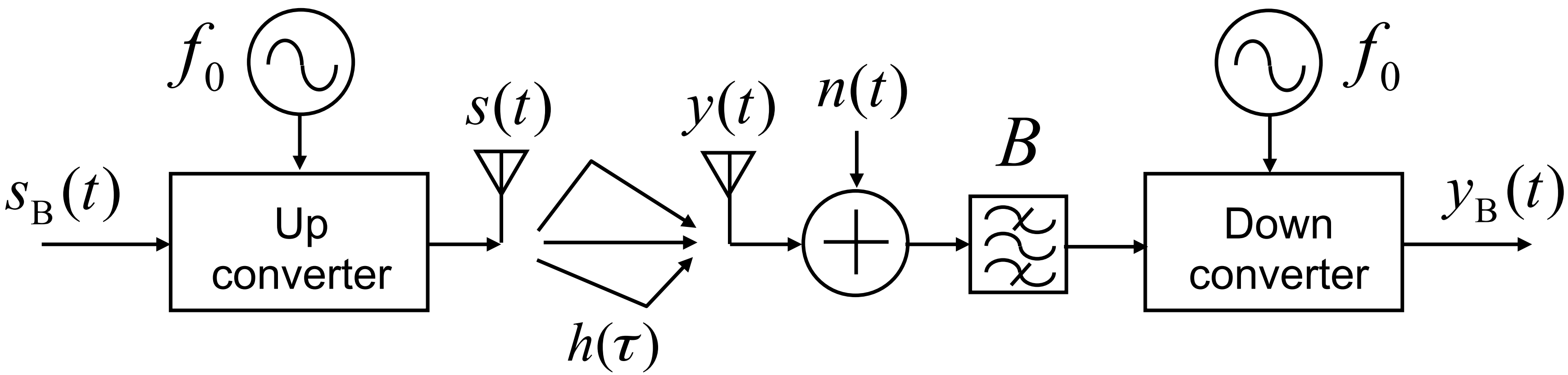
Coherent detection

Compensation of channel response

Digital demodulation

Convert complex signal to message

RF & BB



Transmitter

$$s(t) = \mathrm{Re}\left[s_A(t)\right]$$

$$s_A(t) = s_B(t)e^{j2\pi f_0 t}$$

$$s_B(t)$$

Receiver

$$y(t) = \int h(\tau)s(t-\tau)\mathrm{d}\tau$$

$$y_A(t) = y(t) + j\,\mathrm{hilb}\left(y(t)\right)$$

$$y_B(t) = y_A(t)e^{-j2\pi f_0 t}$$

Analog Demodulation

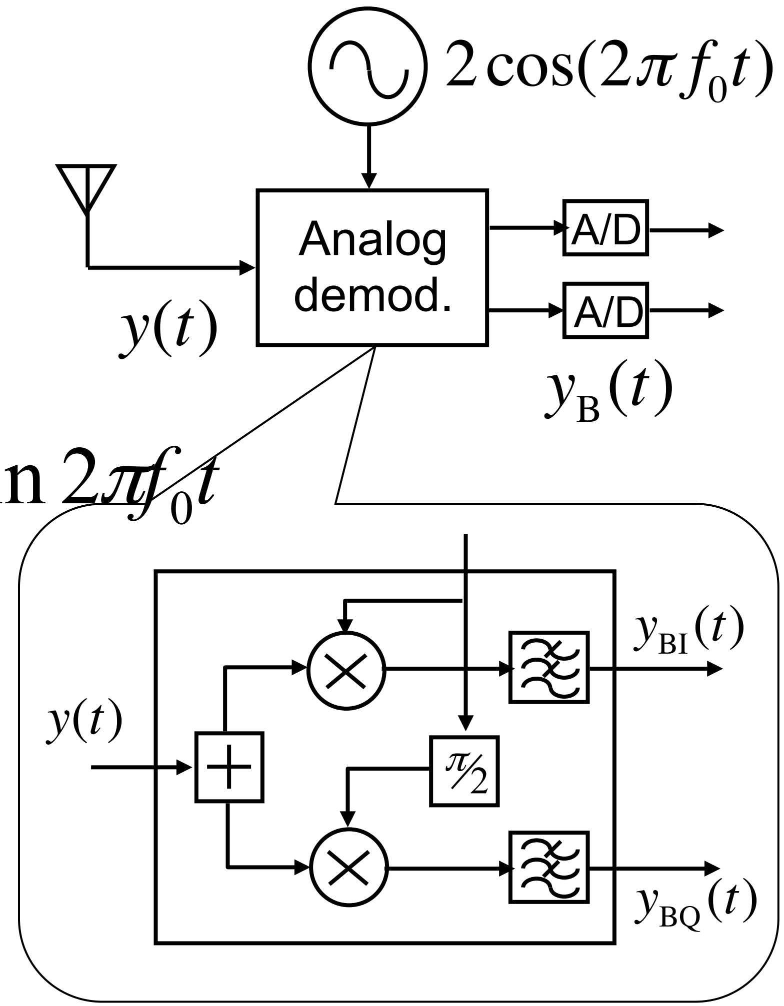
BB receive signal

$$y_{\mathrm{B}}(t) = y_{\mathrm{BI}}(t) + j y_{\mathrm{BQ}}(t)$$

$$y_{\mathrm{A}}(t) = y_{\mathrm{B}}(t) e^{j2\pi f_0 t}$$

$$y(t) = \mathrm{Re}\left[y_{\mathrm{A}}(t) \right]$$

$$= y_{\mathrm{BI}}(t) \cos 2\pi f_0 t - y_{\mathrm{BQ}}(t) \sin 2\pi f_0 t$$



Analog demodulation

Low pass filter

$$y_{\mathrm{BI}}(t) = 2\int y(t) \cos 2\pi f_0 t \, \mathrm{d}t$$

$$y_{\mathrm{BQ}}(t) = -2\int y(t) \sin 2\pi f_0 t \, \mathrm{d}t$$

Narrow Band System

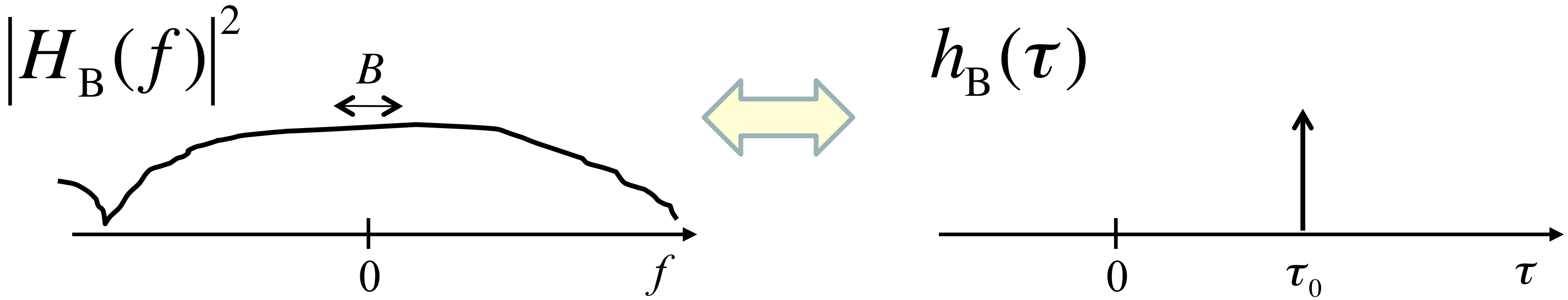
