2017 2Q

Wireless Communication Engineering

#11 Inter Symbol Interference and Adaptive Equalizer

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Course Schedule (2)

	Date	Text	Contents
#9	July 10	4.6	Error correction coding
#10	July 13		Adaptive modulation coding
	July 17		No class
#11	July 20	4.3	Inter symbol interference and adaptive equalizer
#12	July 24	3.6, 4.5	Spread spectrum and code division multiple access (CDMA)
#13	July 27	3.5	Orthogonal frequency division multiplexing (OFDM)
#14	July 31		Collaborative exercise for better understanding 2
#15	Aug 7	All	Final examination @ S421

From Previous Lecture

- Throughput against modulation order

$$TP(\gamma, M) = \log_2 M \left(1 - p_{\mathrm{eb}}(\gamma)\right)^L$$

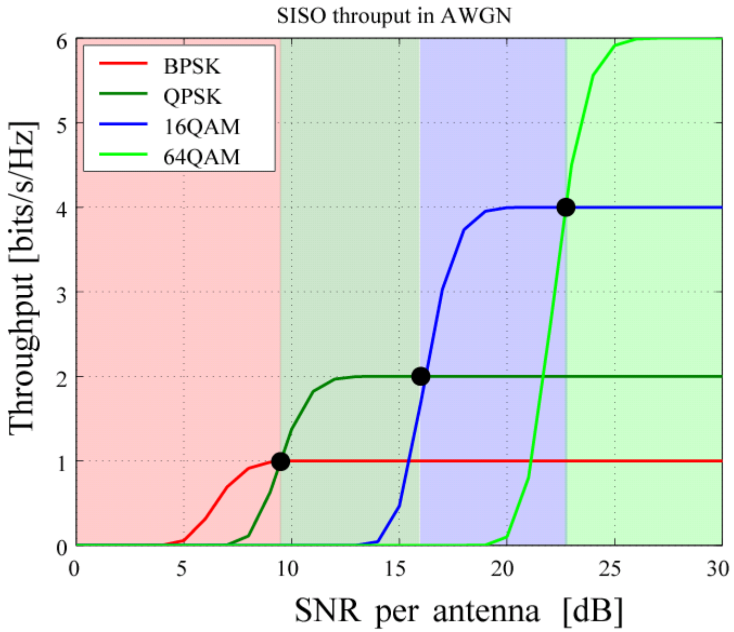
- Adaptive modulation

$$\hat{M} = \arg\max_M TP(\gamma, M)$$

- Throughput performance of AMC

$$\overline{TP}(\bar{\gamma}) = \int_0^{\gamma_1} f(\gamma) TP(\gamma, 2) \mathrm{d}\gamma + \cdots + \int_{\gamma_3}^{\infty} f(\gamma) TP(\gamma, 64) \mathrm{d}\gamma$$

SNR Table for AMC

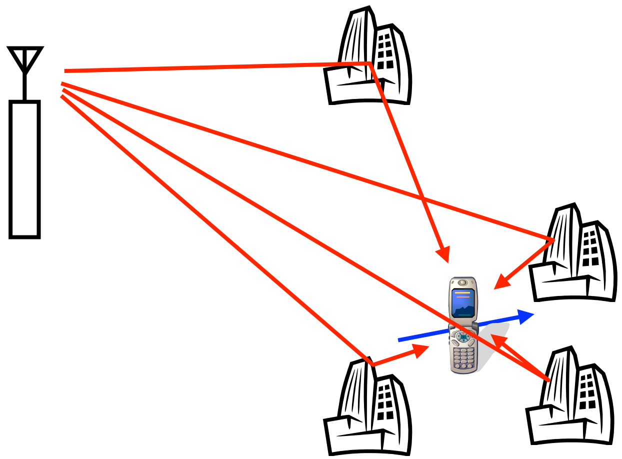


Contents

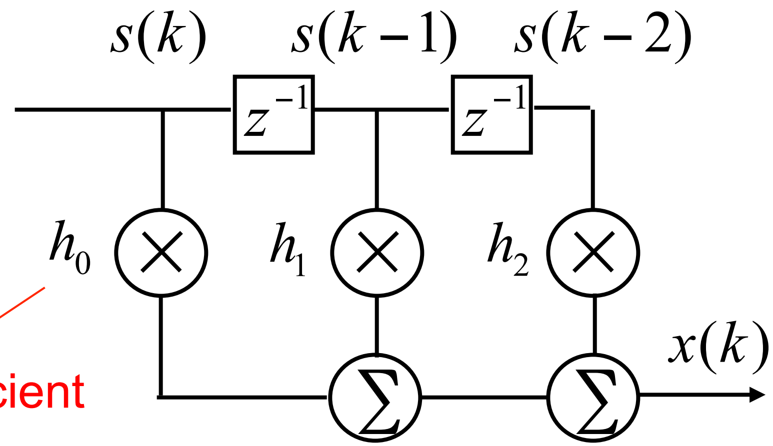
- Delay spread & inter symbol interference
- Classification of equalizer
  - Time domain equalizer (ZF)
  - Frequency domain equalizer (FDE)
  - Maximum likelihood sequence estimation (MLSE)
- Demonstration

Multi-path Channel with Delay Spread

Multi-path channel with delay spread



Convolution with channel coefficients



Channel coefficient

Receive signal model

$$x(t) = \int h(\tau) s(t-\tau) \mathrm{d}\tau + n(t)$$

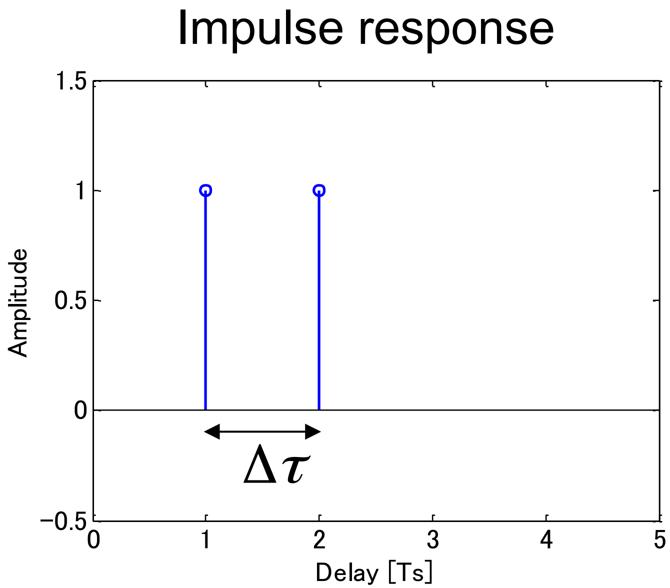
$$x(k) = \sum_{i=0}^{\infty} h_i s(k-i) + n(k)$$

