2017 2Q

Wireless Communication Engineering

#13 Orthogonal Frequency Division Multplexing (OFDM)

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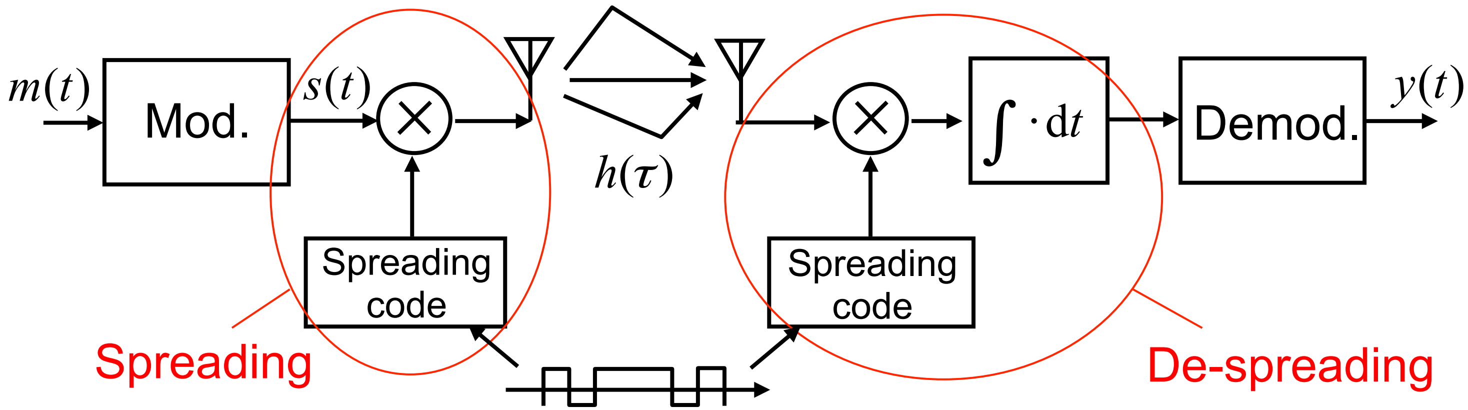
Course Schedule (2)

	Date	Text	Contents
#9	July 10	4.6	Error correction coding
#10	July 13		Adaptive modulation coding
	July 17		No class
#11	July 20	4.3	Inter symbol interference and adaptive equalizer
#12	July 24	3.6, 4.5	Spread spectrum and code division multiple access (CDMA)
#13	July 27	3.5	Orthogonal frequency division multiplexing (OFDM)
#14	July 31		Collaborative exercise for better understanding 2
#15	Aug 7	All	Final examination @ S421

From Previous Lecture

- Spread spectrum

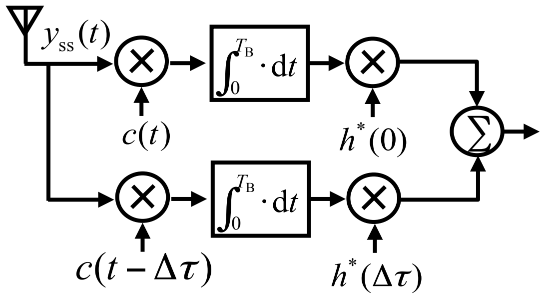


- RAKE receiver

Diversity

$$\gamma = \gamma_1 + \gamma_2$$

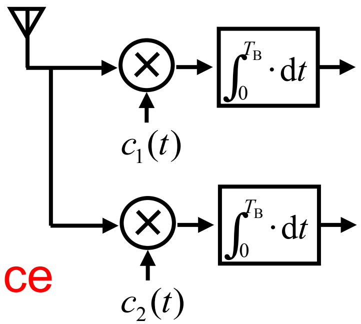
$$\gamma_i = \frac{PR_a^2(0)|h_i|^2}{PR_a^2(\Delta\tau)|h_{j\neq i}|^2 + R_a(0)\sigma^2}$$



- Code Division Multiple Access (CDMA)

$$\gamma = \frac{P|R_a(0)h_1|^2}{P|R_c h_2|^2 + R_a(0)\sigma^2}$$

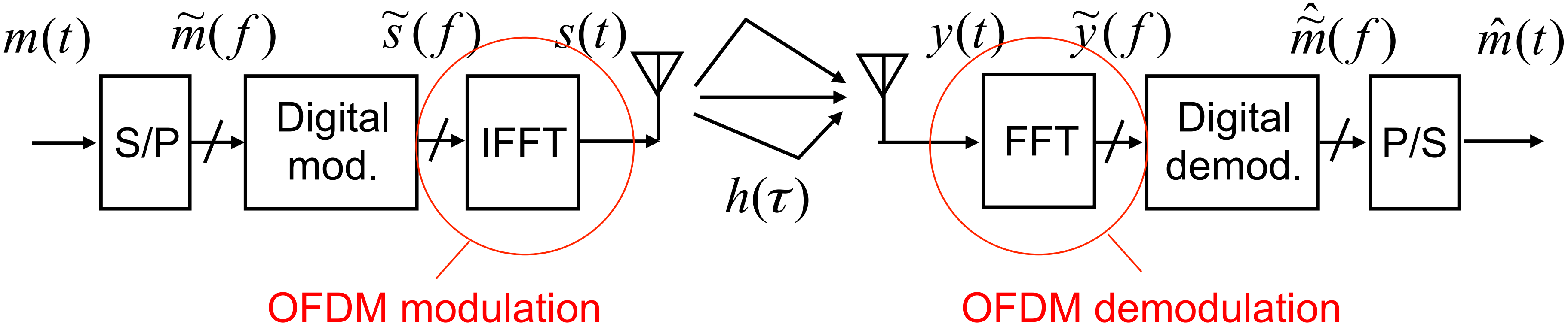
Inter system (user) interference



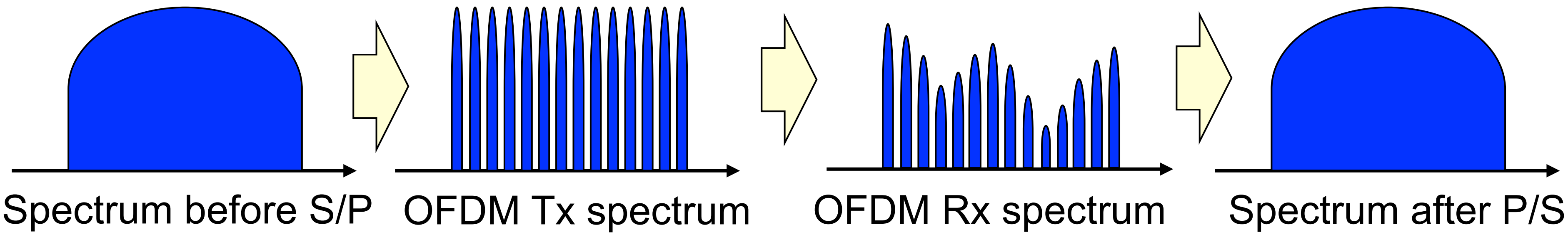
Contents

- OFDM system
- Orthogonal Frequency Division Multiplexing
  - OFDM modulation
  - Frequency domain equalizer (OFDM demodulation)
  - Block (frame) transmission & cyclic prefix
  - Coding over OFDM
  - AMC over OFDM
- IEEE802.11a WLAN
  - Transceiver architecture
  - Channel & subcarrier allocation
  - Frame format
- Demonstration

