

2016 2Q

Wireless Communication Engineering

# #3 Up/Down Conversion and Equivalent Baseband System

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# Course Schedule (1)

	Date	Text	Contents
#1	June 12	1, 7	Introduction to wireless communication systems
#2	June 15	2, 5, etc	Link budget design of wireless access
#3	June 19		Up/down conversion and equivalent baseband system
#4	June 22	3.3, 3.4	Digital modulation and pulse shaping
#5	June 26	3.5	Demodulation and detection error due to noise
#6	June 29		Collaborative exercise for better understanding 1
#7	July 3	4.4	Channel fading and diversity combining
#8	July 6	4.6	Error correction coding

# From Previous Lecture

- Channel capacity

$$C = B \log_2(1 + \gamma) = \alpha \times f_0 \times R \text{ [bps]}$$

- Friis propagation model

$$P_r = \left( \frac{\lambda_0}{4\pi d} \right)^2 G_r G_t P_t \quad \gamma = \left( \frac{\lambda_0}{4\pi d} \right)^2 \cdot \frac{G_r G_t P_t}{P_n}$$

- User rate and multiple access

$$C_{\text{UE}} = \frac{B \log_2(1 + \gamma)}{N_{\text{UE}}} = \frac{B \log_2(1 + \gamma)}{\pi d_0^2 \eta}$$

- Design of wireless access systems

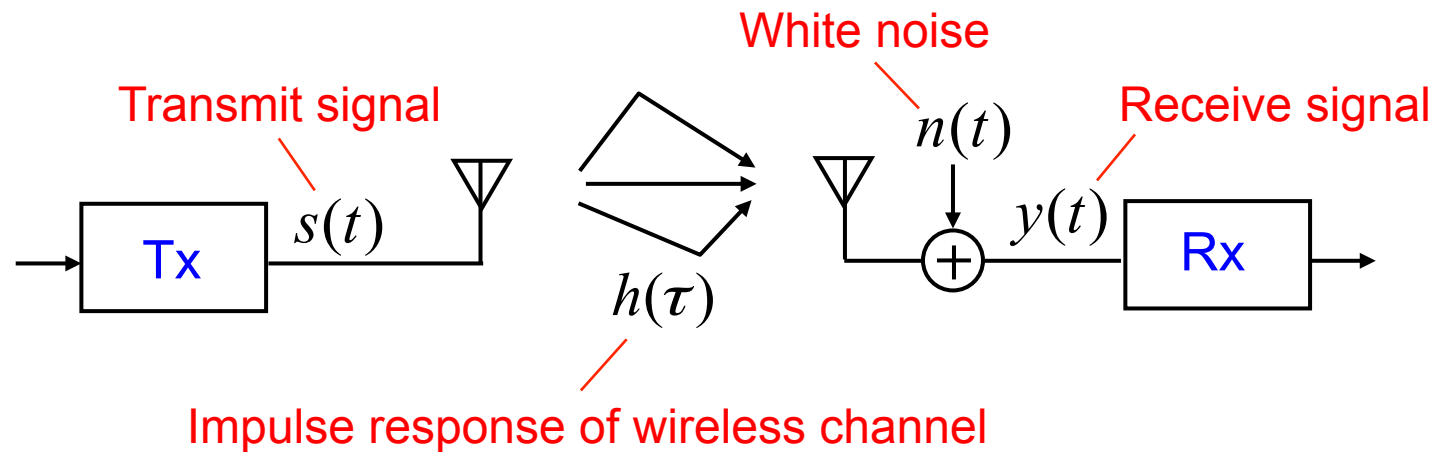
$$C_{\text{UE0}}^{\text{req}} \longrightarrow N_{\text{UE}}, C \longrightarrow d_0, B, R \longrightarrow f_0, P_t, G_t$$

# Contents

- Transmit signal (up conversion)
- Receive signal (down conversion)
- Equivalent baseband signal
- Auto-correlation & power spectrum
- Frequency domain analysis

