2017 2Q
Wireless Communication Engineering

#8 Channel Fading and Diversity Combining

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Course Schedule (1)

	Date	Text	Contents
#1	June 12	1, 7	Introduction to wireless communication systems
#2	June 15	2, 5, etc	Link budget design of wireless access
#3	June 19		Up/down conversion and equivalent baseband system
#4	June 22	3.3, 3.4	Digital modulation and pulse shaping
#5	June 26	3.5	Demodulation and matched filter
#6	June 29		Collaborative exercise for better understanding 1
#7	July 3	3.5	Detection and error due to noise
#8	July 6	4.4	Channel fading and diversity combining

Course Schedule (2)

	Date	Text	Contents
#9	July 10	4.6	Error correction coding
#10	July 13		Adaptive modulation coding
	July 17		No class
#11	July 20	4.3	Inter symbol interference and adaptive equalizer
#12	July 24	3.6, 4.5	Spread spectrum and code division multiple access (CDMA)
#13	July 27	3.5	Orthogonal frequency division multiplexing (OFDM)
#14	July 31		Collaborative exercise for better understanding 2
#15	TBD	All	Final examination

From Previous Lectures

- Channel estimation & coherent detection

$$\hat{h}_{\mathrm{B}} = \frac{1}{K}\sum_{k=1}^{K} y(k) \Big/ s_{\mathrm{TR}}(k) \longrightarrow \hat{s}(k) = y(k)/h_{\mathrm{B}} = s(k) + n(k)/h_{\mathrm{B}}$$

- Error rate of BPSK signal

$$p_{\mathrm{eb}} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{P_{\mathrm{s}}\left|h_{\mathrm{B}}\right|^2}{\sigma^2}}\right) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\gamma}\right)$$

- Error rate of QAM signal

$$p_{\mathrm{eb}} = \frac{2}{\log_2 M}\left(1 - \frac{1}{\sqrt{M}}\right)\mathrm{erfc}\left(\sqrt{\frac{3\gamma}{2(M-1)}}\right)$$

Contents

- Narrow band system
- Gain of propagation channel
- Rayleigh fading & probability distribution
- Error rate in fading channel
- Diversity technologies
- Maximum ratio combining diversity

Narrow Band System

Time invariant narrow band system

$$y_{\mathrm{B}}(t) = \int h_{\mathrm{B}}(\tau)\tilde{s}_{\mathrm{B}}(t-\tau)\mathrm{d}\tau = h_{\mathrm{B}} s_{\mathrm{B}}(t)$$

$$h_{\mathrm{B}} = h_{\mathrm{B}}(\tau_0) = h(\tau_0) e^{-j2\pi f_0 \tau_0}$$

Time variant narrow band system

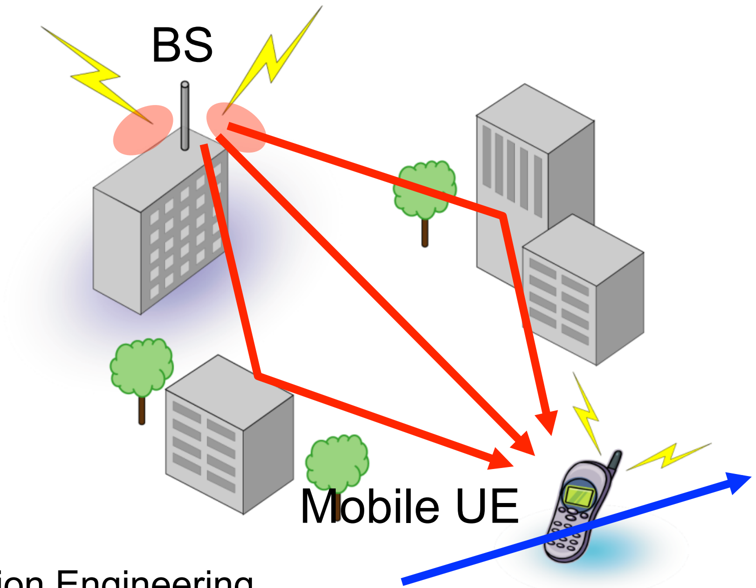
$$h_{\mathrm{B}} \to h_{\mathrm{B}}(d, \phi_{\mathrm{s}}, \phi_{\mathrm{r}}, t)$$

d : Distance between Tx & Rx

ϕ : Tx & Rx antenna angle

t : Mobility of UE

Mobile communication



Gain of Propagation Channel

Gain of narrow band propagation channel

$$G_{\mathrm{prop}} = \left|h_{\mathrm{B}}(d, \phi_{\mathrm{s}}, \phi_{\mathrm{r}}, t)\right|^2$$

$$= G_{\mathrm{rant}}(\phi_{\mathrm{r}})\,G_{\mathrm{fading}}(t)\,G_{\mathrm{shadow}}(t)\,G_{\mathrm{pl}}(d)\,G_{\mathrm{sant}}(\phi_{\mathrm{s}})$$