Equivalent BB system

$$y_{\mathrm{B}}(t) = \int h_{\mathrm{B}}(\tau)\tilde{s}_{\mathrm{B}}(t-\tau)\,\mathrm{d}\tau$$

$$h_{\mathrm{B}}(\tau) = h(\tau)e^{-j2\pi f_0 \tau}$$

Narrow band assumption

$$y_{\mathrm{B}}(t) = h_{\mathrm{B}}(\tau_0)\tilde{s}_{\mathrm{B}}(t-\tau_0) = h_{\mathrm{B}} s_{\mathrm{B}}(t) = |h_{\mathrm{B}}| e^{j\theta_0} s_{\mathrm{B}}(t)$$



Noise

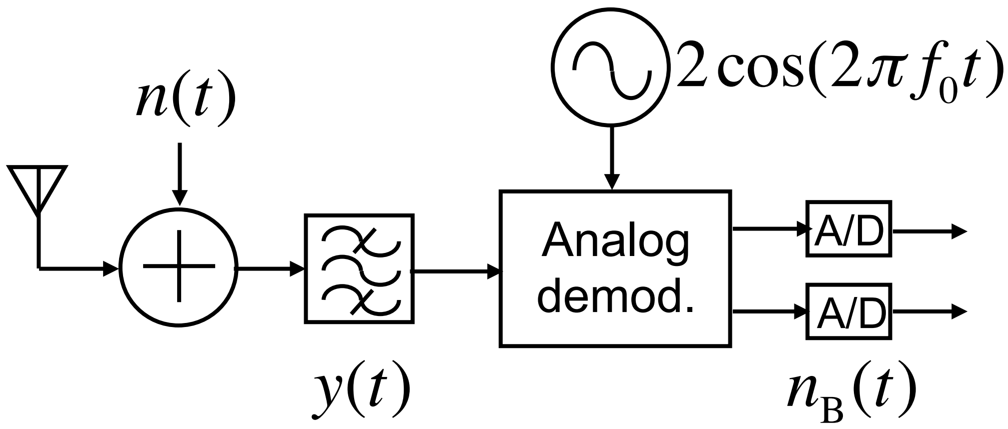
BB noise

$$n_{\mathrm{B}}(t) = n_{\mathrm{BI}}(t) + j n_{\mathrm{BQ}}(t)$$

$$n_{\mathrm{A}}(t) = n_{\mathrm{B}}(t) e^{j2\pi f_0 t}$$

$$n(t) = \mathrm{Re}\left[n_{\mathrm{A}}(t)\right]$$

$$= n_{\mathrm{BI}}(t)\cos 2\pi f_0 t - n_{\mathrm{BQ}}(t)\sin 2\pi f_0 t$$



Analog demodulation

$$n_{\mathrm{BI}}(t) = 2\int n(t)\cos 2\pi f_0 t \, \mathrm{d}t$$

$$n_{\mathrm{BQ}}(t) = -2\int n(t)\sin 2\pi f_0 t \, \mathrm{d}t$$

Property of Noise

Noise power

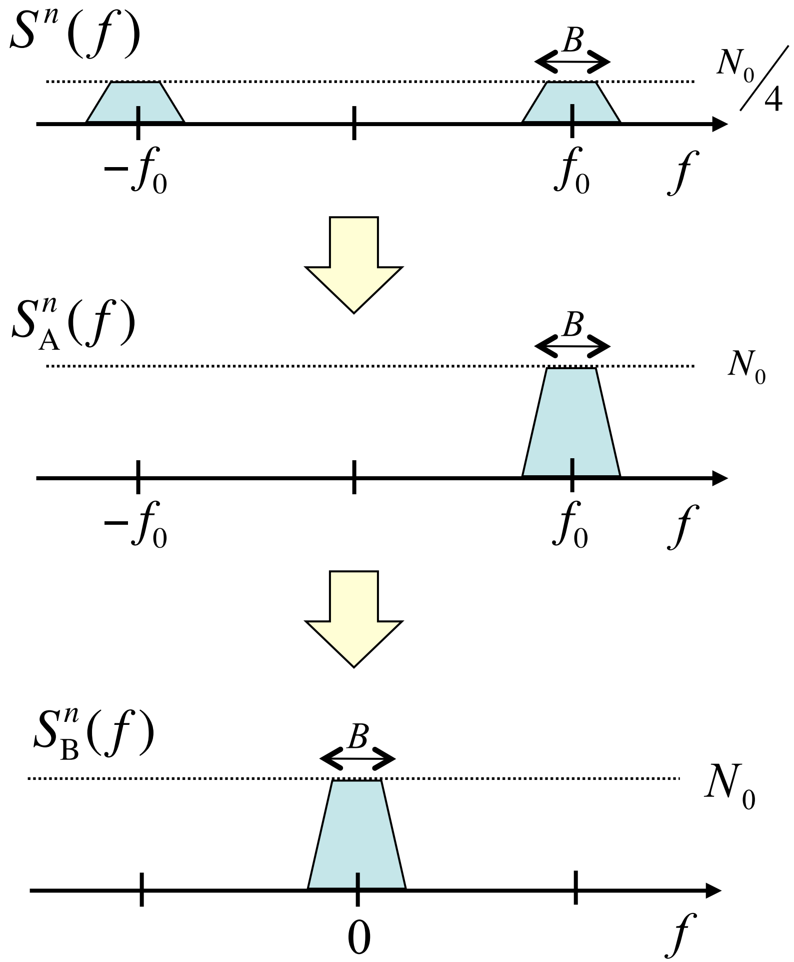
$$P_n = P_{nI} + P_{nQ} = N_0 B = \sigma^2$$