OFDM System



- Block transmission using S/P at Tx and P/S at Rx
- Convert wide band signal to multiple narrow band signals (OFDM)
- Coding and AMC over OFDM subcarriers



Frequency Selective Fading

Receive signal

$$y(t) = \int h(\tau)s(t-\tau)\mathrm{d}t + n(t)$$

2-path model

$$y(t) = h(0)s(t) + h(\Delta\tau)s(t-\Delta\tau) + n(t)$$

Impulse response

$$h(\tau) = h_0\delta(\tau) + h_1\delta(\tau-\Delta\tau)$$

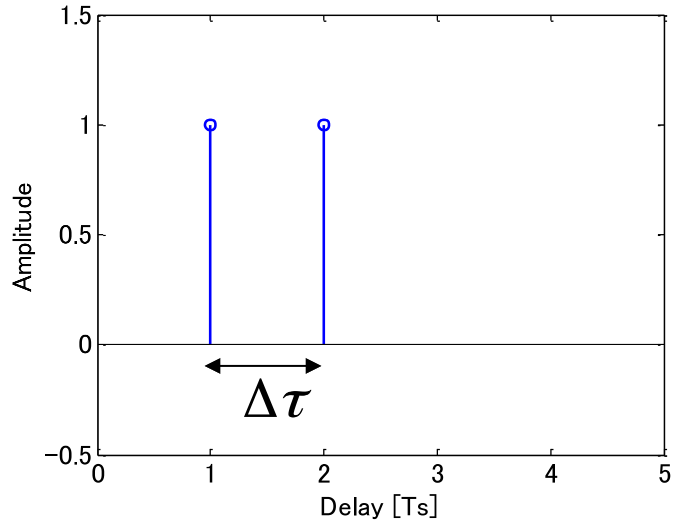
Frequency response

$$\tilde{h}(f) = h_0 + h_1\exp(-j2\pi f\Delta\tau)$$

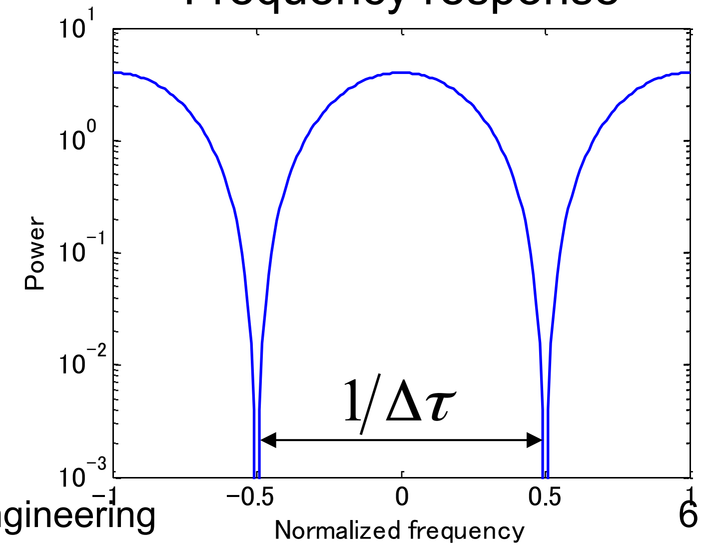
Condition of narrow band system

Bandwidth $\Delta f << 1/\Delta\tau$

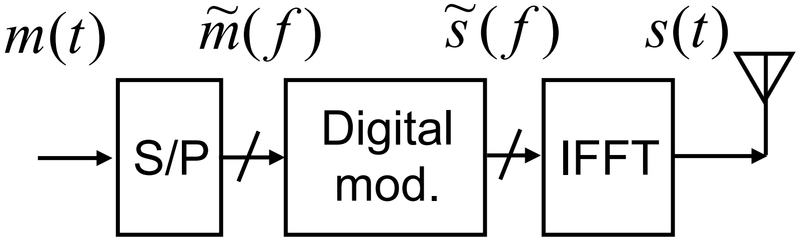
Impulse response

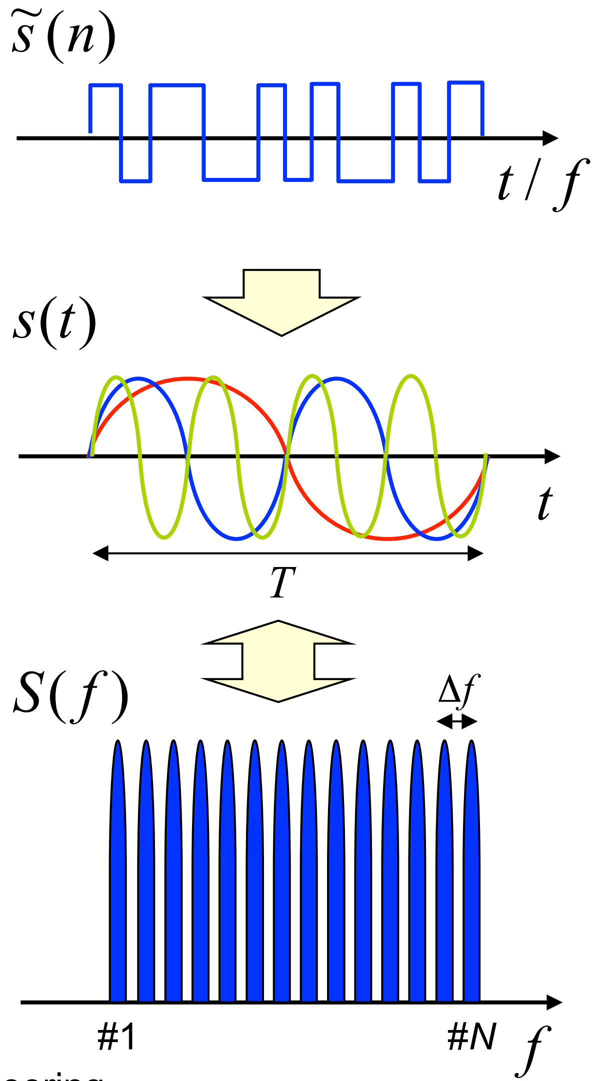


Frequency response



Orthogonal Frequency Division Multiplexing (OFDM)





OFDM mod.