Convolution between transmit signal & channel response

Delay Spread & Frequency Response

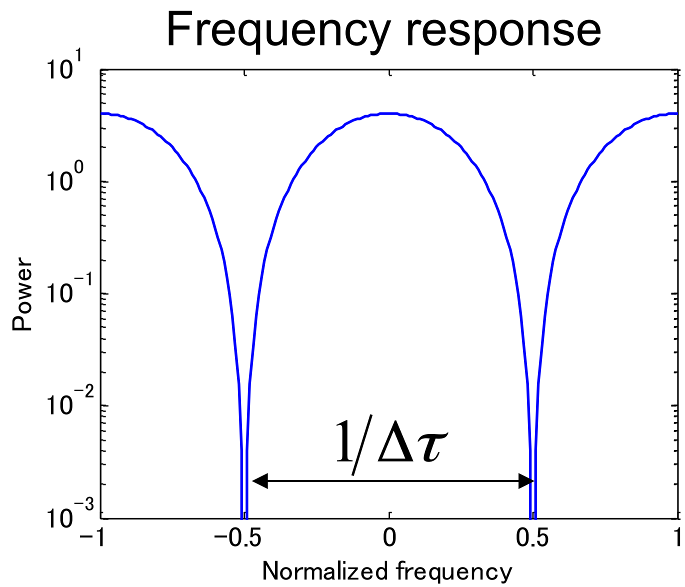
Impulse response

$$h(\tau) = h_0 \delta(\tau) + h_{\Delta\tau} \delta(\tau - \Delta\tau)$$



Frequency response

$$H(f) = h_0 + h_{\Delta\tau} \exp(-j2\pi f \Delta\tau)$$



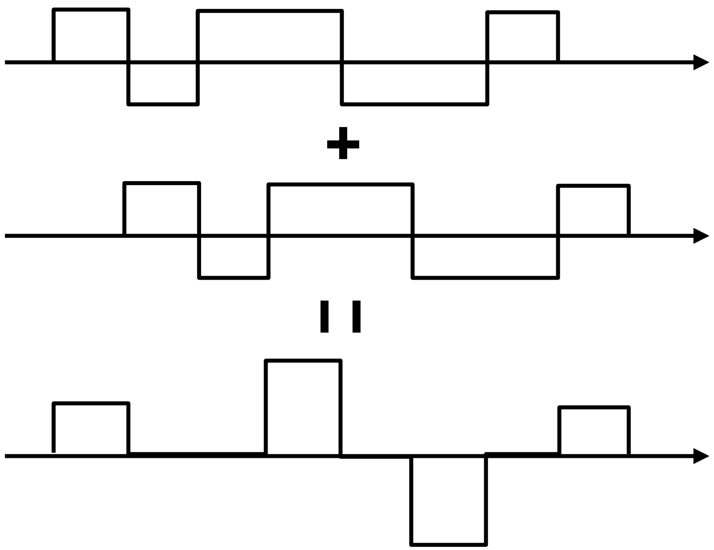
$B > 1/\Delta\tau$: Wide-band

$B << 1/\Delta\tau$: Narrow-band

Inter Symbol Interference (ISI)

2-path model

$$x(t) = h(0)s(t) + h(\tau)s(t-\tau) + n(t)$$

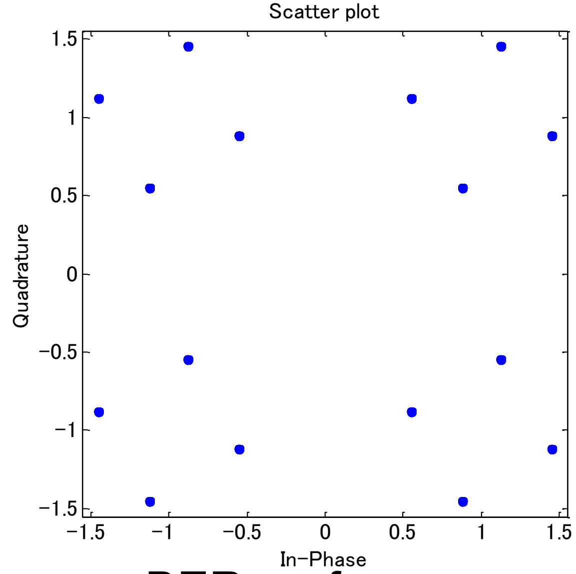


Signal to Interference plus Noise Ratio (SINR)

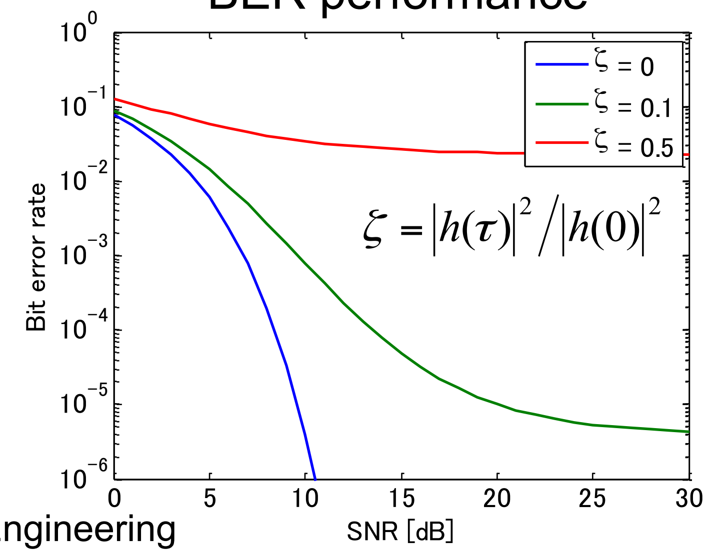
$$\gamma = \frac{|h(0)|^2 P}{|h(\tau)|^2 P + \sigma^2}$$

Interference signal power

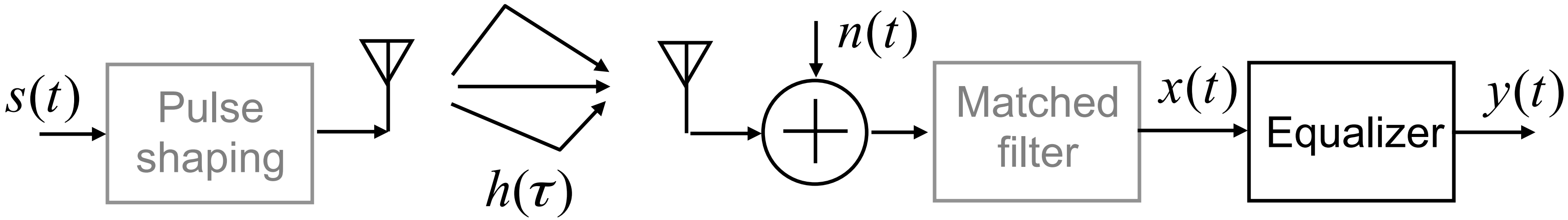
Constellation of QPSK



BER performance

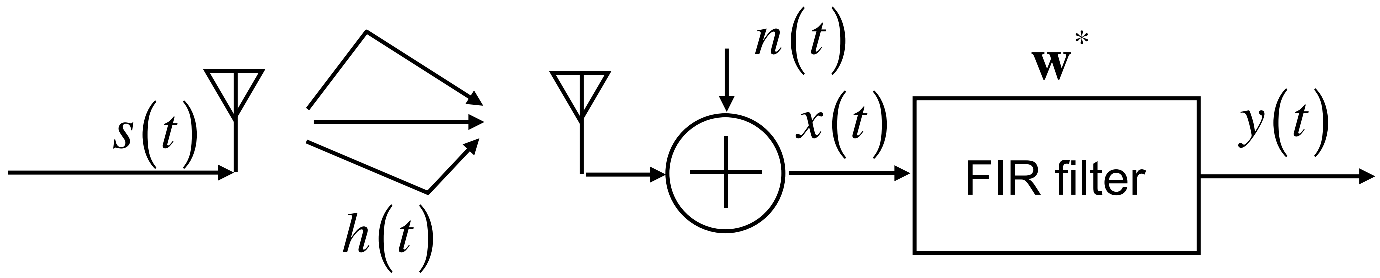


Classification of Equalizer



	Algorithm	Main features
Linear	Zero Forcing (ZF) Minimum Mean Square Error (MMSE) Frequency Domain Equalizer (FDE)	Inverse frequency response Iterative algorithm (LMS) Frame transmission
Nonlinear	Decision Feedback Equalizer (DFE) Maximum Likelihood Sequence Estimation (MLSE)	Infinite Impulse Response (IIR) Viterbi algorithm