PDF of noise

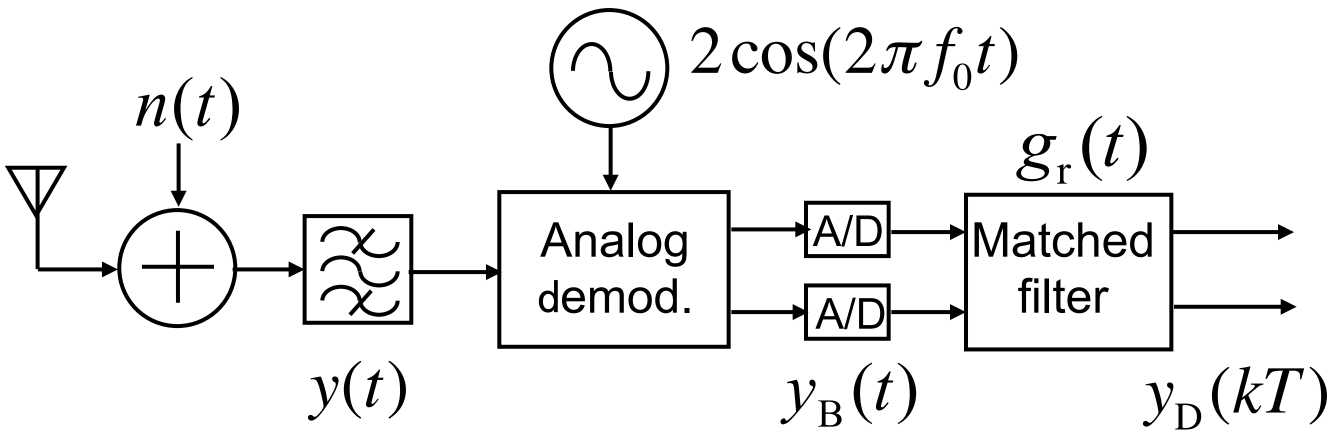
$$p(n_I) = p(n_Q) = \frac{1}{\sqrt{\pi\sigma^2}} e^{-\frac{n_x^2}{\sigma^2}}$$

$$E[n_I] = E[n_Q] = 0$$

$$E\left[|n_I|^2\right] = E\left[|n_Q|^2\right] = \frac{\sigma^2}{2}$$



Matched Filter (1)



BB received signal

$$y_B(t) = h_B s_B(t) + n_B(t)$$

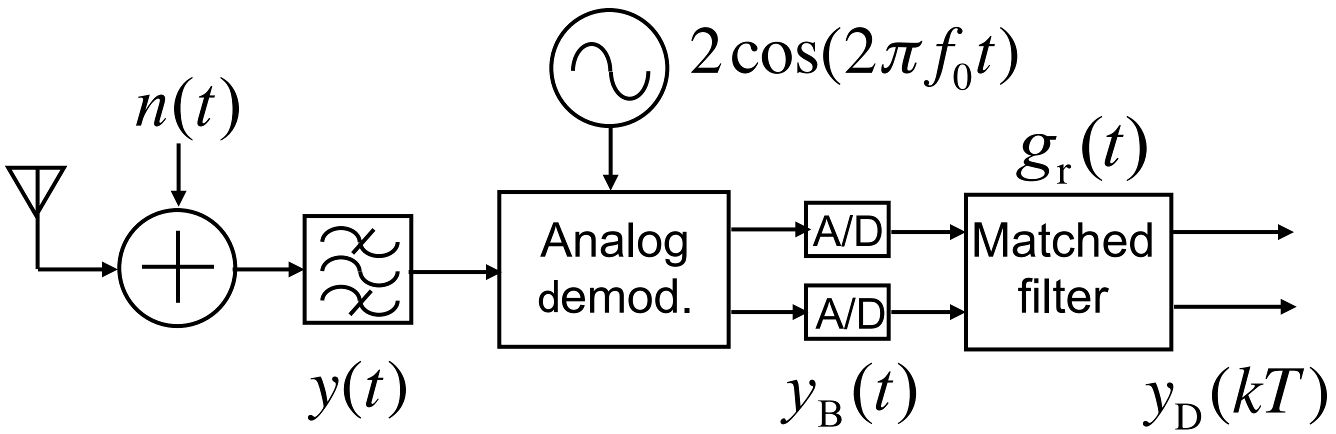
BB transmit signal

$$s_B(t) = g_s(t) \otimes s_D(t) = \sum_n a_n g_s(t - nT)$$

Output of receiver filter

$$y_D(t) = g_r(t) \otimes y_B(t) = g_r(t) \otimes g_s(t) \otimes s_D(t) + g_r(t) \otimes n_B(t)$$

Matched Filter (2)



Receive filter output SNR

$$\gamma = \frac{|g(0)|^2}{E[|n_D(t)|^2]} \qquad g(t) = g_r(t) \otimes g_s(t) \qquad n_D(t) = g_r(t) \otimes n_B(t)$$