# System Model

## ■ System model



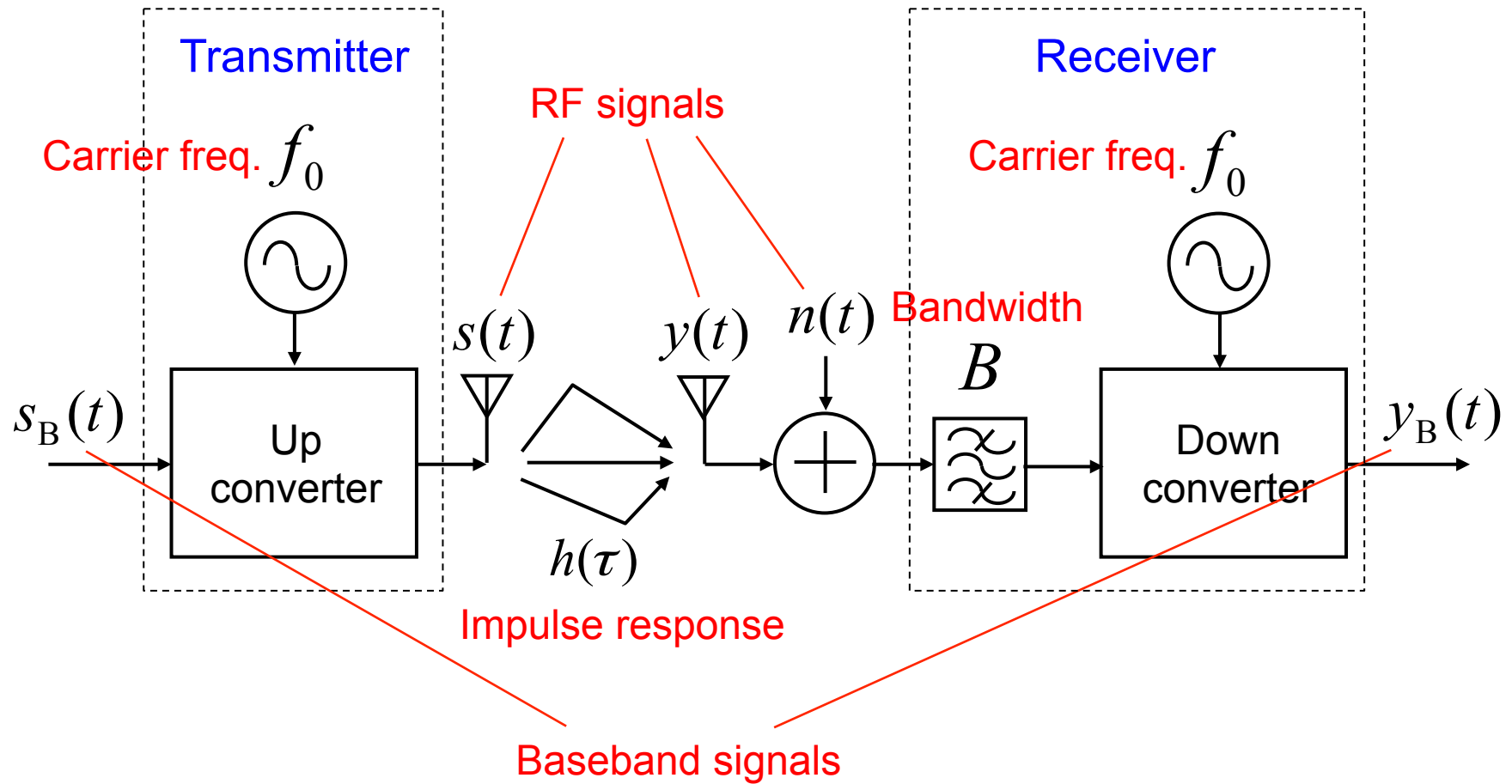
## ■ Receive signal model

$$y(t) = \int h(\tau) s(t - \tau) d\tau + n(t)$$

Assuming delay spread  
(frequency selective fading)

Carrier freq.  $f_0$  & bandwidth  $B$

# Carrier Frequency and Modem



# Transmit Signal (Time Domain)

## ■ Baseband & RF analytic signal

Baseband transmit signal:  $s_B(t) = s_{BR}(t) + js_{BI}(t)$   
In-phase      Quadrature

RF analytic signal:  $s_A(t) = s_B(t)e^{j2\pi f_0 t}$   
Up conversion (BB  $\rightarrow$  RF)