Rx antenna gain

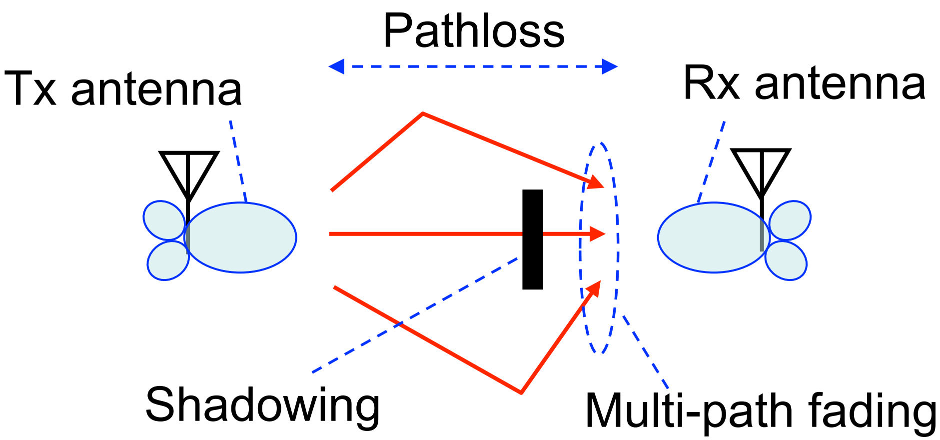
Multi-path fading

Shadowing

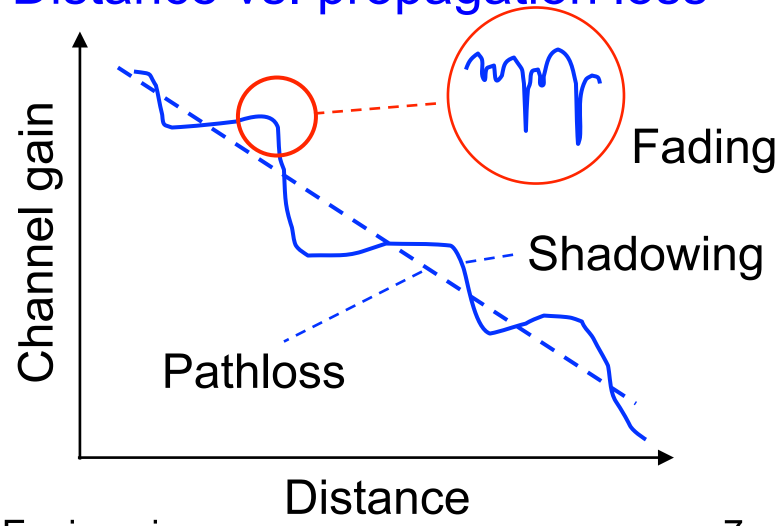
Pathloss

Tx antenna gain

Narrow band propagation channel



Distance vs. propagation loss



Standing Wave & Fading

Multi-path propagation channel

$$h_{\mathrm{B}}(t) = \sum_i h_{\mathrm{B}i}(t)$$

$$h_{\mathrm{B}i}(t) = h_{\mathrm{pl}}(d)\, e^{-j2\pi f_0 \tau_i(t)}$$

Doppler shift (phase shift due to mobility)

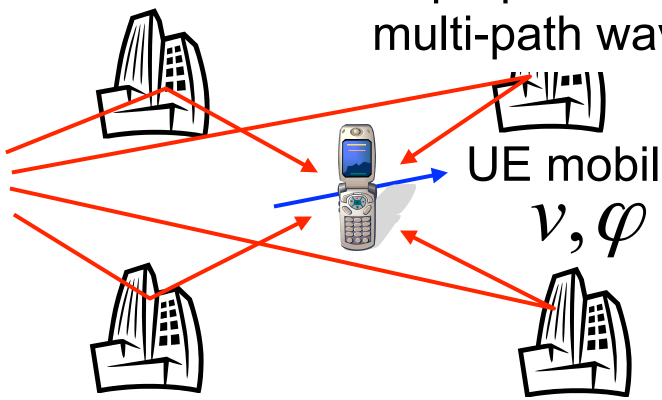
$$\tau_i(t) = \tau_0 + \frac{vt\cos\varphi_i}{c}$$

$$h_{\mathrm{B}i}(t) = h_{\mathrm{pl}}(d)\, e^{j\theta_0 - 2\pi f_{\mathrm{d}i} t}$$