Combined pulse of transmitter & receiver

Frequency domain analysis

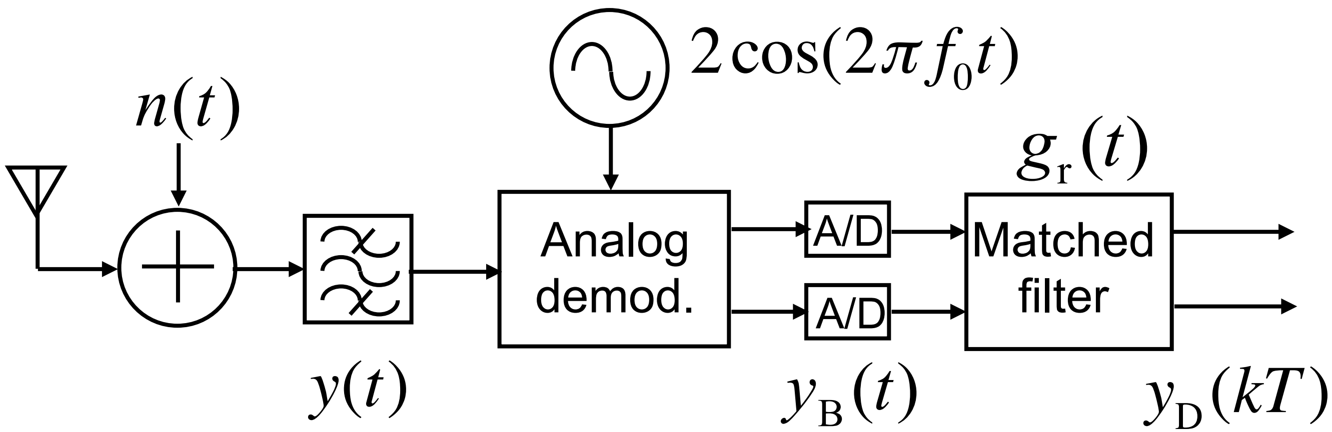
Signal power

$$|g(0)|^2 = \left| \int G_r(f) G_s(f) df \right|^2$$

Noise power

$$E[|n_D(t)|^2] = N_0 \int |G_r(f)|^2 df$$

Matched Filter (3)



Schwarz inequality

$$|g(0)|^2 = \left|\int G_r(f) G_s(f) \mathrm{d}f\right|^2 \le \int |G_r(f)|^2 \mathrm{d}f \int |G_s(f)|^2 \mathrm{d}f$$

Matched filter (SNR maximization)

$$G_r(f) = (G_s(f))^* \qquad g_r(t) = g_s(-t)$$

Maximum SNR

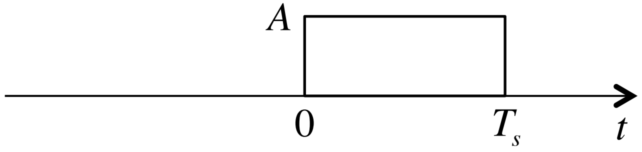
Parseval's theorem

Energy of transmit pulse

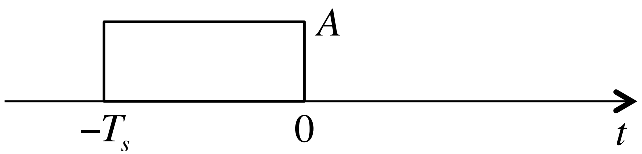
$$\gamma = \frac{|g(0)|^2}{\mathrm{E}[|n(t)|^2]} \le \frac{1}{N_0} \int |G_s(f)|^2 \mathrm{d}f = \frac{1}{N_0} \int |g_s(t)|^2 \mathrm{d}t = \frac{E_s}{N_0} = \frac{P_s T_s}{N_0} = \frac{P_s}{N_0 B} = \frac{P_s}{\sigma^2}$$

Example of Matched Filter

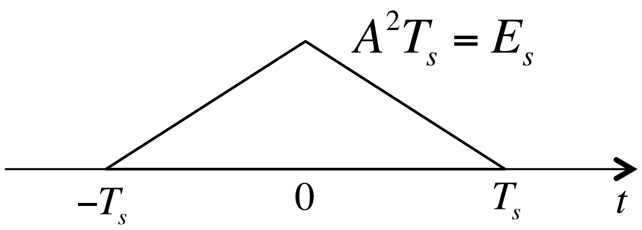
$g_{\mathrm{s}}(t)$



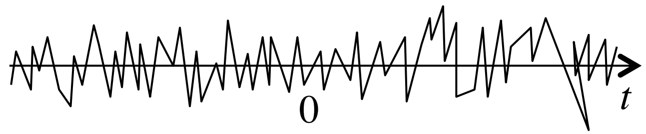
$g_{\mathrm{r}}(t) = g_{\mathrm{s}}(-t)$



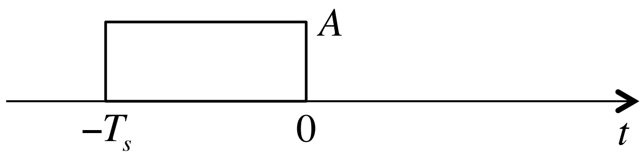
$$g(t) = \int g_{\mathrm{r}}(\tau) g_{\mathrm{s}}(t-\tau)\,\mathrm{d}t$$



$n_{\mathrm{B}}(t)$

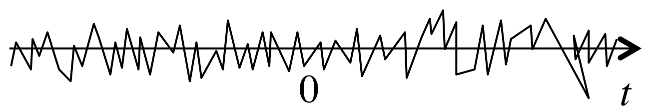


$g_{\mathrm{r}}(t) = g_{\mathrm{s}}(-t)$



$$n_{\mathrm{D}}(t) = \int g_{\mathrm{r}}(\tau) n_{\mathrm{B}}(t-\tau)\,\mathrm{d}t$$

$$\mathrm{E}\left[\left|n_{\mathrm{D}}(t)\right|^2\right] = N_0 \int \left|G_{\mathrm{r}}\left(f\right)\right|^2 \mathrm{d}f = A^2 T_s N_0$$