Time Domain Equalizer



Time domain

$$x(k) = \sum_{i=0}^{\infty} h_i s(k-i) + n(k)$$

$$y(k) = \sum_{i=-\infty}^{\infty} w_i^* x(k-i)$$

$$= w^* \otimes h \otimes s + w^* \otimes n$$

Frequency (Z) domain

$$Y(z) = \sum_{k=-\infty}^{\infty} y(k) z^{-k}, \quad z = e^{j2\pi f}$$

$$Y(z) = W^*(z)H(z)S(z) + W^*(z)N(z)$$

$$W^*(z) = 1/H(z)$$

Channel inversion

$$W^*(z) = \frac{1}{h_0 + h_1 z^{-1} + h_2 z^{-2} + \cdots}$$

Channel inversion requires IIR filter

Transversal Filter (FIR Filter)

Transversal (FIR) filter

$$y(k) = \sum_{i=-N}^{N} w_i^* x(k-i)$$

Z transformation

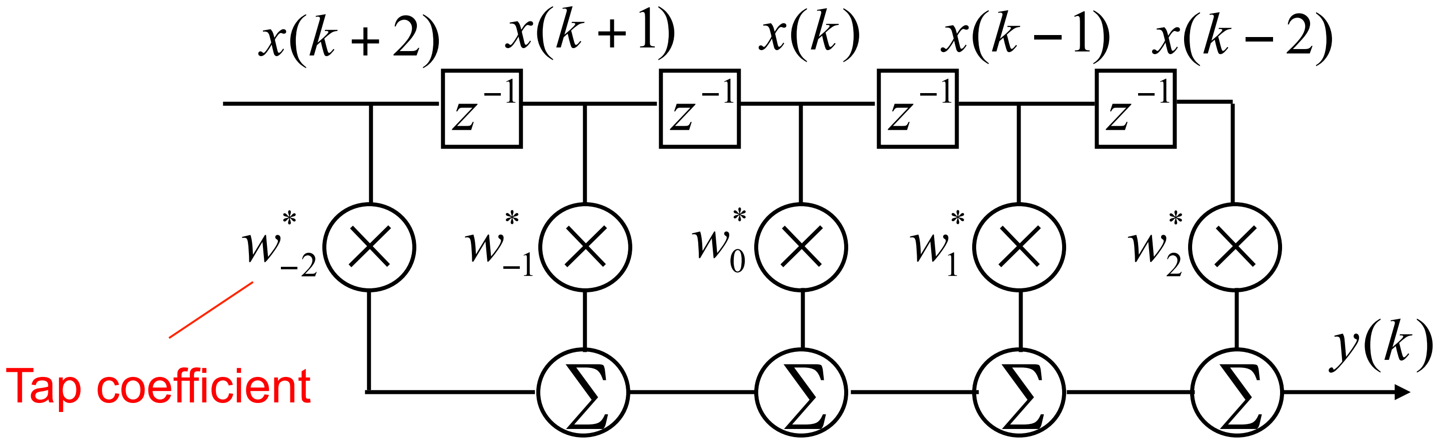
$$W^*(z) = w_{-2}^* + w_{-2}^* z^{-1} + \cdots$$

Convolution Matrix

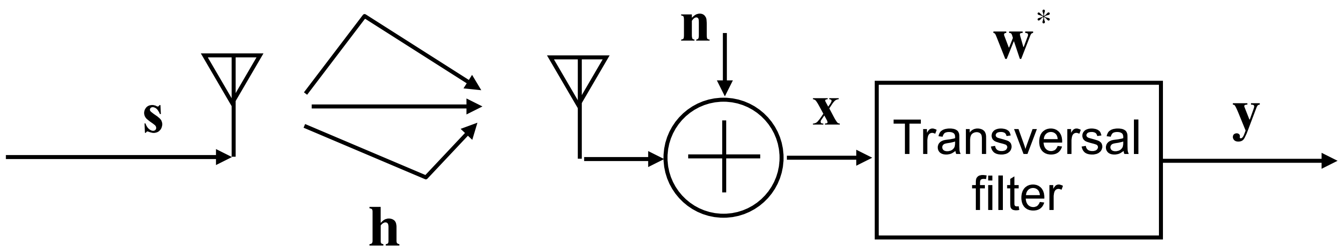
$$\begin{bmatrix} y(-2) \\ y(-1) \\ y(0) \\ y(1) \\ y(2) \end{bmatrix} = \begin{bmatrix} x(0) & x(-1) & x(-2) & x(-3) & x(-4) \\ x(1) & x(0) & x(-1) & x(-2) & x(-3) \\ x(2) & x(1) & x(0) & x(-1) & x(-2) \\ x(3) & x(2) & x(1) & x(0) & x(-1) \\ x(4) & x(3) & x(2) & x(1) & x(0) \end{bmatrix} \begin{bmatrix} w_{-2}^* \\ w_{-1}^* \\ w_0^* \\ w_1^* \\ w_2^* \end{bmatrix}$$

$$\mathbf{y} = \overleftrightarrow{\mathbf{X}} \mathbf{w}^*$$

Cyclic shift matrix



Linear FIR Equalizer (ZF)



Time domain

$$x = h \otimes s + n$$

$$y = w^* \otimes h \otimes s + w^* \otimes n$$

Matrix domain

$$\mathbf{x} = \overleftrightarrow{\mathbf{S}}\mathbf{h} + \mathbf{n}$$

$$\mathbf{y} = \overleftrightarrow{\mathbf{S}}\overleftrightarrow{\mathbf{H}}\mathbf{w}^* + \overleftrightarrow{\mathbf{N}}\mathbf{w}^*$$

ZF linear equalizer

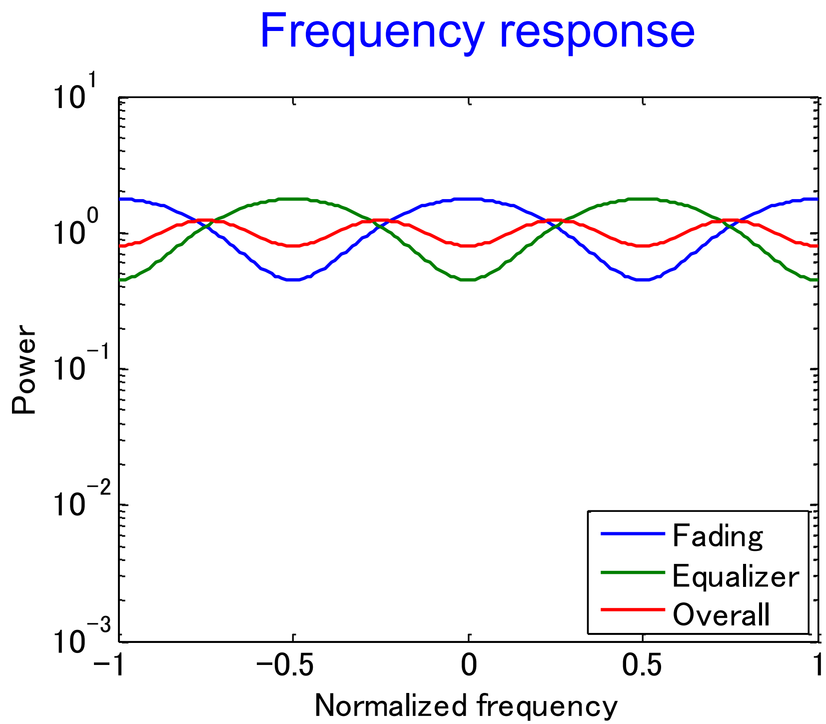
$$\mathbf{w}^* = \overleftrightarrow{\mathbf{H}}^{-1}\mathbf{e}_0$$