$$s(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \tilde{s}(n) \exp\left(j 2\pi n \Delta f t\right)$$

$$0 \le t \le T = 1/\Delta f$$

N: # of subcarriers
T: OFDM symbol period
Δf: Subcarrier spacing

s.t. $\Delta f \ll 1/\Delta\tau \leftrightarrow T \gg \Delta\tau$

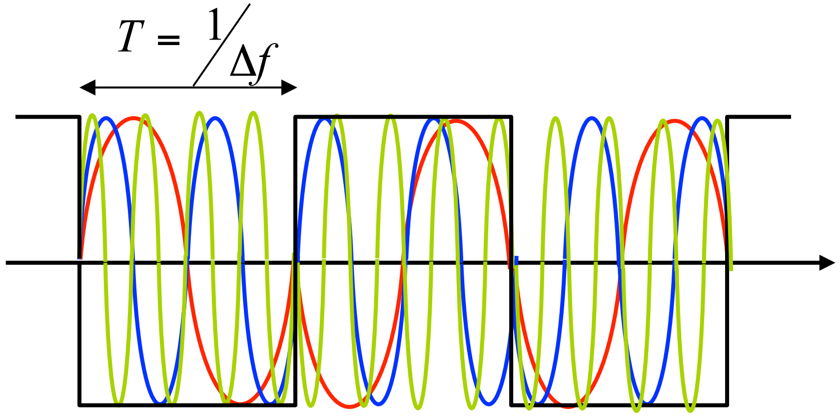
Inverse Discrete Fourier Transform (IDFT)

$$s(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \tilde{s}(n) \exp\left(j 2\pi \frac{nk}{N} \right)$$

$$t = k\Delta t \qquad \Delta t = 1/N\Delta f$$

$N\Delta f$: Bandwidth

Frequency Spectrum of OFDM



OFDM modulated signal

$$s(t)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}\tilde{s}(n)\exp\left(j2\pi n\Delta f t\right)$$

Auto-correlation

$$R_S(\tau)=\mathrm{E}\left[s^*(t)s(t+\tau)\right]$$

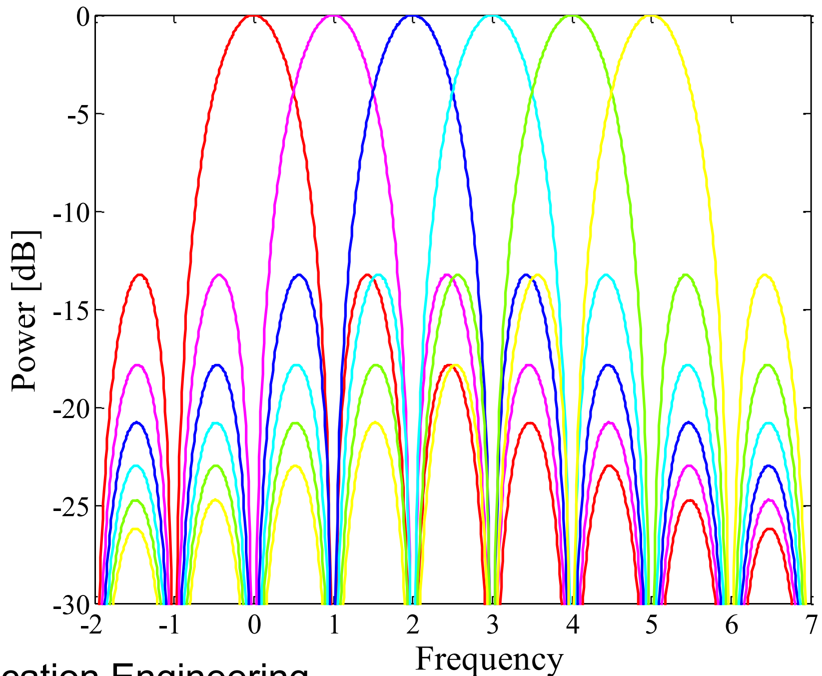
$$=\frac{1}{N}\sum_{n=0}^{N-1}\left(1-\frac{|\tau|}{T}\right)\exp\left(j2\pi n\Delta f\tau\right)$$

Auto-correlation of random pulse seq.

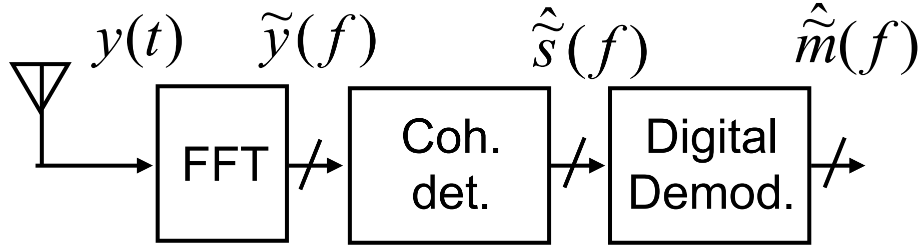
Power spectrum

Power spectrum

$$S_S(f)=\frac{1}{N}\sum_{n=0}^{N-1}\mathrm{sinc}^2\left(T(f-n\Delta f)\right)$$



OFDM Demodulation (FDE)



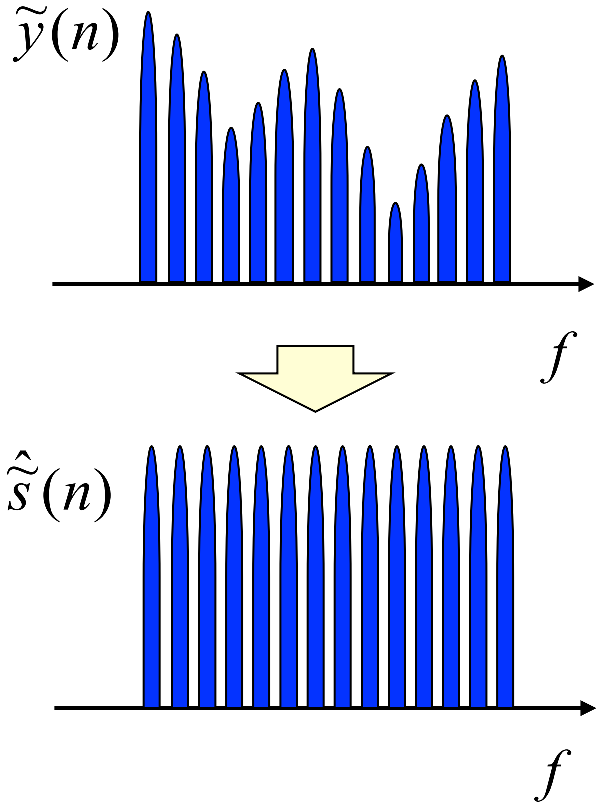
Receive signal

$$y(t) = \int h(\tau)s(t-\tau)\mathrm{d}t + n(t)$$

OFDM demodulation

$$\tilde{y}(n) = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} y(k)\exp\left(-j2\pi\frac{kn}{N}\right)$$