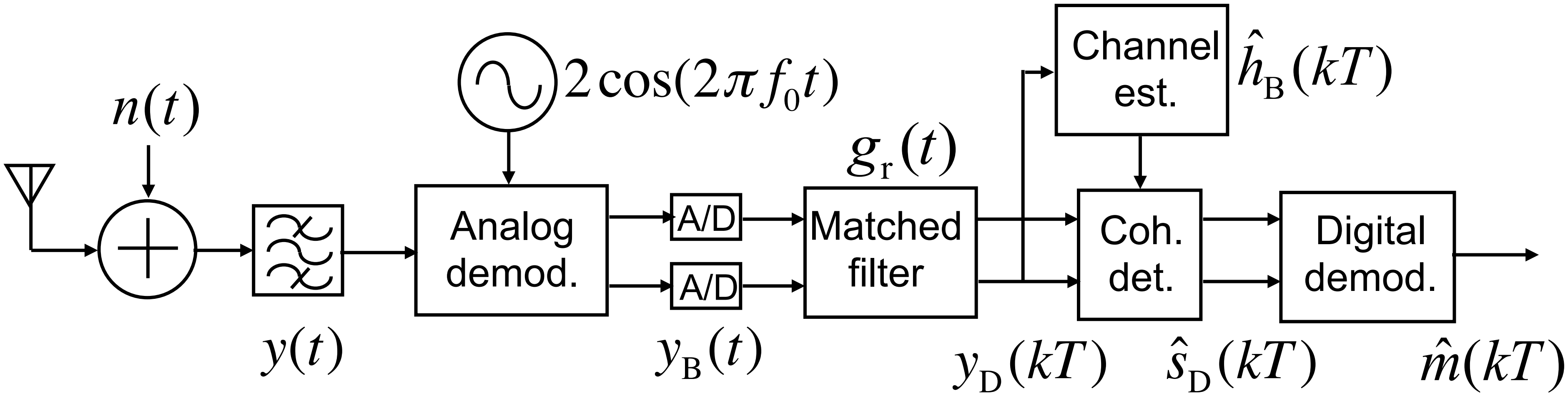
Detection Schemes

Received signal

$$y_D(k) = h_B s_D(k) + n_D(k)$$

Detection	Modulation	Demodulation
Envelope	ASK	$\hat{s}_D(k) = \lvert y_D(k)\rvert / \lvert h_B\rvert$
Correlation	FSK	$\left\lvert\int y_B(t)\exp(j\pi\Delta f t)\mathrm{d}t\right\rvert \gtrless \left\lvert\int y_B(t)\exp(-j\pi\Delta f t)\mathrm{d}t\right\rvert$
Differential	Differential mod. $\tilde{\theta}(k) = \theta(k) + \theta(k-1)$	$\hat{s}_D(k) = y_D(k) / y_D(k-1)$
Coherent	PSK, QAM, MSK	$\hat{s}_D(t) = y_D(t) / h_B$

Coherent Detection



Output of matched filter

$$y_D(t) = g(t) \otimes h_B s_D(t) + g_r(t) \otimes n_B(t)$$

$$y_D(k) = h_B s_D(k) + n_D(k)$$

$$\Rightarrow \quad y(k) = h_B s(k) + n(k)$$

Assume
$$g(0) = \int G(f)\,\mathrm{d}f = 1$$

Coherent detection

$$\hat{s}(k) = y(k) / \hat{h}_B$$

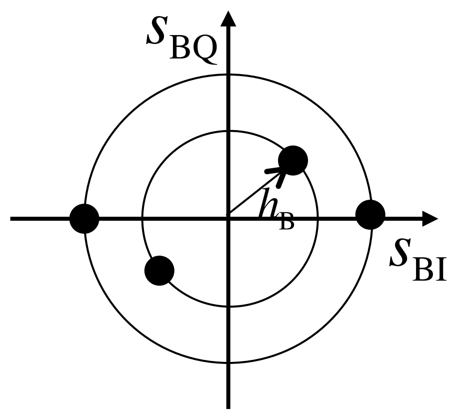
Compensation of channel response

Digital demodulation

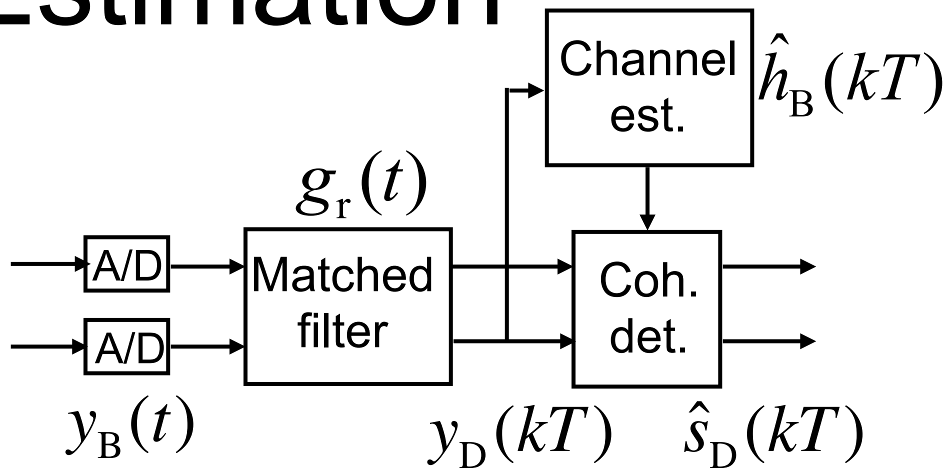
$$\hat{m}(k) = f^{-1}\left(\hat{s}(k)\right)$$