## ■ Transmit signal

$$\begin{aligned} s(t) &= \text{Re}[s_A(t)] \\ &= s_{BR}(t) \cos 2\pi f_0 t - s_{BI}(t) \sin 2\pi f_0 t \\ &= \sqrt{s_{BR}^2(t) + s_{BI}^2(t)} \cos(2\pi f_0 t + \angle s_{BI}/s_{BR}) \end{aligned}$$

# Receive Signal (Time Domain)

## ■ Receive signal

$$\begin{aligned}y(t) &= \int h(\tau) s(t - \tau) d\tau \\&= \operatorname{Re} \left[ \int h(\tau) s_R(t - \tau) d\tau \right] \\&= \operatorname{Re} \left[ \int h(\tau) s_B(t - \tau) e^{j2\pi f_0(t - \tau)} d\tau \right]\end{aligned}$$

## ■ RF analytic & baseband (BB) receive signal

Analytic receive signal:  $y_A(t) = y(t) + j \operatorname{hilb}(y(t))$

Hilbert transformation

Baseband receive signal:  $y_B(t) = y_A(t) e^{-j2\pi f_0 t}$

Down conversion (RF  $\rightarrow$  BB)

# Equivalent Baseband Signal

- RF and baseband (BB) receive signal

$$y(t) = \text{Re} \left[ \int h(\tau) s_B(t - \tau) e^{j2\pi f_0(t - \tau)} d\tau \right]$$

$$y_B(t) = y_A(t) e^{-j2\pi f_0 t} = \left( y(t) + j \text{hilb}(y(t)) \right) e^{-j2\pi f_0 t}$$

- Equivalent baseband system

$$y_B(t) = \int h(\tau) s_B(t - \tau) e^{-j2\pi f_0 \tau} d\tau = \int h_B(\tau) s_B(t - \tau) d\tau$$

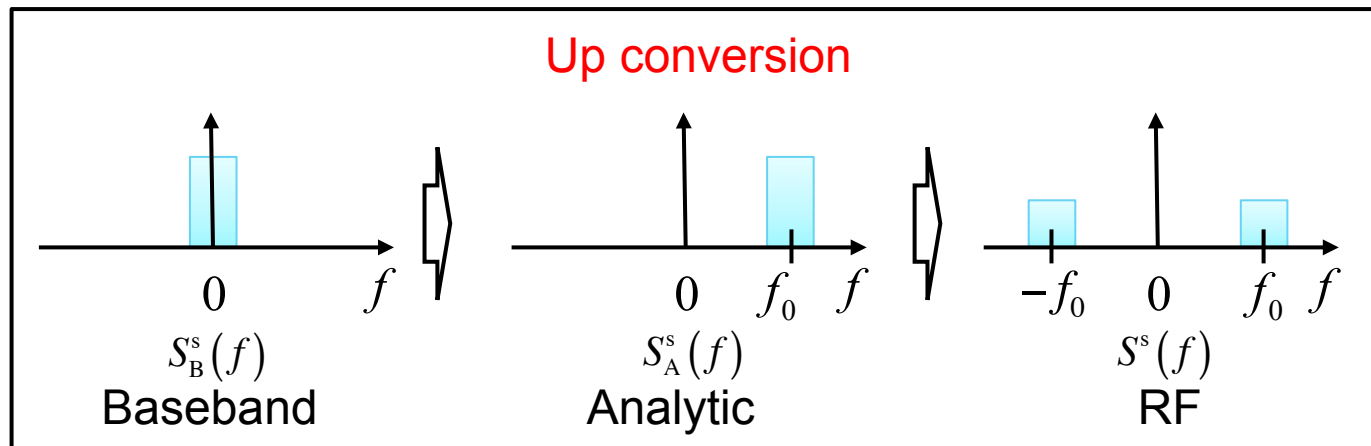
Separation from carrier freq.

Equivalent baseband impulse response:  $h_B(\tau) = h(\tau) e^{-j2\pi f_0 \tau}$

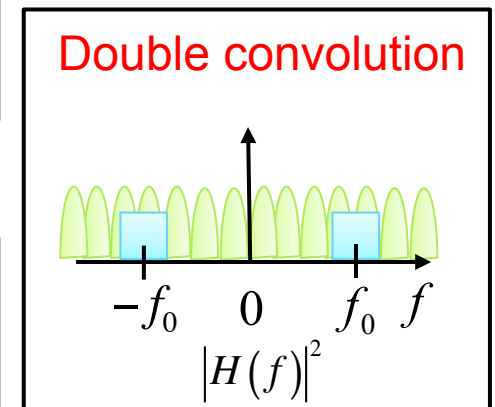
Phase rotation depending on carrier freq.