$$= \tilde{h}(n)\tilde{s}(n) + \tilde{n}(n)$$



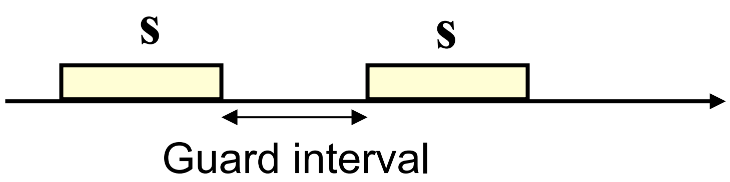
Frequency Domain Equalizer (ZF) (FDE)

$$\hat{\tilde{s}}(n) = \tilde{y}(n) / \tilde{h}(n)$$

Per subcarrier
coherent detection

Matrix Representation of OFDM

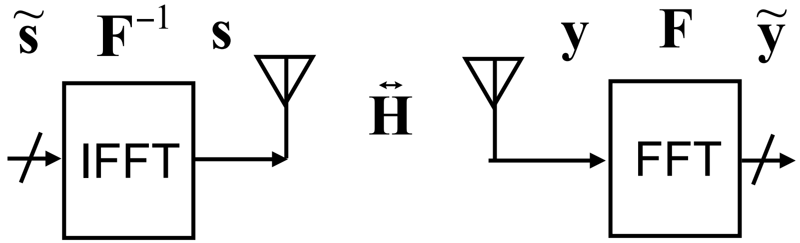
OFDM mod. (block transmission)



$$\tilde{\mathbf{s}} = \begin{bmatrix} \tilde{s}_0 & \tilde{s}_1 & \cdots & \tilde{s}_{N-1} \end{bmatrix}^T$$

$$\mathbf{s} = \mathbf{F}^{-1}\tilde{\mathbf{s}} \qquad \mathbf{F}_{nk} = \frac{1}{\sqrt{N}} \exp\left(-j2\pi \frac{kn}{N} \right)$$

Inverse DFT matrix



Property of cyclic shift matrix

$$\mathbf{F}\overleftrightarrow{\mathbf{H}}\mathbf{F}^{-1} = \mathbf{F}\mathbf{h} \bullet \mathbf{F}\mathbf{F}^{-1} = \mathrm{diag}[\tilde{\mathbf{h}}]$$

Hadamard product   Diagonal matrix

Receive signal vector

$$\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{N-2} \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} h_o & 0 & \cdots & 0 & 0 \\ h_1 & h_o & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ h_{M-1} & \cdots & h_1 & h_o & 0 \\ 0 & h_{M-1} & \cdots & h_1 & h_o \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \\ \vdots \\ s_{N-2} \\ s_{N-1} \end{bmatrix} + \begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{N-2} \\ n_N \end{bmatrix}$$

$$\mathbf{y} = \overleftrightarrow{\mathbf{H}}\mathbf{s} + \mathbf{n}$$

Cyclic shift matrix

OFDM demodulation

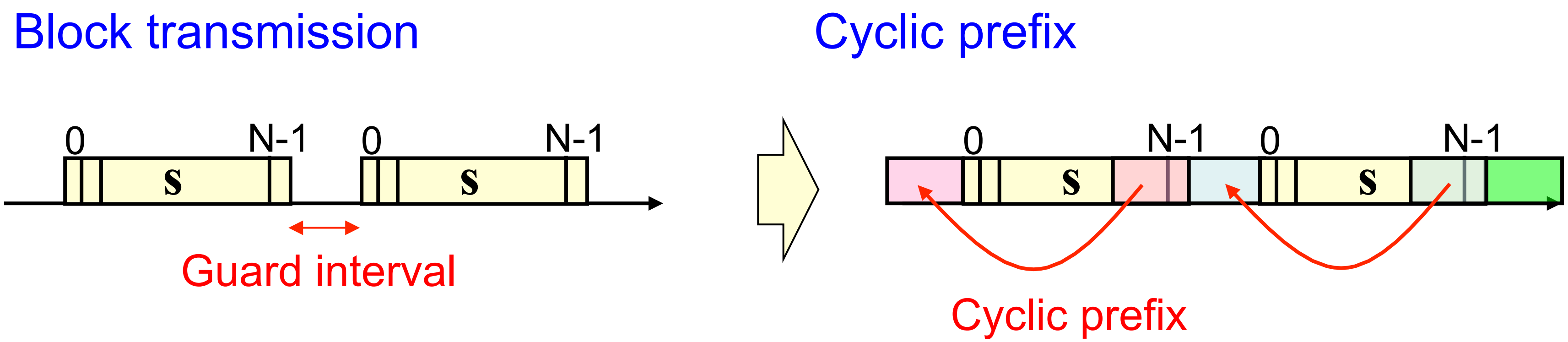
DFT matrix

$$\tilde{\mathbf{y}} = \mathbf{F}\mathbf{y} = \mathbf{F}\overleftrightarrow{\mathbf{H}}\mathbf{s} + \mathbf{F}\mathbf{n}$$

$$= \tilde{\mathbf{h}} \bullet \tilde{\mathbf{s}} + \tilde{\mathbf{n}} = \mathrm{diag}[\tilde{\mathbf{h}}]\tilde{\mathbf{s}} + \tilde{\mathbf{n}}$$

N parallel transmissions

Block Transmission & Cyclic Prefix



Receiver signal vector

$$\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{N-2} \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} h_o & 0 & \cdots & 0 & 0 \\ h_1 & h_o & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ h_{M-1} & \cdots & h_1 & h_o & 0 \\ 0 & h_{M-1} & \cdots & h_1 & h_o \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \\ \vdots \\ s_{N-2} \\ s_{N-1} \end{bmatrix}$$

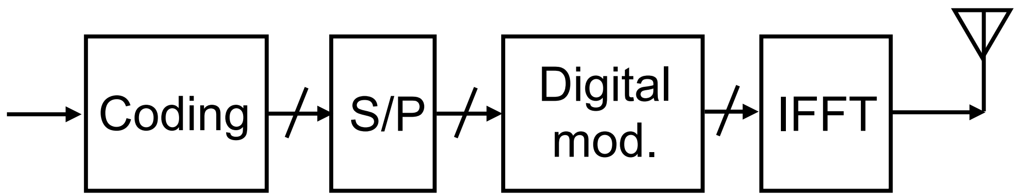
Still cyclic shift matrix

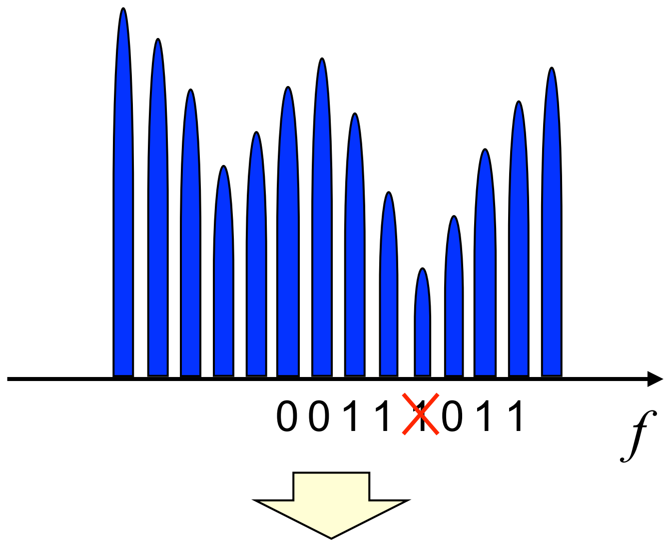
$$\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{N-2} \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} h_o & 0 & h_{M-1} & \cdots & h_1 \\ h_1 & h_o & \cdots & h_{M-1} & \vdots \\ \vdots & \vdots & \ddots & \vdots & h_{M-1} \\ h_{M-1} & \cdots & h_1 & h_o & 0 \\ 0 & h_{M-1} & \cdots & h_1 & h_o \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \\ \vdots \\ s_{N-2} \\ s_{N-1} \end{bmatrix}$$