for each modulation method of ASK, PSK, FSK

Constellation



Channel Estimation



Output of matched filter

$$y(k) = h_B s_{TR}(k) + n(k)$$

Training signal

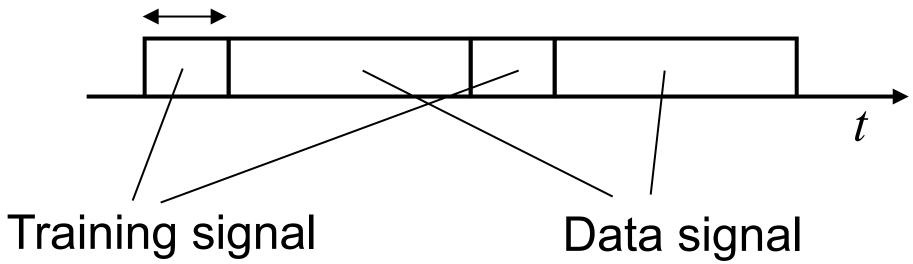
Channel estimation

$$\tilde{h}_B = y(k) \big/ s_{TR}(k)$$

$$\hat{h}_B = \frac{1}{K} \sum_{k=1}^{K} \tilde{h}_B(k)$$

Frame structure of transmit signal

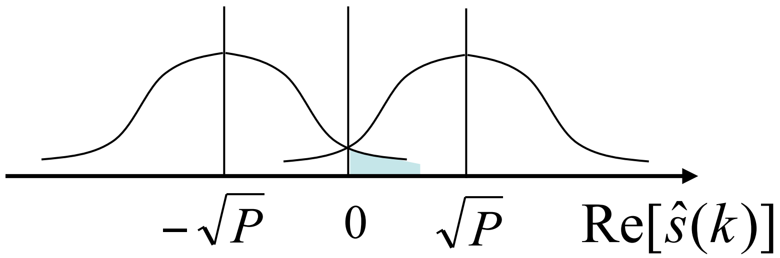
# of training symbols: K



Error Rate of BPSK Signal

Output of coherent detection

$$\hat{s}(k) = y(k)/h_B$$
$$= s(k) + n(k)/h_B$$



Complex Gaussian with variance $\sigma^2/|h_B|^2$

Transmit power

$$E[|s(k)|^2] = P$$

Complementary error function

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} \exp(-z^2)\mathrm{d}z$$

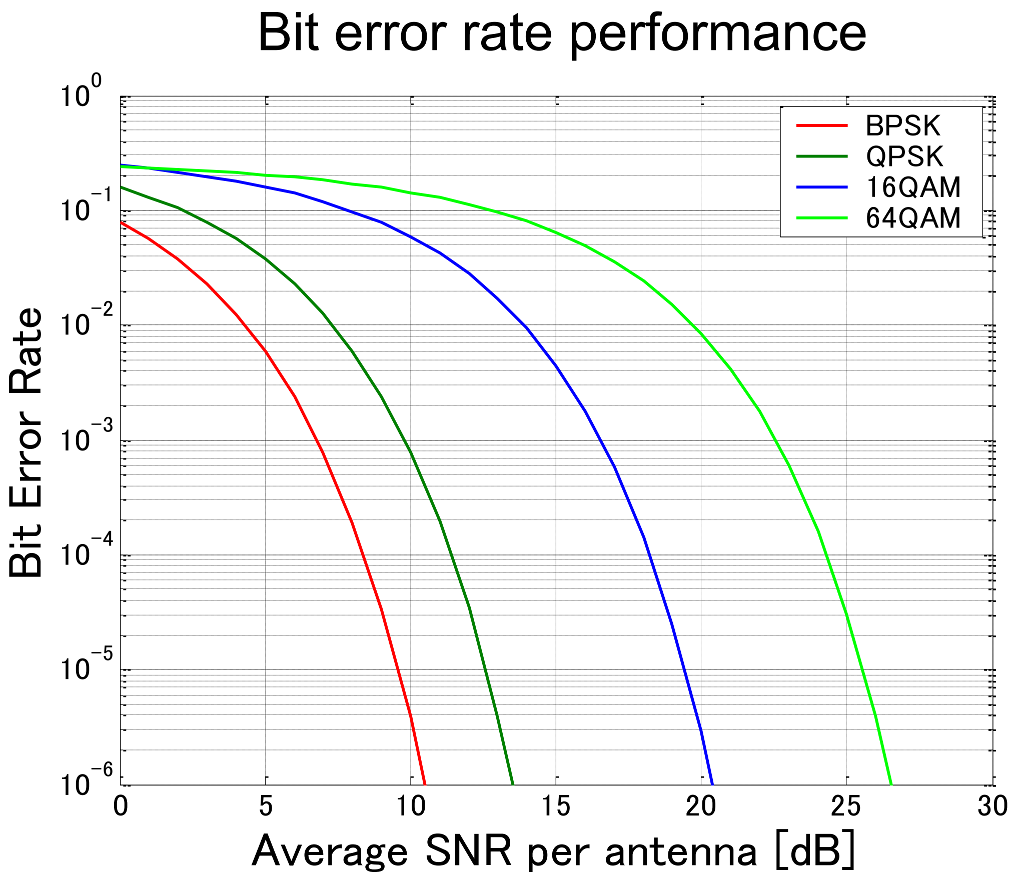
Error rate of BPSK signal

$$P_{eb} = \frac{1}{\sqrt{\pi\sigma^2/|h_B|^2}}\int_0^{\infty} \exp\left(-\frac{(x+\sqrt{P})^2}{\sigma^2/|h_B|^2}\right)\mathrm{d}x$$
$$= \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{P|h_B|^2}{\sigma^2}}\right) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\gamma}\right)$$

Receive SNR

$$\gamma = \frac{P|h_B|^2}{\sigma^2}$$

Error Rate Performance



Summary

- Analog demodulation & matched fileter

$$y(t) = \mathrm{Re}[h s_{\mathrm{B}}(t)\exp(j2\pi f_0 t)] \longrightarrow y_{\mathrm{B}}(t) = h_{\mathrm{B}} s_{\mathrm{B}}(t) + n_{\mathrm{B}}(t)$$

$$y_{\mathrm{D}}(t) = g(t) \otimes h_{\mathrm{B}} s_{\mathrm{D}}(t) + g_{\mathrm{r}}(t) \otimes n_{\mathrm{B}}(t)$$

- Channel estimation & coherent detection

$$\hat{h}_{\mathrm{B}} = \frac{1}{K}\sum_{k=1}^{K} y(k) \Big/ s_{\mathrm{TR}}(k) \longrightarrow \hat{s}(k) = y(k)/h_{\mathrm{B}} = s(k) + n(k)/h_{\mathrm{B}}$$