# Frequency Domain Analysis

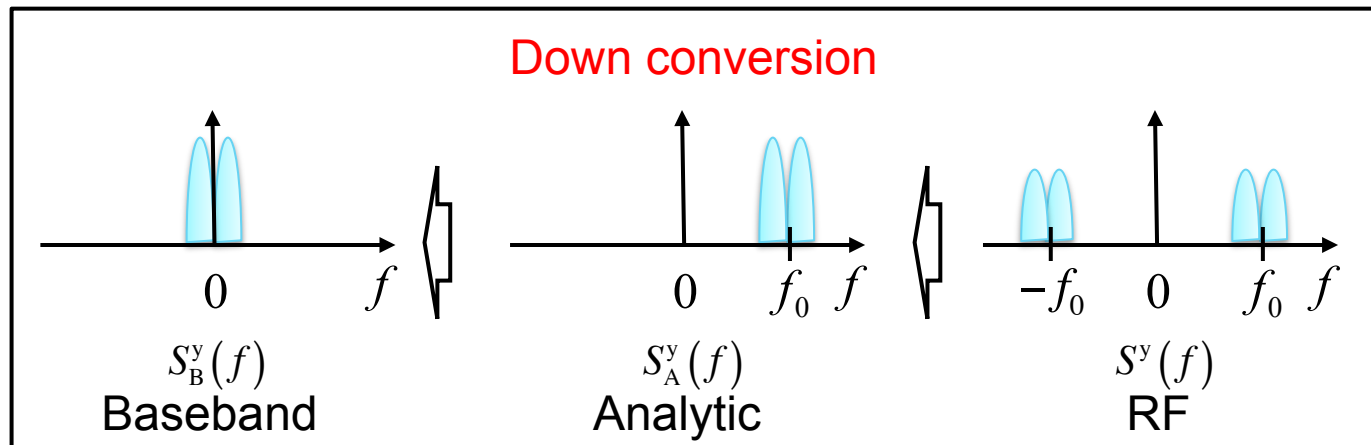
## Transmitter



## RF channel



## Receiver



# Auto-correlation & Power Spectrum

## ■ Random pulse sequence

$$s_B(t)$$



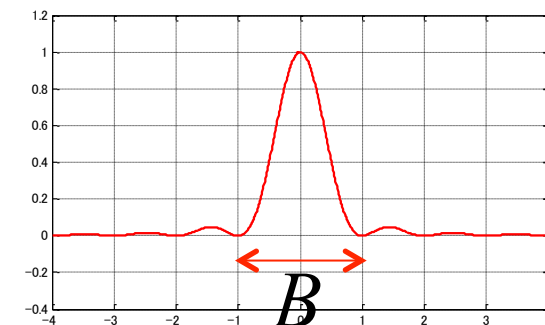
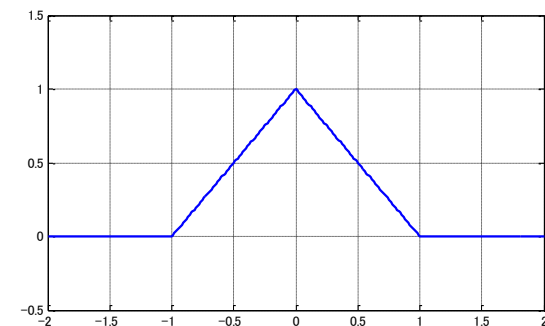
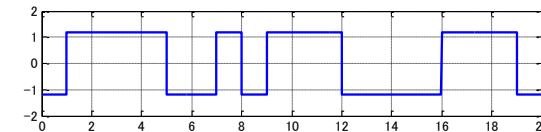
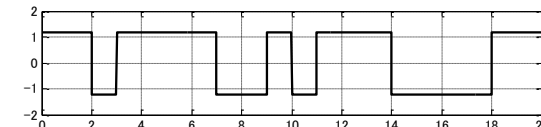
## ■ Auto-correlation