Time continuous transmission by keeping orthogonal property of subcarriers

Coding over OFDM

Coding over subcarriers





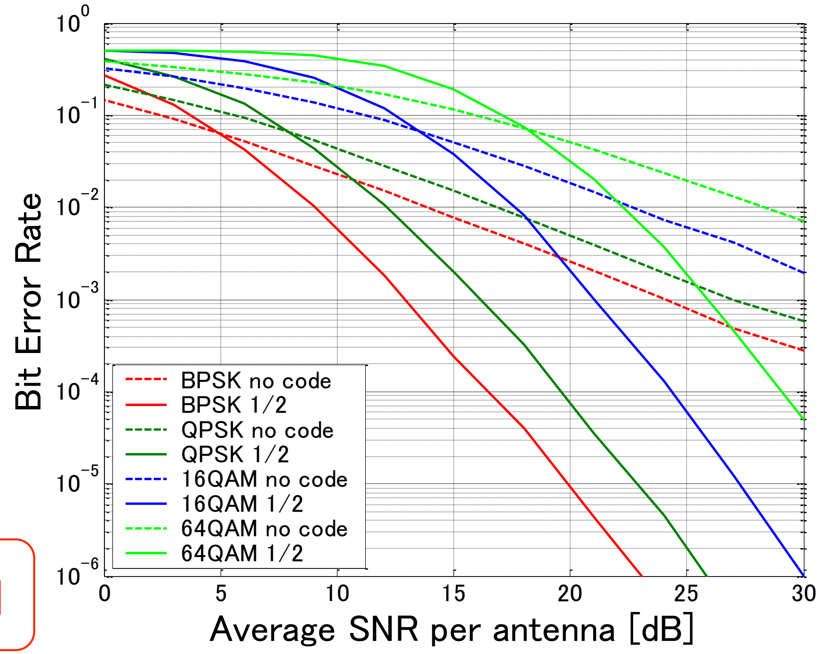
Error correction at subcarriers at fading dip

Frequency diversity using coding over OFDM

BER of Viterbi decoding

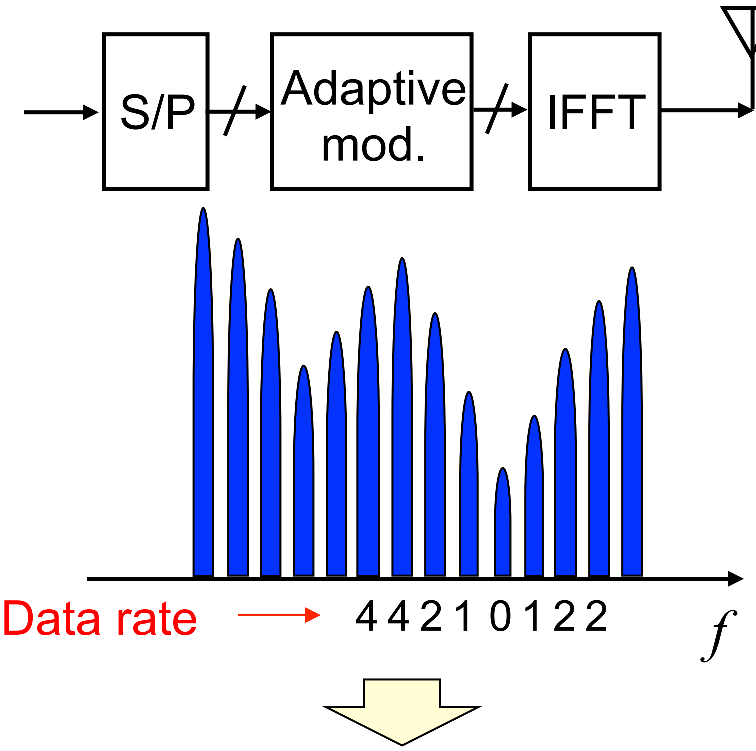
$$P_e < \sum_{d=d_{\min}}^{\infty} 2^{d-d_{\min}} P_2(d) \quad P_2(d) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\sum_{i=1}^{d}\gamma_i}\right)$$

BER performance



AMC over OFDM

Adaptive modulation per subcarrier



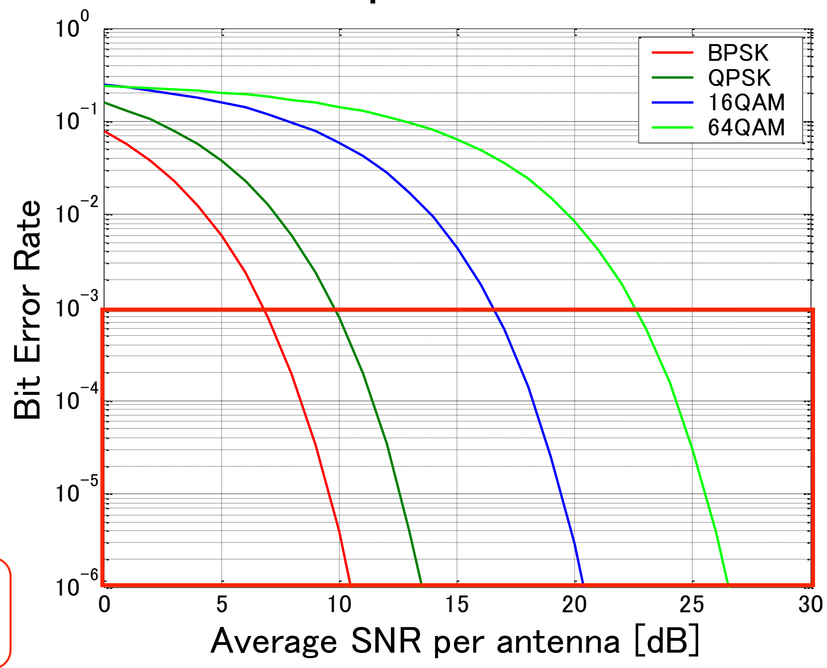
Adaptive data rate control per subcarrier