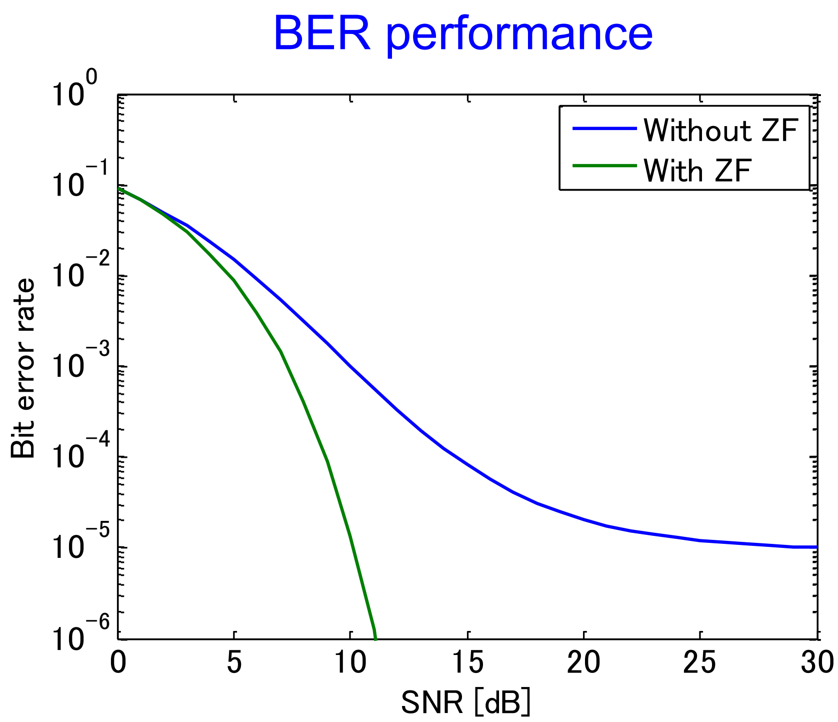
$$\begin{bmatrix} w_{-2}^* \\ w_{-1}^* \\ w_0^* \\ w_1^* \\ w_2^* \end{bmatrix} = \begin{bmatrix} h_o & 0 & 0 & 0 & 0 \\ h_1 & h_o & 0 & 0 & 0 \\ h_2 & h_1 & h_o & 0 & 0 \\ h_3 & h_2 & h_1 & h_o & 0 \\ h_4 & h_3 & h_2 & h_1 & h_o \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Forcing zeros

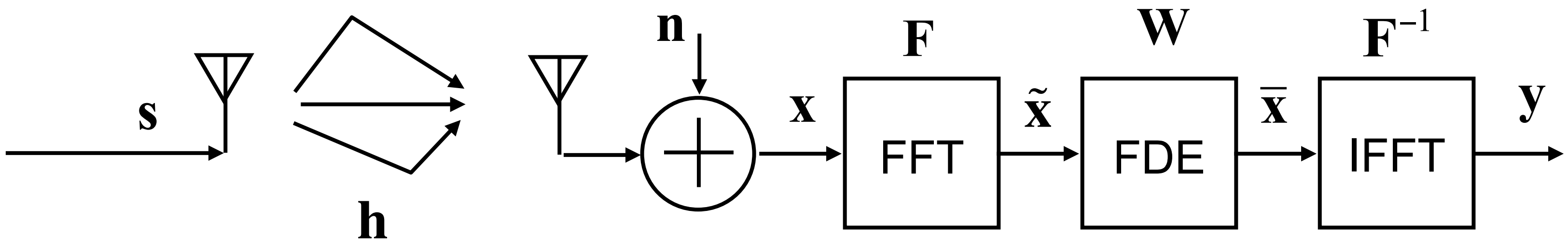
Desired output

Performance of ZF Equalizer



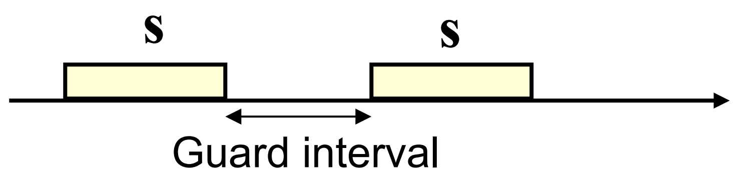


Frequency Domain Equalizer (FDE)



Frame transmission with guard interval

$$\mathbf{s} = \begin{bmatrix} s_0 & s_1 & \cdots & s_{L-1} \end{bmatrix}^T$$



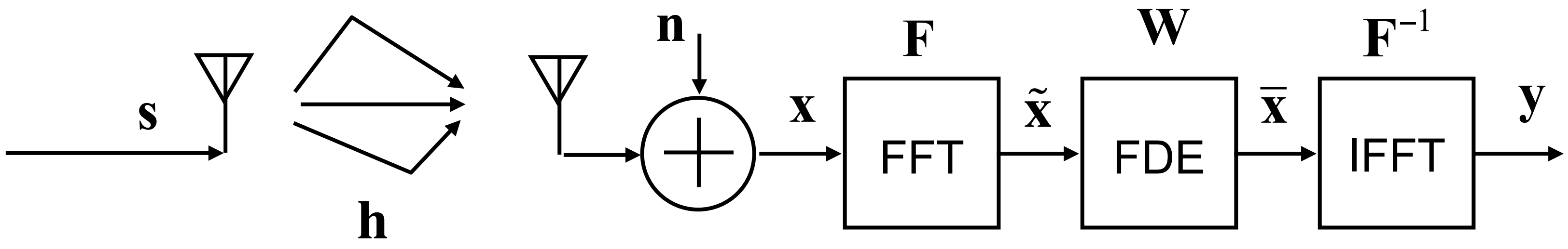
Fourier transformation matrix

$$\tilde{\mathbf{s}} = \mathbf{F}\mathbf{s}, \qquad \mathbf{F}_{kl} = \frac{1}{\sqrt{L}} \exp\left(-j\frac{2\pi}{L}kl\right)$$

$\mathbf{F}$: DFT matrix

$\mathbf{F}^{-1} = \mathbf{F}^H$: IDFT matrix

Frequency Domain Equalizer (FDE)



Time domain

$$x = h \otimes s + n$$

$$\mathbf{x} = \overleftrightarrow{\mathbf{H}}\mathbf{s} + \mathbf{n}$$

Frequency domain

$$\tilde{\mathbf{x}} = \mathbf{F}\mathbf{x} = \mathbf{F}\overleftrightarrow{\mathbf{H}}\mathbf{s} + \mathbf{F}\mathbf{n}$$

Convolution

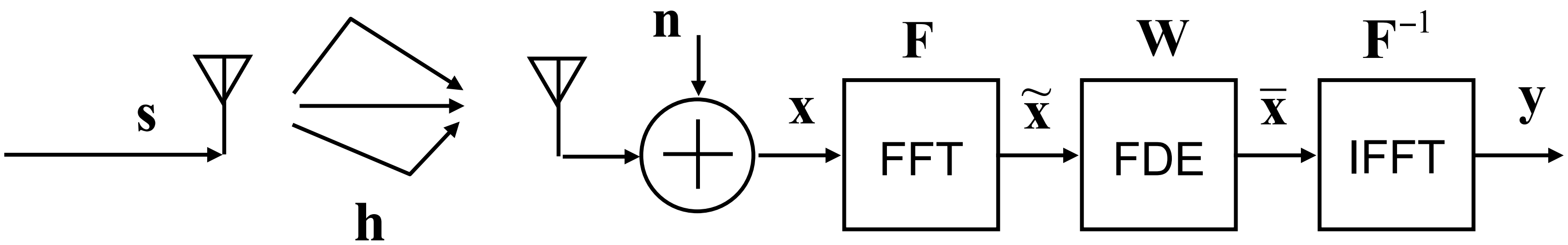
$$\mathbf{F}\overleftrightarrow{\mathbf{H}}\mathbf{s} = \mathbf{F}\mathbf{h} \bullet \mathbf{F}\mathbf{s} = \tilde{\mathbf{h}} \bullet \tilde{\mathbf{s}}$$