$$R_B^s(\tau) = E[s_B^*(t)s_B(t+\tau)]$$



## ■ Power spectrum

$$S_B^s(f) = \int_{-\infty}^{\infty} R_B^s(\tau) e^{-j2\pi f\tau} d\tau$$



# Analytic Signal (Freq. Domain)

## ■ Auto-correlation of analytic signal

$$\begin{aligned} R_R^s(\tau) &= E[s_A^*(t)s_A(t+\tau)] \\ &= E[s_B^*(t)s_B(t+\tau)]e^{j2\pi f_0\tau} = R_B^s(\tau)e^{j2\pi f_0\tau} \end{aligned}$$

Up conversion (BB  $\rightarrow$  RF)

## ■ Power spectrum of analytic signal

$$S_A^s(f) = \int_{-\infty}^{\infty} R_A^s(\tau)e^{-j2\pi f\tau} d\tau = S_B^s(f - f_0)$$

Frequency conversion

# Transmit Signal (Freq. Domain)

## ■ Auto-correlation of transmit signal

$$s(t) = \text{Re}[s_A(t)] = \frac{1}{2}(s_A(t) + s_A^*(t))$$

$$R^s(\tau) = E[s^*(t)s(t+\tau)]$$

Assuming independency  
between in-phase & quadrature signals

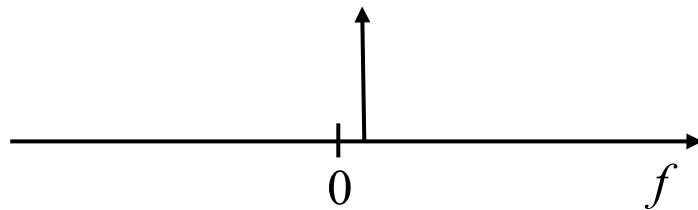
$$= \frac{1}{4}(R_A^s(\tau) + R_A^{s*}(\tau)) = \frac{1}{4}(R_B^s(\tau)e^{j2\pi f_0\tau} + R_B^{s*}(\tau)e^{-j2\pi f_0\tau})$$

## ■ Power spectrum of transmit signal

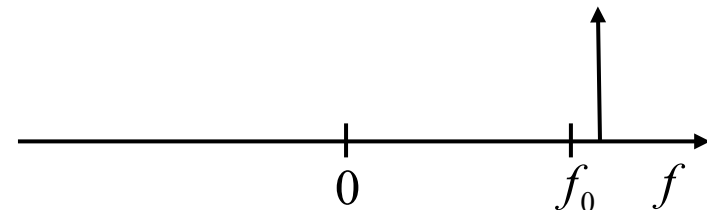
$$S^s(f) = \int_{-\infty}^{\infty} R^s(\tau)e^{-j2\pi f\tau} d\tau = \frac{1}{4}(S_B^s(f - f_0) + S_B^s(-f - f_0))$$

Positive freq.                      Negative freq.

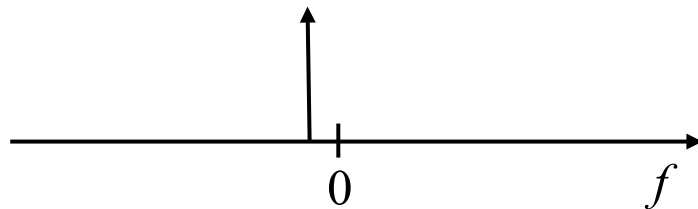
# Example of Transmit Spectrum



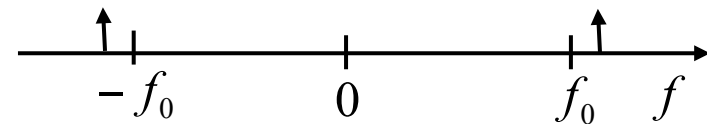
$$S_B^s(f)$$



$$S_A^s(f) = S_B^s(f - f_0)$$



$$S_B^s(-f)$$



$$S^s(f) = \frac{1}{4} \left( S_B^s(f - f_0) + S_B^s(-f - f_0) \right)$$

# Receive Signal (Freq. Domain)

## ■ Auto-correlation of receive signal

$$y_B(t) = \int h_B(\tau) s_B(t - \tau) d\tau$$

$$\begin{aligned} R_B^y(\tau) &= E[y_B^*(t) y_B(t + \tau)] \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_B^*(\tau_1) h_B(\tau_2) R_B^s(\tau - \tau_1 + \tau_2) d\tau_1 d\tau_2 \end{aligned}$$