Objective function

$$\min_{M_i} P_e = \frac{1}{R}\sum_{i=1}^{N} \log_2 M_i P_e(M_i, \gamma_i)$$

$$\text{subject to } R = \sum_{i=1}^{N} \log_2 M_i$$

BER performance



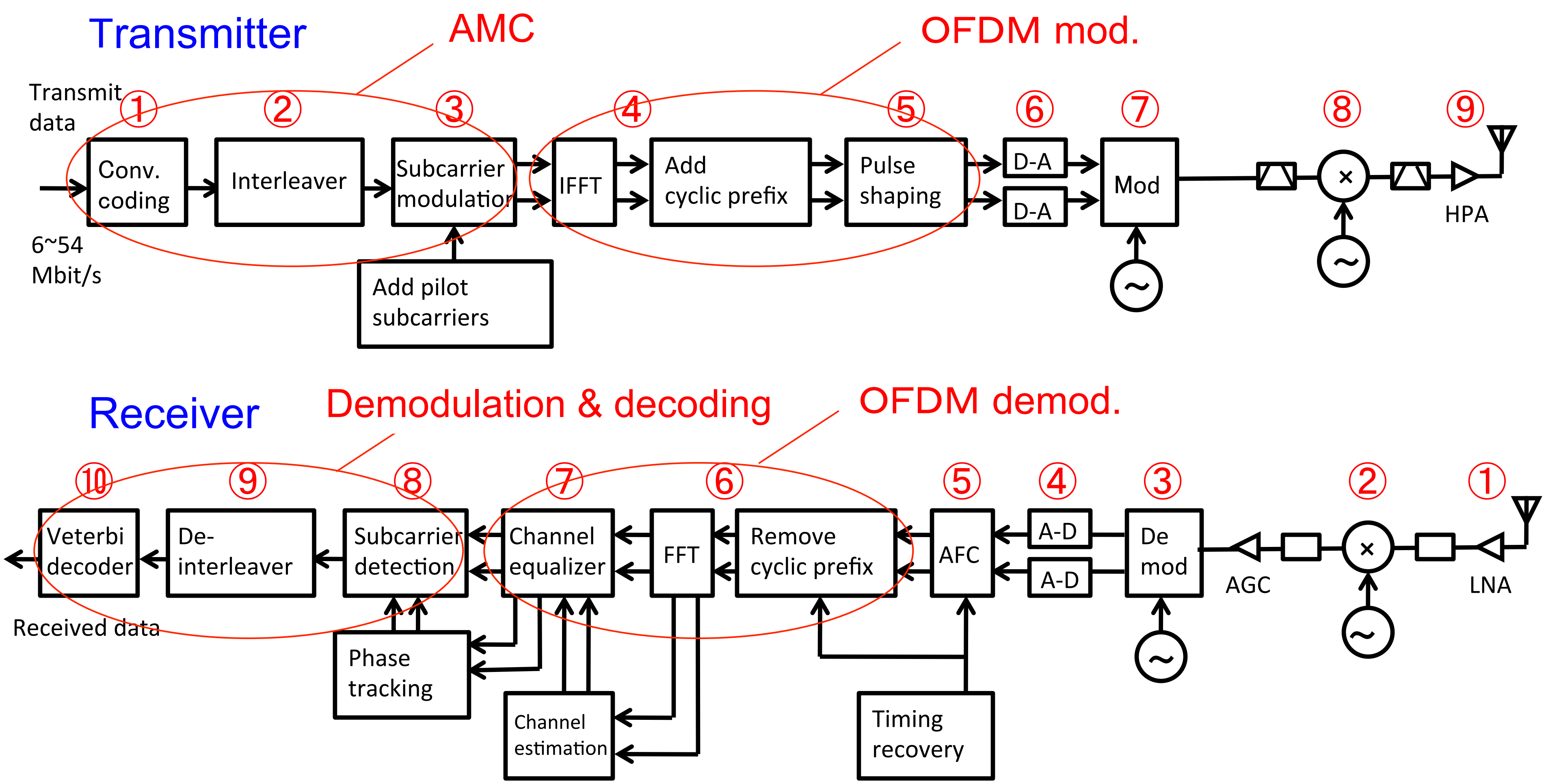
Frequency diversity using AMC over OFDM

IEEE802.11 WLAN

WLAN standardized by IEEE 802 committee working group (WG) 11

	802.11b	802.11a	802.11g	802.11n	802.11ac
Year of approval	1999	1999	2003	2009	2014
RF band	2.4GHz	5GHz	2.4GHz	2.4 & 5GHz	5GHz
Channel bandwidth	20MHz	20MHz	20MHz	20/40MHz	20/40/80/ 160MHz
Modulation	DSSS, CCK	OFDM, AMC	OFDM, AMC, CCK	MIMO-OFDM, AMC, CCK	MIMO-OFDM, AMC256Q, MU-MIMO
Max data rate	11Mbps	54Mbps	54Mbps	600Mbps	6.93Gbps
MAC	CSMA/CA	CSMA/CA	CSMA/CA	CSMA/CA	CSMA/CA+ MU-MIMO

IEEE802.11a WLAN



Specification of IEEE802.11a

Modulation	OFDM (Orthogonal Frequency Division Multiplexing) (Subcarrier modulation: BPSK, QPSK, 16QAM, 64QAM)
# of subcarriers	52 subcarriers (including 4 pilot subcarriers) 64 point FFT is assumed
Error Correction Coding	Convolutional coding (K=7, R=1/2, 2/3, 3/4) Viterbi decoder Subcarrier interleaver
Data rate	6 Mbit/s (BPSK, R=1/2) mandatory 9 Mbit/s (BPSK, R=3/4) option 12 Mbit/s (QPSK, R=1/2) mandatory 18 Mbit/s (QPSK, R=3/4) option 24 Mbit/s (16QAM, R=1/2) mandatory 36 Mbit/s (16QAM, R=3/4) option 48 Mbit/s (64QAM, R=2/3) option 54 Mbit/s (64QAM, R=3/4) option
OFDM symbol	4.0 μs
Guard interval	0.8 μs
Occupied BW	16.6 MHz
# of channels	4 (Frequency range: 5.15～5.25 GHz [Japan]) Channel interval: 20MHz

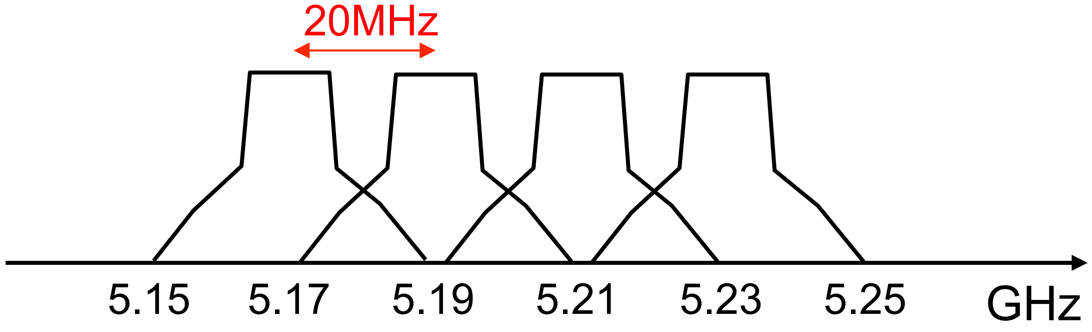
bit rate, code rate

$$6 \times \frac{3}{4} \times 20 \times \frac{48}{64} \times \frac{3.2}{4} = 54$$