- Error rate of BPSK signal

$$P_{\mathrm{eb}} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{P|h_{\mathrm{B}}|^2}{\sigma^2}}\right) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\gamma}\right)$$

Nyquist matched fileter

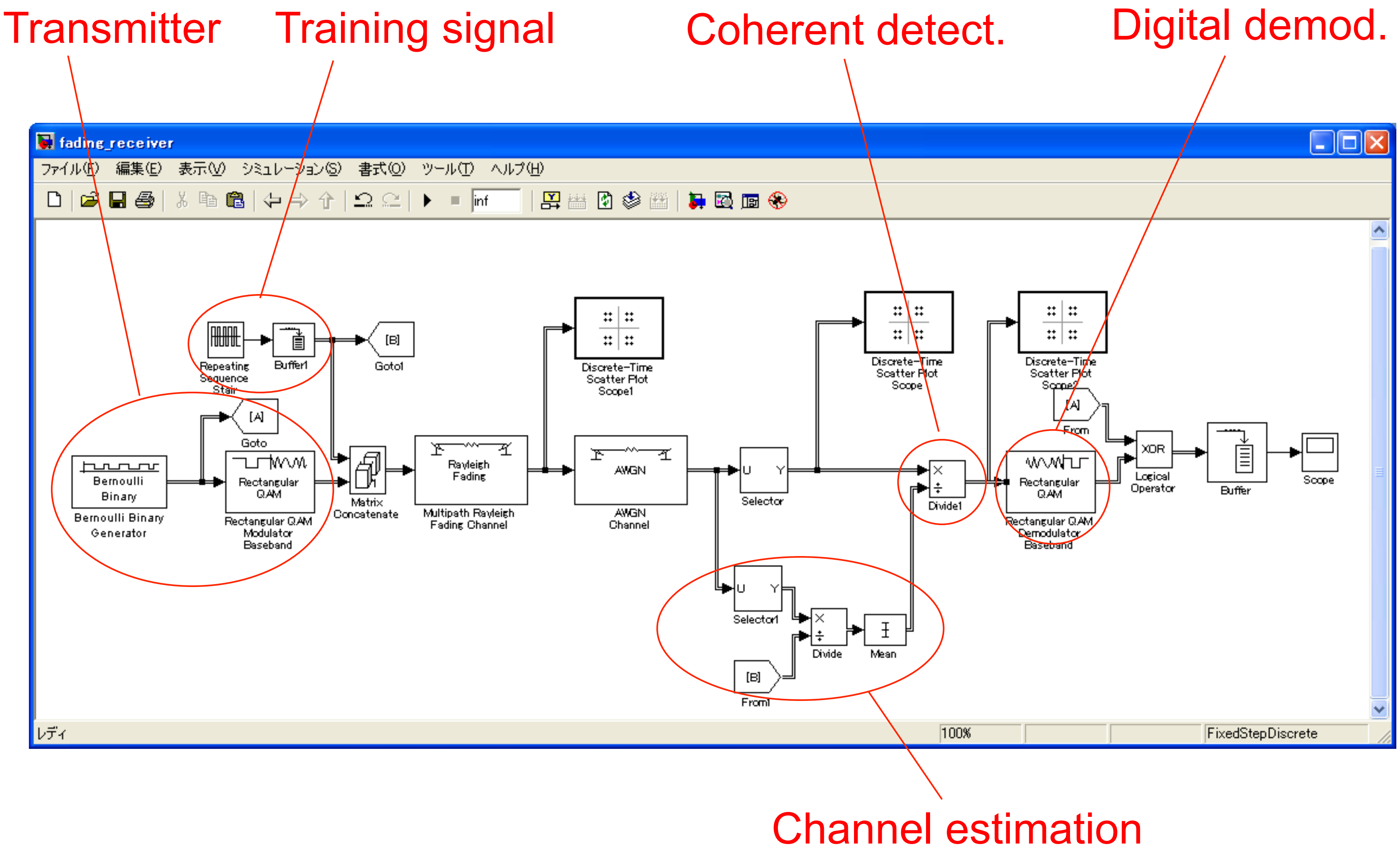
$$g(t) = g_{\mathrm{r}}(t) \otimes g_{\mathrm{s}}(t) = g_{\mathrm{s}}(-t) \otimes g_{\mathrm{s}}(t)$$

$$g(0) = \int G(f)\mathrm{d}f = 1 \qquad \int |G_{\mathrm{r}}(f)|^2 \mathrm{d}f = 1$$

- Error rate of QAM signal

$$P_{\mathrm{eb}} \cong \frac{2}{\log_2 M}\left(1 - \frac{1}{\sqrt{M}}\right)\mathrm{erfc}\left(\sqrt{\frac{E_0|h_{\mathrm{B}}|^2}{\sigma^2}}\right)$$

Demo


Transmitter
Training signal
Coherent detect.
Digital demod.
fading_receiver
ファイル(F) 編集(E) 表示(V) シミュレーション(S) 書式(O) ツール(T) ヘルプ(H)
inf
Repeating Sequence Stair
Buffer1
[B]
Goto1
[A]
Goto
Bernoulli Binary
Bernoulli Binary Generator
Rectangular QAM
Rectangular QAM Modulator Baseband
Matrix Concatenate
Rayleigh Fading
Multipath Rayleigh Fading Channel
Discrete-Time Scatter Plot Scope1
AWGN
AWGN Channel
Selector
Discrete-Time Scatter Plot Scope
Divide1
Discrete-Time Scatter Plot Scope2
From
Rectangular QAM
Rectangular QAM Demodulator Baseband
XOR
Logical Operator
Buffer
Scope
Selector1
Divide
Mean
[B]
From1
レディ
100%
FixedStepDiscrete
Channel estimation