Double convolution

## ■ Power spectrum of receive signal

$$S_B^y(f) = \int_{-\infty}^{\infty} R_B^y(\tau) e^{-j2\pi f\tau} d\tau = |H_B(f)|^2 S_B^s(f)$$

$$H_B(f) = \int_{-\infty}^{\infty} h_B(\tau) e^{-j2\pi f\tau} d\tau$$

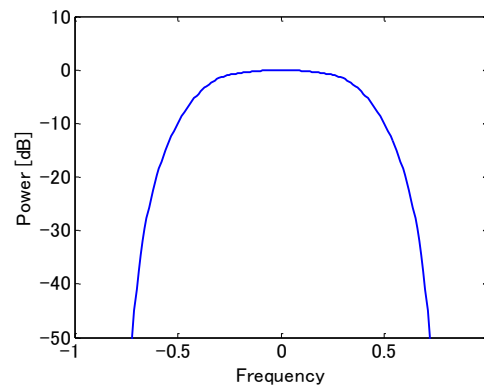
Feature of double convolution

# Example of Receive Spectrum

$$S_B^y(f) = |H_B(f)|^2 S_B^s(f)$$

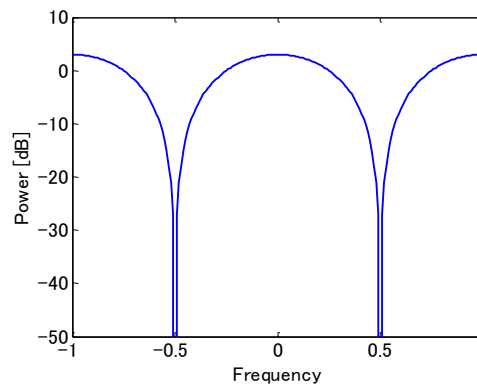
$$S_B^s(f)$$

Transmit spectrum



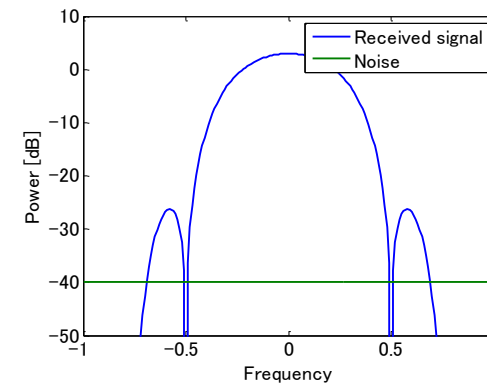
$$|H_B(f)|^2$$

Channel response



$$S_B^y(f)$$

Receive spectrum



# Summary

- Equivalent baseband system

$$y_B(t) = \int h_B(\tau) s_B(t - \tau) d\tau$$

$$y_B(t) = y_A(t) e^{-j2\pi f_0 t} \quad y_A(t) = y(t) + j \text{hilb}(y(t))$$

- Power spectrum of transmit signal

$$S^s(f) = \frac{1}{4} \left( S_B^s(f - f_0) + S_B^s(-f - f_0) \right)$$

- Power spectrum of receive signal

$$S_B^y(f) = |H_B(f)|^2 S_B^s(f)$$

# White Noise

## ■ Receive signal

$$y(t) = \int h(\tau) s(t - \tau) d\tau + n(t)$$

White noise

Transmit (depends on frequency & bandwidth)

## ■ Auto-correlation of white noise

$$R^n(\tau) = E[n^*(t) n(t + \tau)] = \frac{N_0}{2} \delta(0)$$

## ■ Power spectrum of white noise

$$S^n(f) = \int_{-\infty}^{\infty} R^n(\tau) e^{-j2\pi f\tau} d\tau = \frac{N_0}{2}$$