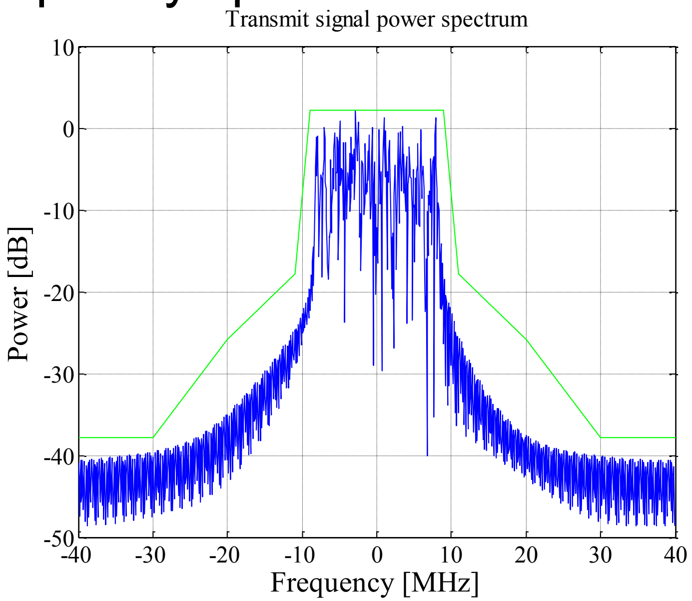
BW, guard band, GI

Frequency Spectrum of IEEE802.11a

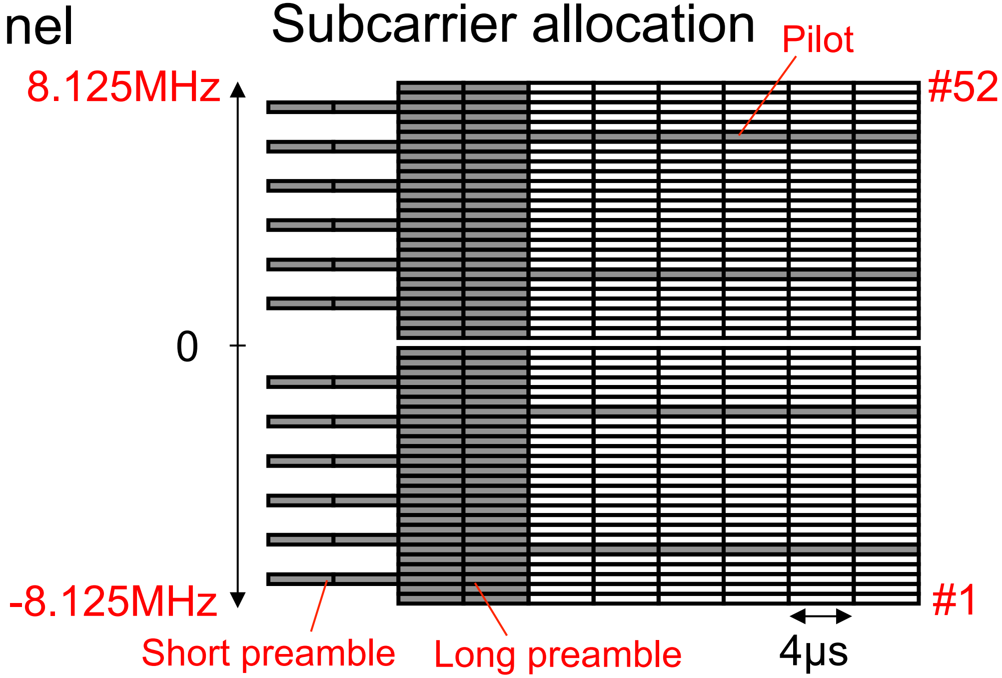
Channel allocation for 5GHz WLAN



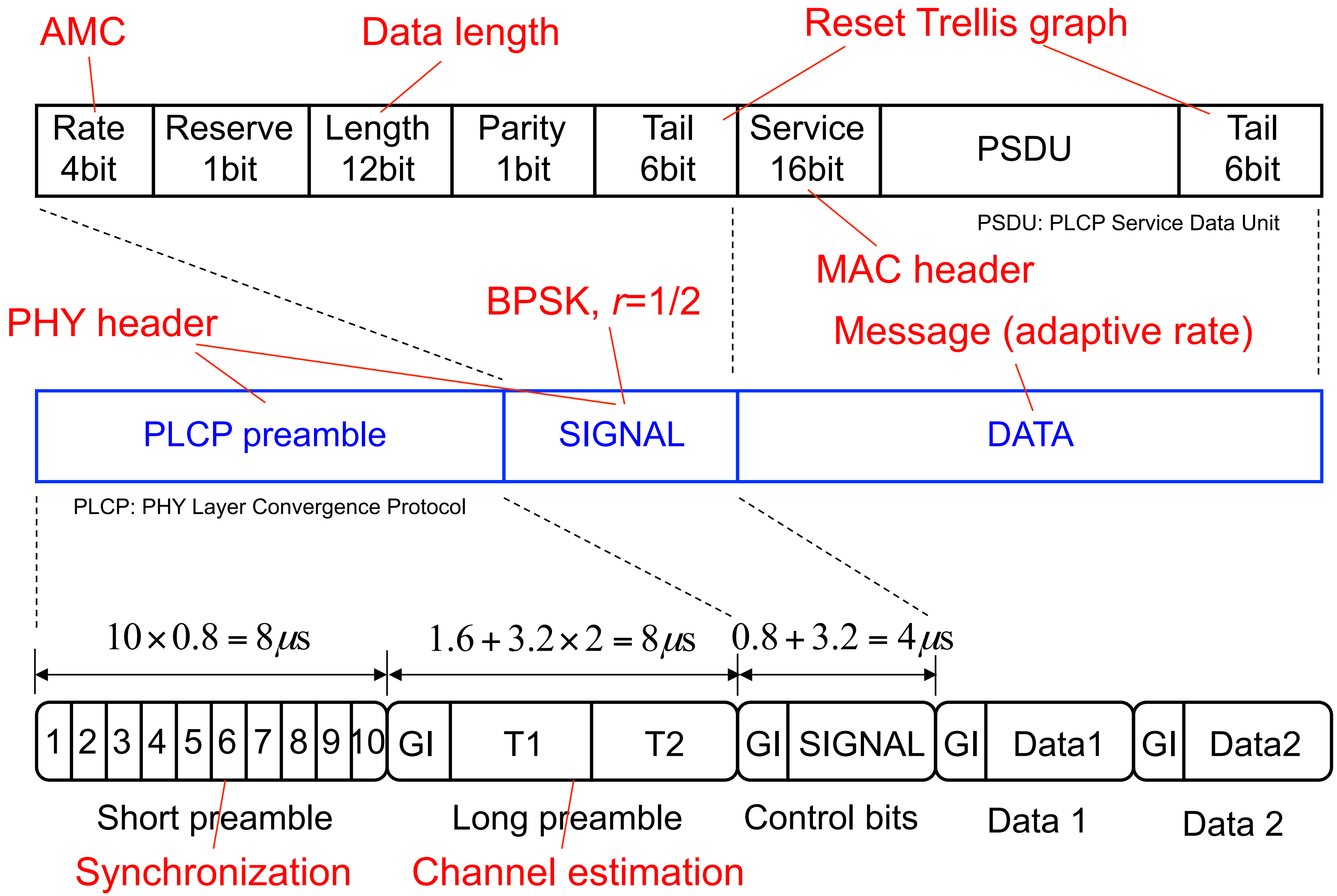
Frequency spectrum of each channel



Subcarrier allocation

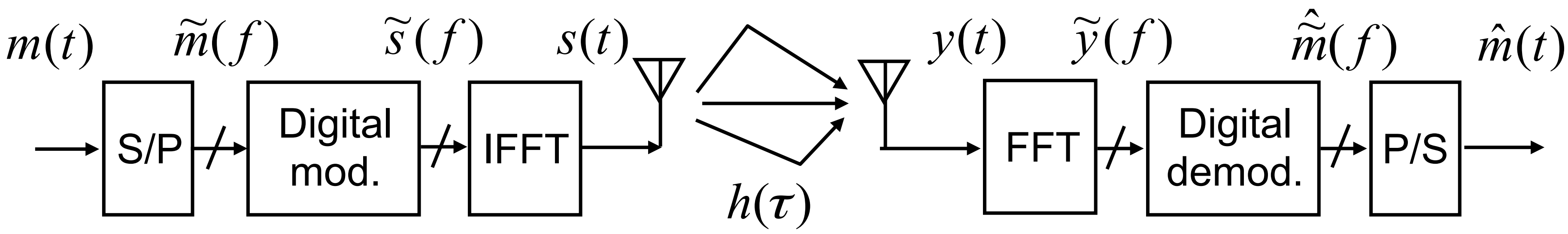


Frame Format of IEEE802.11a



Summary

- Orthogonal Frequency Division Multiplexing (OFDM)



OFDM mod.

$$s(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \tilde{s}(n) \exp\left(j2\pi k \frac{n}{N} \right)$$