Hadamard product

Frequency domain equalizer

$$\tilde{\mathbf{h}} \bullet \tilde{\mathbf{s}} = \mathrm{diag}\left(\tilde{\mathbf{h}}\right)\tilde{\mathbf{s}} = \begin{bmatrix} \tilde{h}_0 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \tilde{h}_{L-1} \end{bmatrix} \begin{bmatrix} \tilde{s}_0 \\ \vdots \\ \tilde{s}_{L-1} \end{bmatrix}, \qquad \mathbf{W} = \begin{bmatrix} \tilde{h}_0 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \tilde{h}_{L-1} \end{bmatrix}^{-1}$$

Frequency Domain Equalizer (FDE)



Frequency domain receive signal

$$\tilde{\mathbf{x}} = \mathbf{F}\mathbf{x} = \mathbf{F}\overleftrightarrow{\mathbf{H}}\mathbf{s} + \mathbf{F}\mathbf{n} = \tilde{\mathbf{h}} \bullet \tilde{\mathbf{s}} + \tilde{\mathbf{n}}$$

Frequency domain ZF equalizer

$$\mathbf{W} = \begin{bmatrix} \tilde{h}_0 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \tilde{h}_{L-1} \end{bmatrix}^{-1}$$

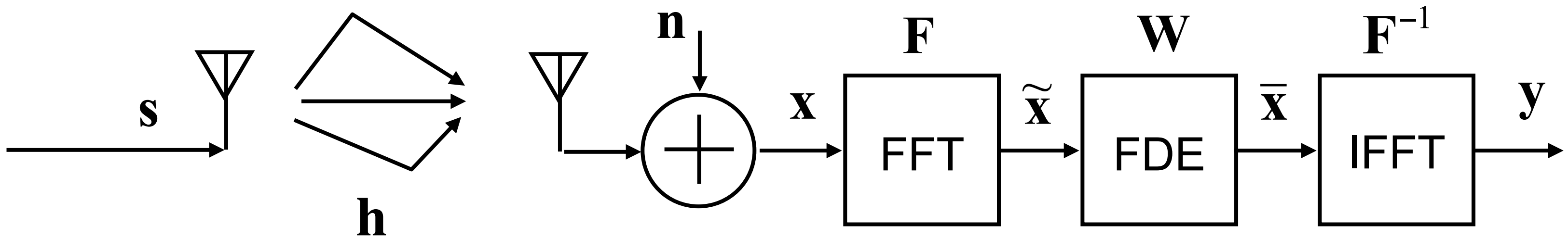
Output of FDE

$$\mathbf{y} = \mathbf{F}^{-1}\mathbf{W}\mathbf{F}\mathbf{x} = \mathbf{F}^{-1}\tilde{\mathbf{s}} + \mathbf{F}^{-1}\mathbf{W}\mathbf{F}\mathbf{n} = \mathbf{s} + \mathbf{n}'$$

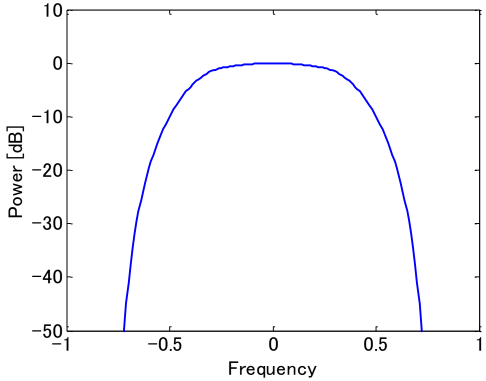
ISI free

Colored noise

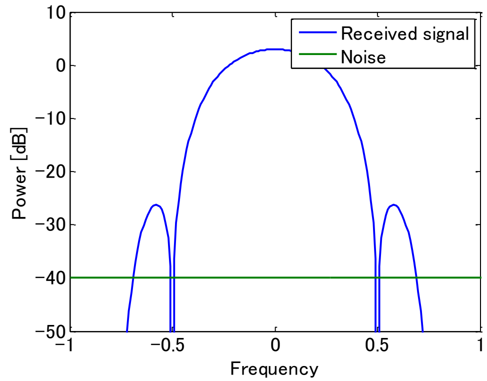
Frequency Domain Equalizer (FDE)



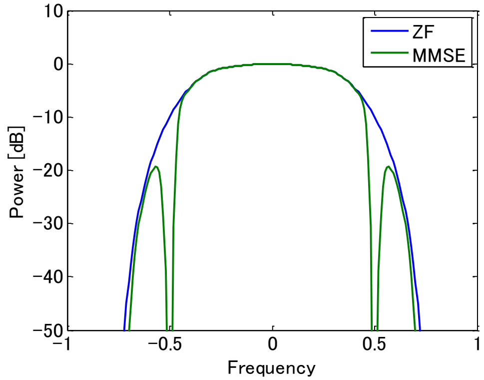
Transmit spectrum



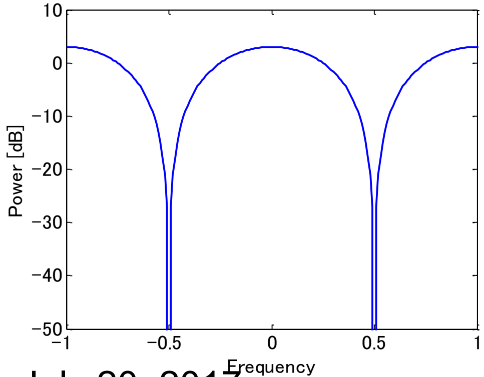
Receive spectrum



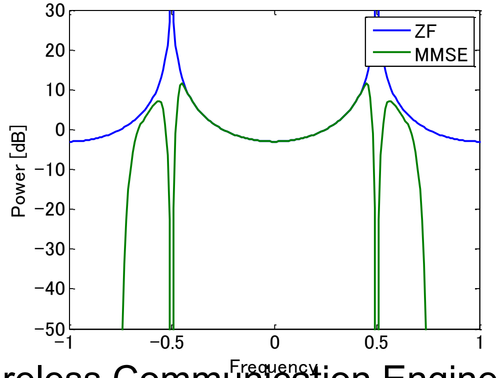
Equalized spectrum



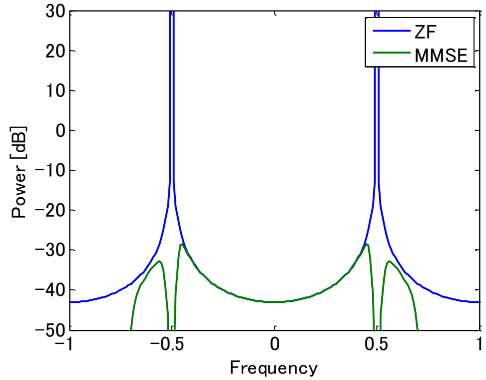
Channel response



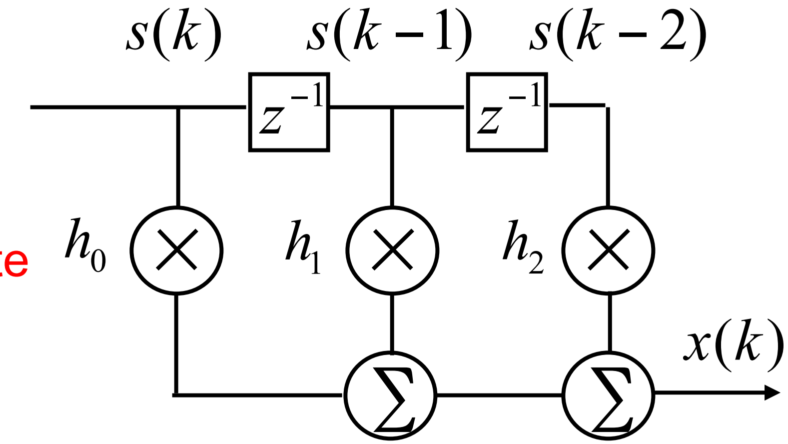
FDE response



Noise spectrum (ZF, MMSE)