# Equivalent Baseband Noise

- Noise in equivalent baseband system

$$n_A(t) = n(t) + j \text{hilb}(n(t))$$

$$n_B(t) = n_A(t)e^{-j2\pi f_0 t}$$

- Power spectrum and auto-correlation

$$S_B^n(f) = \begin{cases} N_0, & -\frac{B}{2} \leq f \leq \frac{B}{2} \\ 0, & \text{otherwise} \end{cases}$$

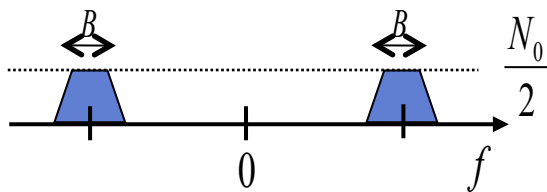
$$R_B^n(\tau) = \int_{-\infty}^{\infty} S_B^n(f) e^{j2\pi f \tau} df = N_0 B \text{sinc}(\tau B)$$

# Power Spectrum of Bandpass Noise

$$P_n = N_0 B = kT_{\text{emp}} B = \alpha N_0 f_0$$

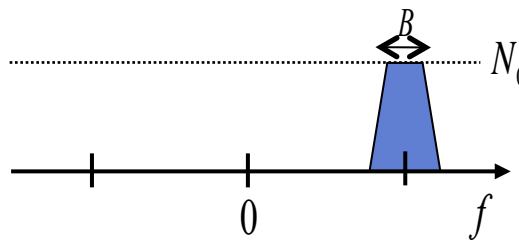
$$S^n(f)$$

Bandpass noise



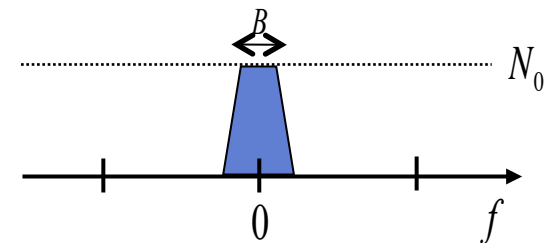
$$S_A^n(f)$$

Analytic signal of noise



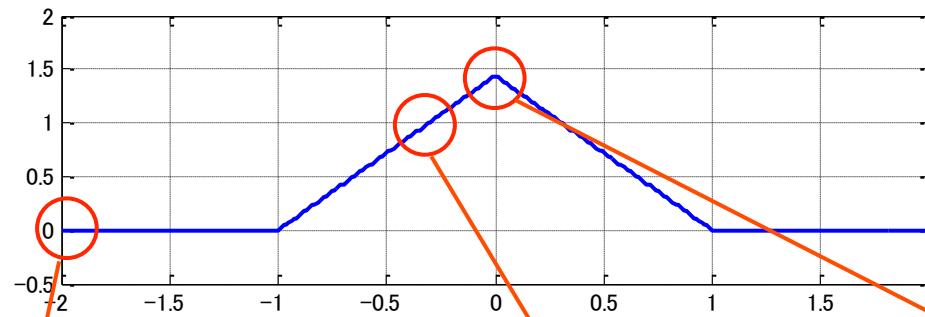
$$S_B^n(f)$$

Baseband noise