OFDM demod. (FDE)

$$\tilde{y}(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} y(k) \exp\left(-j2\pi n \frac{k}{N} \right)$$

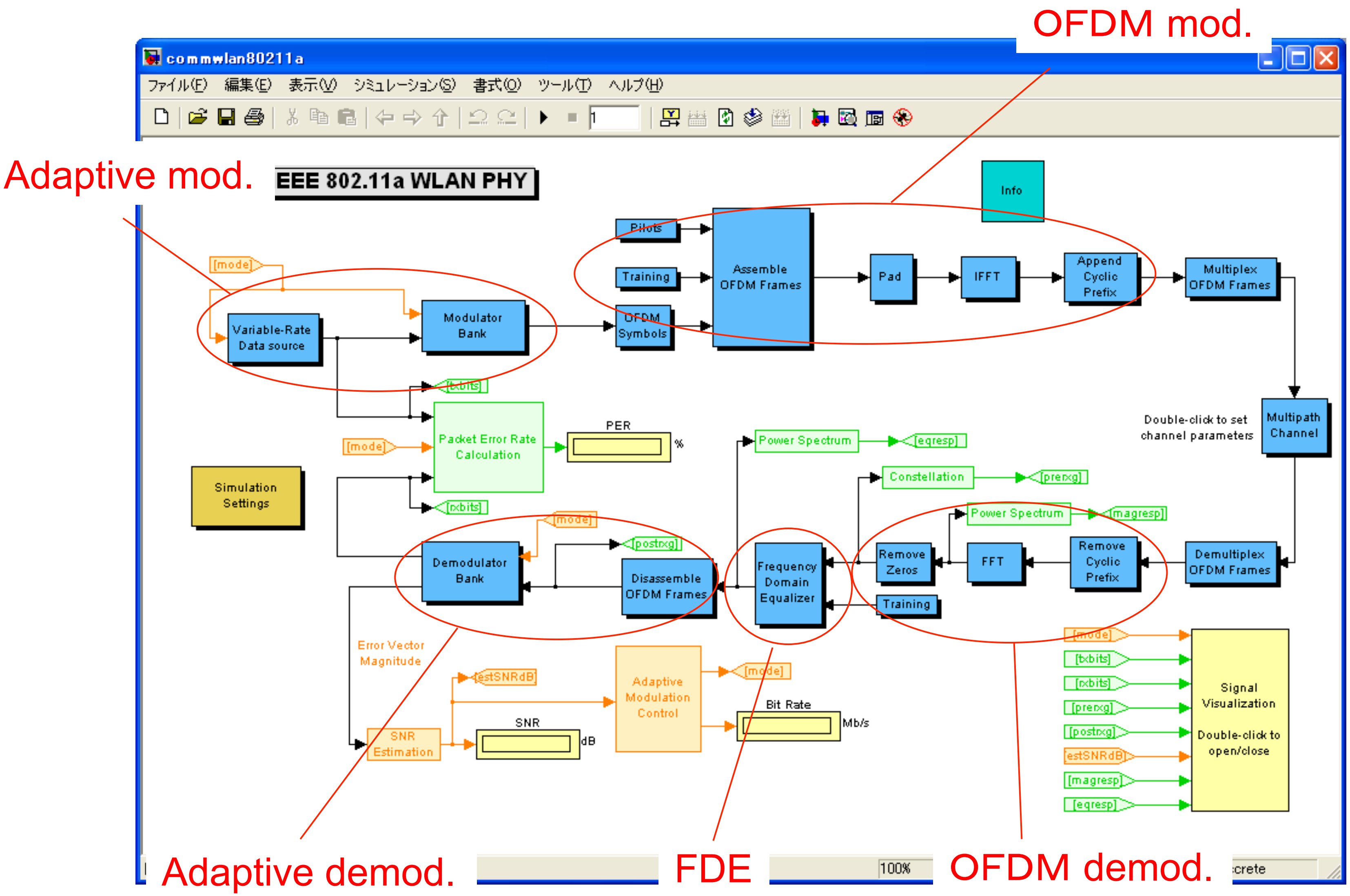
$$= \tilde{h}(n)\tilde{s}(n) + \tilde{n}(n)$$

$$\hat{\tilde{s}}(n) = \tilde{y}(n) \big/ \tilde{h}(n)$$

- AMC over OFDM

Coding: Error correction of subcarriers at fading dip
Adaptive mod.: Adaptive data rate control per subcarrier

Demo (IEEE802.11a)



Demo (IEEE802.11a)