Maximum Likelihood Estimation



Receive signal

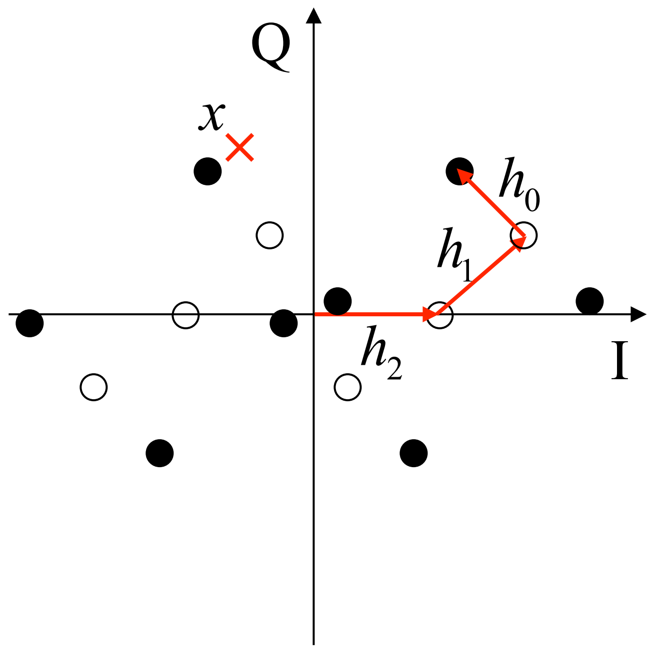
$$x(k) = \sum_{i=0}^{2} h_i s(k-i) + n(k)$$

Likelihood function

$$J(k) = \left| x(k) - \sum_{i=0}^{2} h_i \tilde{s}(k-i) \right|^2$$

Symbol candidate

Replica signal



Maximum likelihood estimation

$$\hat{s}(k), \hat{s}(k-1), \hat{s}(k-2) = \arg \min_{\tilde{s}(k), \tilde{s}(k-1), \tilde{s}(k-2)} J(k)$$

Complexity

Modulation order M

Number of taps L

M^L search problem

Maximum Likelihood Sequence Estimation (MLSE)

Receive signal

$$x(k) = \sum_{i=0}^{2} h_i s(k-i) + n(k)$$

Branch metric

$$B_{AA}(k) = x(k) - h_0 - h_1 - h_2$$

$$B_{AC}(k) = x(k) - h_0 - h_1 + h_2$$

Path metric

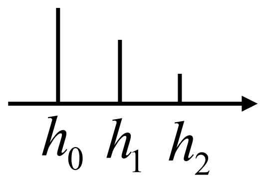
$$P_{AA}(k) = P_A(k-1) + B_{AA}(k)$$

$$P_{AC}(k) = P_A(k-1) + B_{AC}(k)$$

Survived path

$$P_A(k) = \min\left[P_{AA}(k), P_{AC}(k)\right]$$

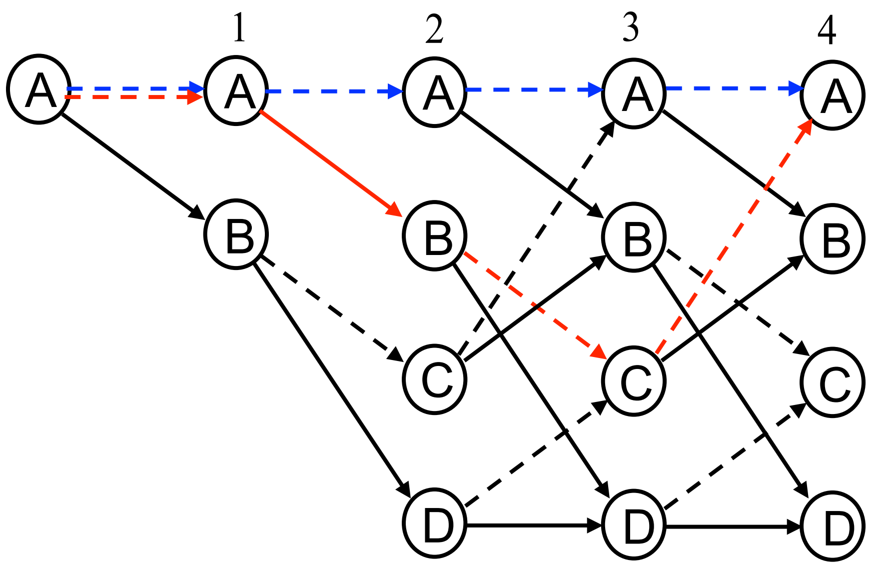
3-path model



Trellis diagram in the case of BPSK

$$A = \begin{pmatrix}-1 & -1\end{pmatrix} \quad B = \begin{pmatrix}1 & -1\end{pmatrix}$$

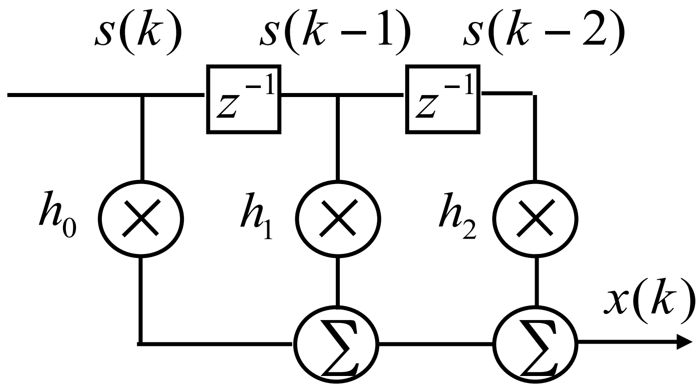
$$C = \begin{pmatrix}-1 & 1\end{pmatrix} \quad D = \begin{pmatrix}1 & 1\end{pmatrix}$$



Summary

- Multi-path channel with delay spread

$$x(k)=\sum_{i=0}^{\infty}h_i s(k-i)+n(k)$$



- Linear equalizer（ZF, MMSE）

$$y(k)=\sum_{i=-\infty}^{\infty}w_i^* x(k-i)$$

ZF: $\mathbf{w}^*=\widetilde{\mathbf{H}}^{-1}\mathbf{e}_0$　　MMSE: $\mathbf{w}=\mathbf{R}_x^{-1}\mathbf{h}$