# Auto-correlation of Rectangular Random Pulse

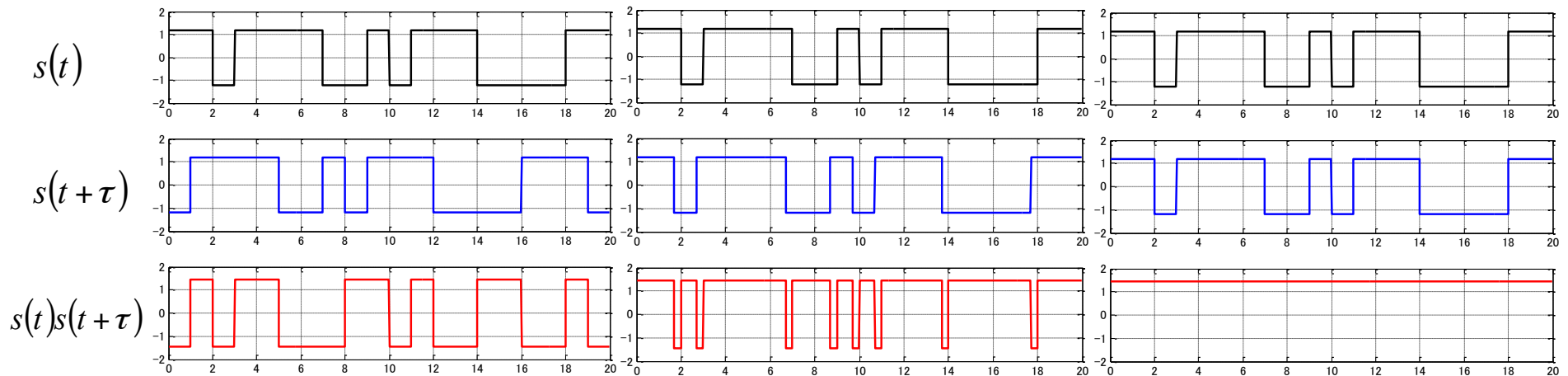
$$R_s(\tau) = E[s(t + \tau)s(t)]$$



$$|\tau| = 2T$$

$$|\tau| = 0.3T$$

$$|\tau| = 0$$



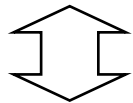
# Power Spectrum of Rectangular Random Pulse

$$R_t(\tau) = \begin{cases} 1 - \frac{|\tau|}{T}, & |\tau| < T \\ 0, & |\tau| \geq T \end{cases}$$

II

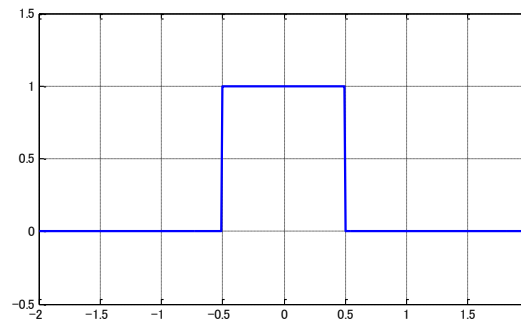
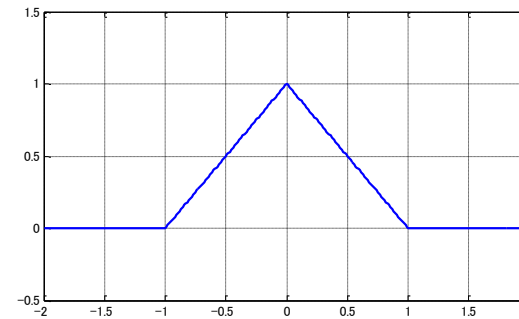
$$R_t(\tau) = \int_{-\infty}^{\infty} R_r(t) R_r(\tau - t) dt$$

$$R_r(\tau) = \begin{cases} 1, & |\tau| \leq T/2 \\ 0, & |\tau| > T/2 \end{cases}$$

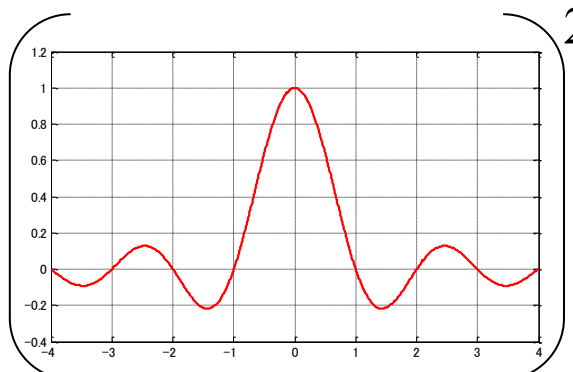
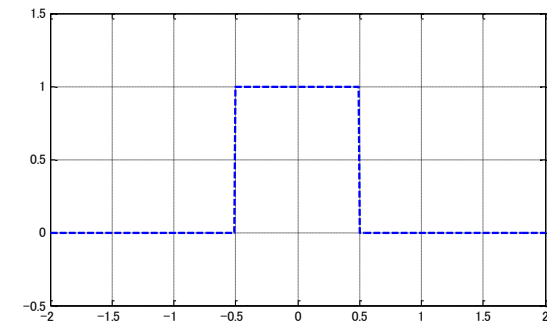


$$S_t(f) = S_r(f) S_r(f)$$

$$S_r(f) = T \operatorname{sinc}(fT)$$



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