Error Rate of MSK

Output of coherent detection

$$\hat{s}(k) = s(k) + n(k)/h_B$$

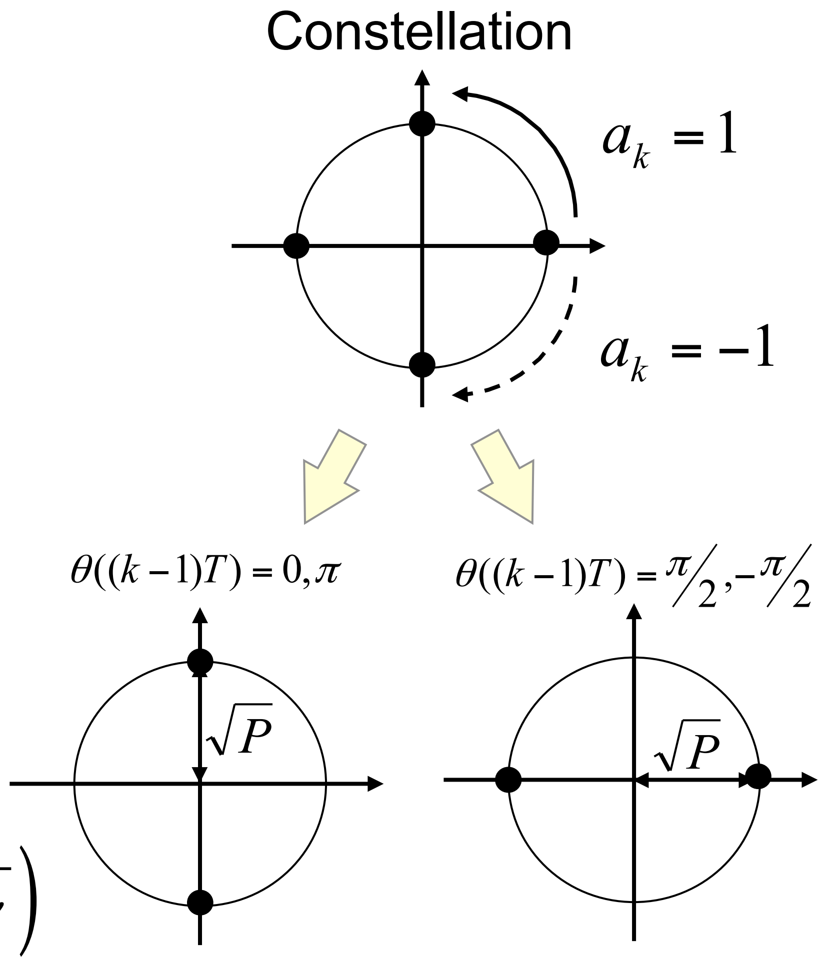
MSK modulation

$$s(k) = \exp(j\theta(k))$$

$$\theta(k) = \pi a_k/2 + \theta(k-1)$$

BER of MSK signal

$$P_{eb} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{P|h_B|^2}{\sigma^2}}\right) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\gamma}\right)$$



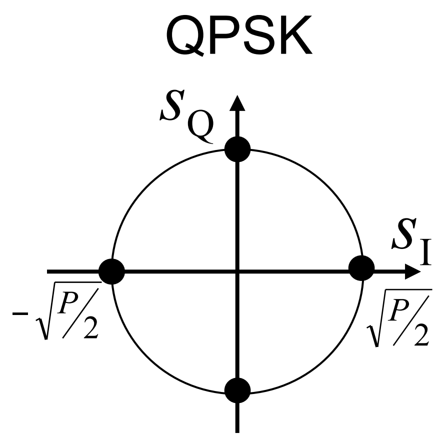
Error Rate of QPSK Signal

Output of coherent detection

$$\hat{s}(k) = y(k)/h_B$$
$$= s_I(k) + js_Q(k) + n(k)/h_B$$

Transmit power

$$E[|s_I(k)|^2] = E[|s_Q(k)|^2] = P/2$$



Bit error rate

$$P_{eb} = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{P|h_B|^2}{2\sigma^2}}\right) = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{\gamma}{2}}\right)$$

Proportional to SNR per bit

Symbol error rate

$$P_{es} = 1 - \underbrace{(1 - P_{eb})}_{I}\underbrace{(1 - P_{eb})}_{Q} = 2P_{eb} - P_{eb}^2 \cong 2P_{eb}$$

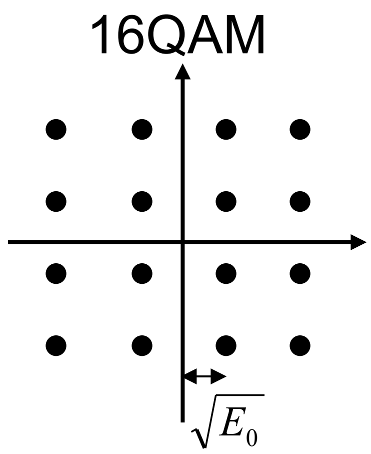
Error Rate of QAM Signal

Symbol error rate

$$P_{\mathrm{es}} = 1-(1-P_{\mathrm{esI}})(1-P_{\mathrm{esQ}}) \cong 2P_{\mathrm{esI}} = 2P_{\mathrm{esQ}}$$

$$P_{\mathrm{esI}} = \underbrace{\left(\frac{\sqrt{M}-2}{\sqrt{M}}\right)\times 2P_{\mathrm{eb}}^{\mathrm{BPSK}}}_{\text{Center}} + \underbrace{\left(\frac{2}{\sqrt{M}}\right)\times P_{\mathrm{eb}}^{\mathrm{BPSK}}}_{\text{Two edges}}$$

$$= \left(1-\frac{1}{\sqrt{M}}\right)\mathrm{erfc}\left(\sqrt{\frac{E_0|h_{\mathrm{B}}|^2}{\sigma^2}}\right)$$



Transmit power

$$P = 2\left(\frac{2E_0}{\sqrt{M}}\sum_{i=1}^{\sqrt{M}/2}(2i-1)^2\right) = \frac{2(M-1)E_0}{3}$$

Bit error rate

$$P_{\mathrm{eb}} \cong \frac{1}{\log_2 M}P_{\mathrm{es}}$$

Symbol error corresponds to one bit error owing to Gray coding