- Frequency domain equalizer（FDE）

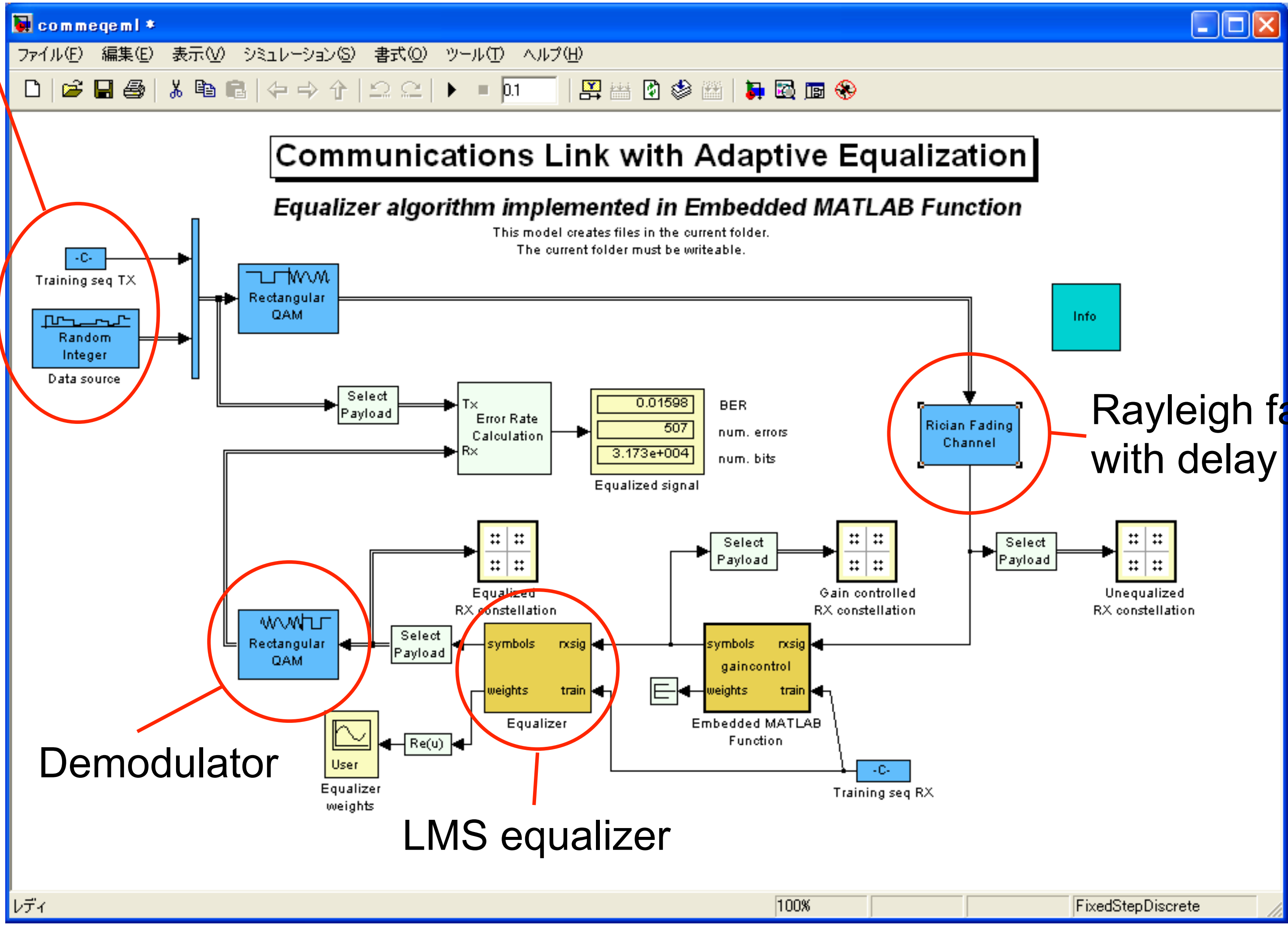
$$\mathbf{y}=\mathbf{F}^{-1}\mathbf{W}\mathbf{F}\mathbf{x}\qquad \mathbf{W}=\mathrm{diag}\left[\begin{array}{cccc}1/\tilde{h}_0 & 1/\tilde{h}_1 & \cdots & 1/\tilde{h}_{N-1}\end{array}\right]$$

- Maximum likelihood sequence estimation（MLSE）

$$\hat{s}(k),\hat{s}(k-1),\hat{s}(k-2)=\arg\min_{\tilde{s}(k),\tilde{s}(k-1),\tilde{s}(k-2)}\left|x(k)-\sum_{i=0}^{\infty}h_i\tilde{s}(k-i)\right|^2$$

Demo

Modulator＋Training sequence

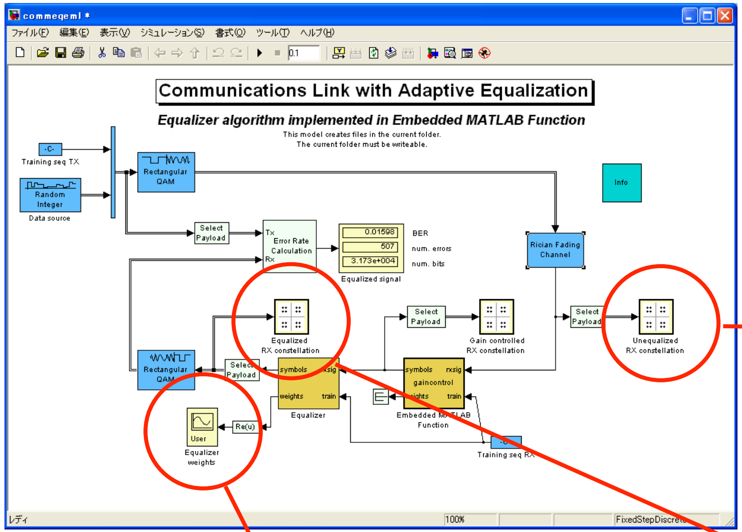


Rayleigh fading with delay spread

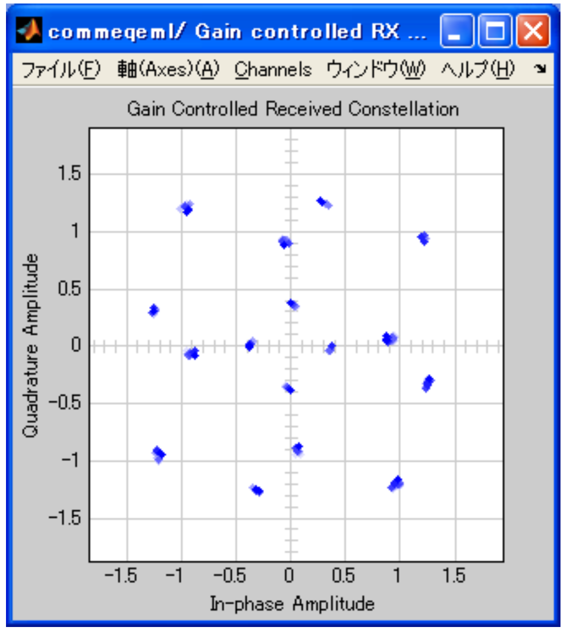
Demodulator

LMS equalizer

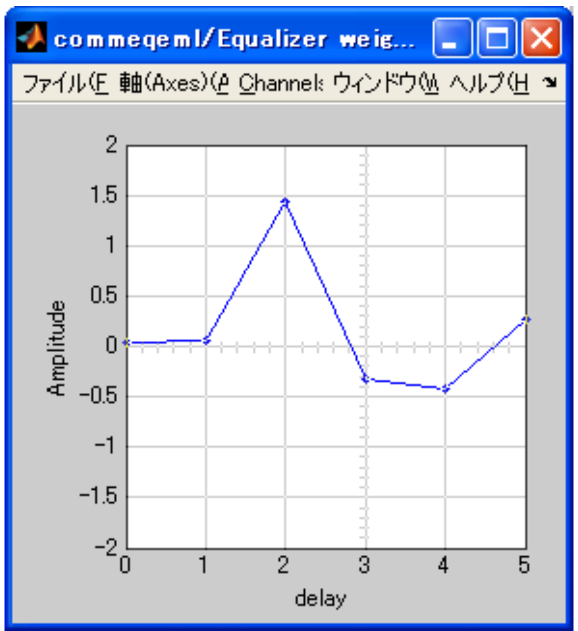
Demo



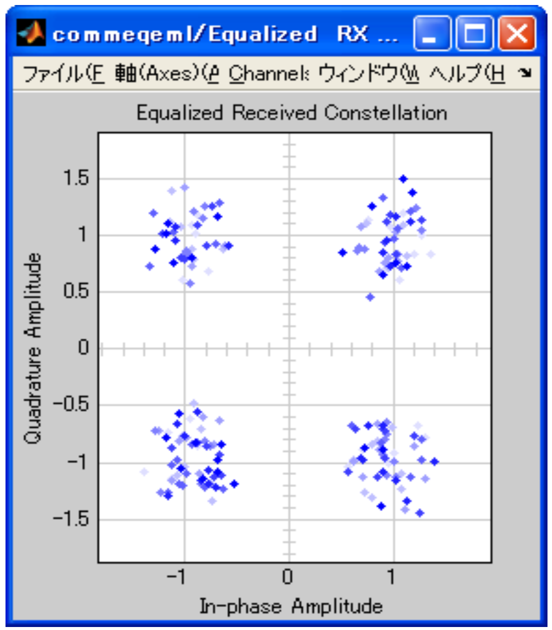
Before equalizer



Tap coefficients



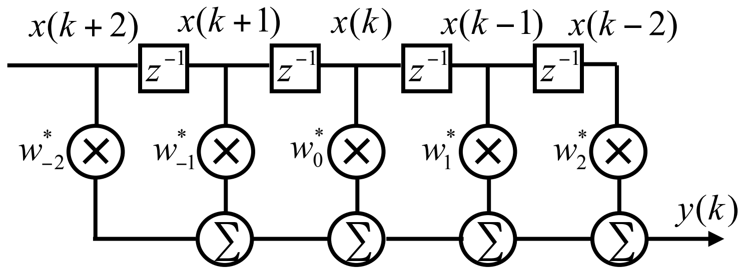
After equalizer



Linear Equalizer (MMSE)

Mean Square Error (MSE) of symbol

$$\hat{s}(k) = \sum_{i=-\infty}^{\infty} w_i^* x(k-i) = \mathbf{w}^H \mathbf{x}$$



$$J = \mathrm{E}[|e(k)|^2] = E[|s(k) - \hat{s}(k)|^2]$$

$$= E[\mathbf{w}^H \mathbf{x}\mathbf{x}^H \mathbf{w} - 2\,\mathrm{Re}[\mathbf{w}^H \mathbf{x} s^*(k)] + |s(k)|^2]$$

$$= \mathbf{w}^H \mathbf{R}_x \mathbf{w} - 2P\,\mathrm{Re}[\mathbf{w}^H \mathbf{h}] + P$$

$$\mathbf{R}_x = \mathrm{E}[\mathbf{x}\mathbf{x}^H] \qquad P\mathbf{h} = \mathrm{E}[\mathbf{x} s^*(k)]$$

Covariance matrix — Channel estimation

MMSE equalizer

$$\frac{\partial}{\partial \mathbf{w}} J = 2\mathbf{R}_x \mathbf{w} - 2P\mathbf{h} = 0$$

$$\mathbf{w} = P\mathbf{R}_x^{-1}\mathbf{h}$$

Winner filter

Derivation of complex matrix

$$\frac{\partial \alpha}{\partial w} = \frac{\partial \alpha}{\partial w_{\mathrm{I}}} + j\frac{\partial \alpha}{\partial w_{\mathrm{Q}}}$$

$$\frac{\partial}{\partial \mathbf{w}} \mathbf{w}^H \mathbf{h} = 2\mathbf{h}$$

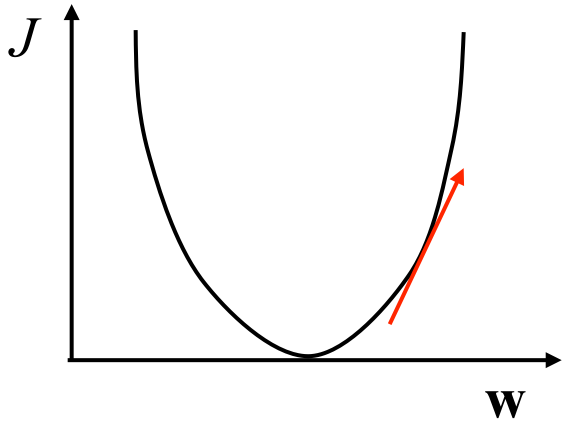
$$\frac{\partial}{\partial \mathbf{w}} \mathbf{w}^H \mathbf{R}_x \mathbf{w} = 2\mathbf{R}_x \mathbf{w}$$

Least Mean Square (LMS)

Derivative of MSE

$$\frac{\partial}{\partial \mathbf{w}} J = 2\mathbf{R}_x \mathbf{w} - 2\mathbf{h}$$
$$= 2\,\mathrm{E}\left[\mathbf{x}\left(\mathbf{x}^H \mathbf{w} - s^*(k)\right)\right]$$
$$= -2\,\mathrm{E}\left[\mathbf{x} e^*(k)\right]$$

Iterative optimization



With optimal weight

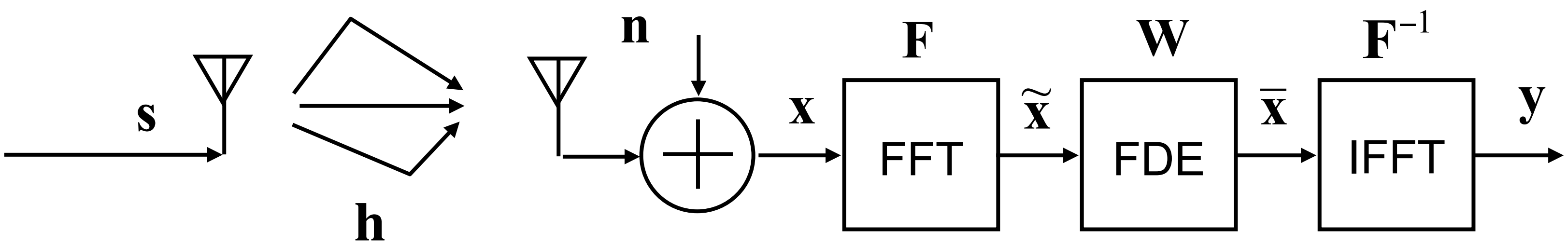
$$\mathrm{E}\left[\mathbf{x}^* e(k)\right] = 0$$

Iterative MMSE Optimization (LMS)

$$\mathbf{w}(k+1) = \mathbf{w}(k) - \mu \frac{\partial J}{\partial \mathbf{w}}$$
$$= \mathbf{w}(k) - \mu \mathbf{x}(k+1) e^*(k)$$

Without calculation of
covariance matrix and channel vector

FDE (MMSE)



Frequency domain receive signal

$$\tilde{\mathbf{x}} = \mathbf{F}\mathbf{x} = \mathbf{F}\overleftrightarrow{\mathbf{H}}\mathbf{s} + \mathbf{F}\mathbf{n} = \tilde{\mathbf{h}} \bullet \tilde{\mathbf{s}} + \tilde{\mathbf{n}}$$

$$\tilde{\mathbf{h}} = \mathbf{F}\mathbf{h} = \begin{bmatrix} \tilde{h}_0 & \tilde{h}_1 & \cdots & \tilde{h}_{L-1} \end{bmatrix}^T \quad \tilde{\mathbf{s}} = \mathbf{F}\mathbf{s} = \begin{bmatrix} \tilde{s}_0 & \tilde{s}_1 & \cdots & \tilde{s}_{L-1} \end{bmatrix}^T$$

ZF equalizer

$$\mathbf{w} = \begin{bmatrix} 1/\tilde{h}_0 & 1/\tilde{h}_1 & \cdots & 1/\tilde{h}_{L-1} \end{bmatrix}^T$$

MMSE equalizer

$$\mathbf{w} = \begin{bmatrix} \dfrac{\tilde{P}\tilde{h}_0^*}{\tilde{P}\left|\tilde{h}_0\right|^2 + \tilde{\sigma}^2} & \cdots & \dfrac{\tilde{P}\tilde{h}_{L-1}^*}{\tilde{P}\left|\tilde{h}_{L-1}\right|^2 + \tilde{\sigma}^2} \end{bmatrix}^T$$

Output of FDE

$$\mathbf{y} = \mathbf{F}^{-1}\mathbf{W}\mathbf{F}\mathbf{x} \qquad \mathbf{W} = \mathrm{diag}\left[\mathbf{w}\right]$$