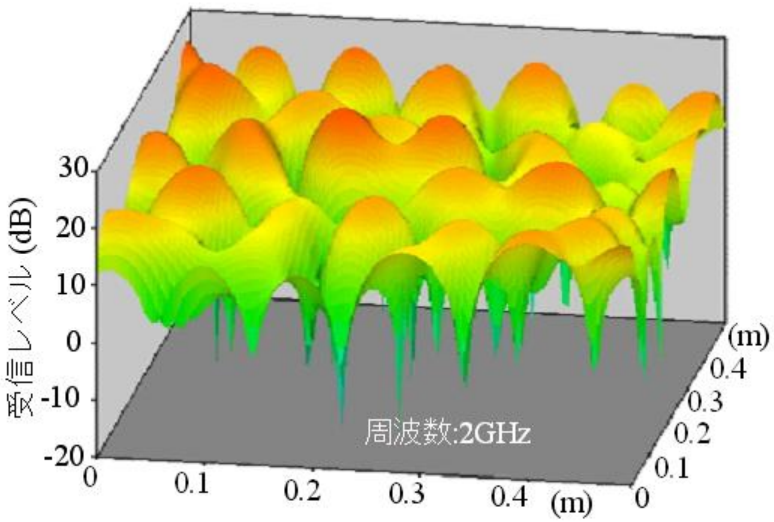
$$f_{\mathrm{d}i} = \frac{v\cos\theta_i}{\lambda_0}$$

Multi-path fading

Superposition of multi-path waves



Standing wave



Rayleigh Fading

Multi-path channel

$$h_{\mathrm{B}}(t) = \sum_i h_{\mathrm{pl}}(d) e^{j\theta_i(t)} = x + jy$$

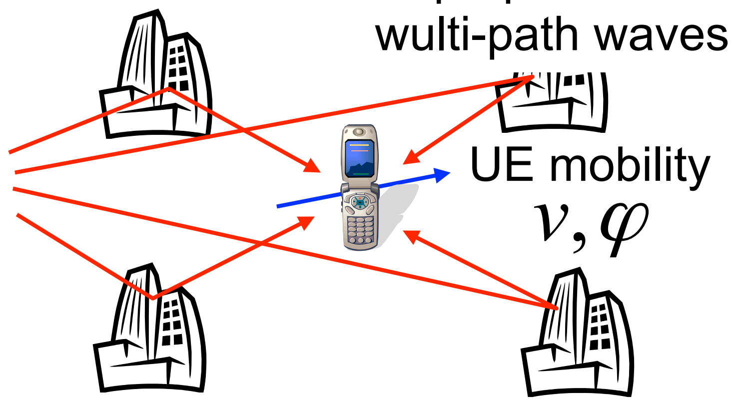
Central limit theorem

Sum of independent random variables

⇩

Gaussian distribution

Multi-path fading

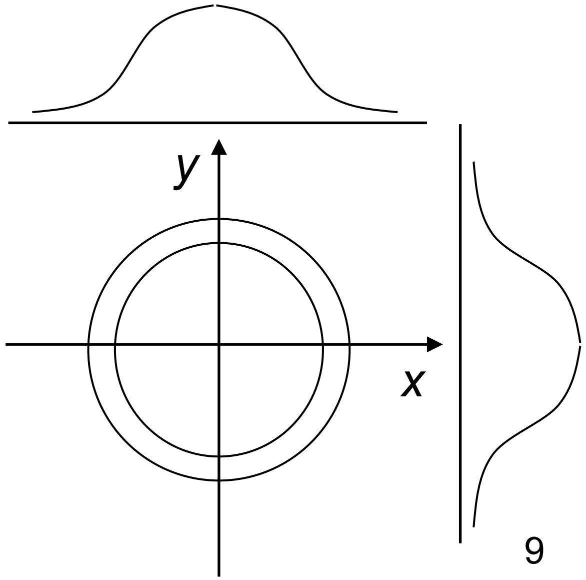


Complex Gaussian distribution

$$f(x) = \frac{1}{\sqrt{\pi\sigma_{\mathrm{h}}^2}} \exp\left(-\frac{x^2}{\sigma_{\mathrm{h}}^2}\right)$$

$$f(x, y) = \frac{1}{\pi\sigma_{\mathrm{h}}^2} \exp\left(-\frac{x^2 + y^2}{\sigma_{\mathrm{h}}^2}\right)$$

Complex Gaussian dist.



Probability of Fading

Cartesian to polar conversion

$$f(x,y)=\frac{1}{\pi\sigma_{\mathrm{h}}^2}\exp\left(-\frac{x^2+y^2}{\sigma_{\mathrm{h}}^2}\right)$$

$$x=r\cos\phi \qquad y=r\sin\phi$$

$$f(r,\phi)=f(x,y)\left|\det\mathbf{J}\right|=\frac{r}{\pi\sigma_{\mathrm{h}}^2}\exp\left(-\frac{r^2}{\sigma_{\mathrm{h}}^2}\right)$$

Jacobian

$$\mathrm{d}x\,\mathrm{d}y=\begin{vmatrix}\frac{\partial x}{\partial r} & \frac{\partial x}{\partial \phi}\\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \phi}\end{vmatrix}\mathrm{d}r\,\mathrm{d}\phi$$

$$=\left|\det\mathbf{J}\right|\mathrm{d}r\,\mathrm{d}\phi$$

Uniform phase distribution

Rayleigh distribution (amplitude & power)

$$f(r)=\int_0^{2\pi}f(r,\phi)\,\mathrm{d}\phi=\frac{2r}{\sigma_{\mathrm{h}}^2}\exp\left(-\frac{r^2}{\sigma_{\mathrm{h}}^2}\right)$$

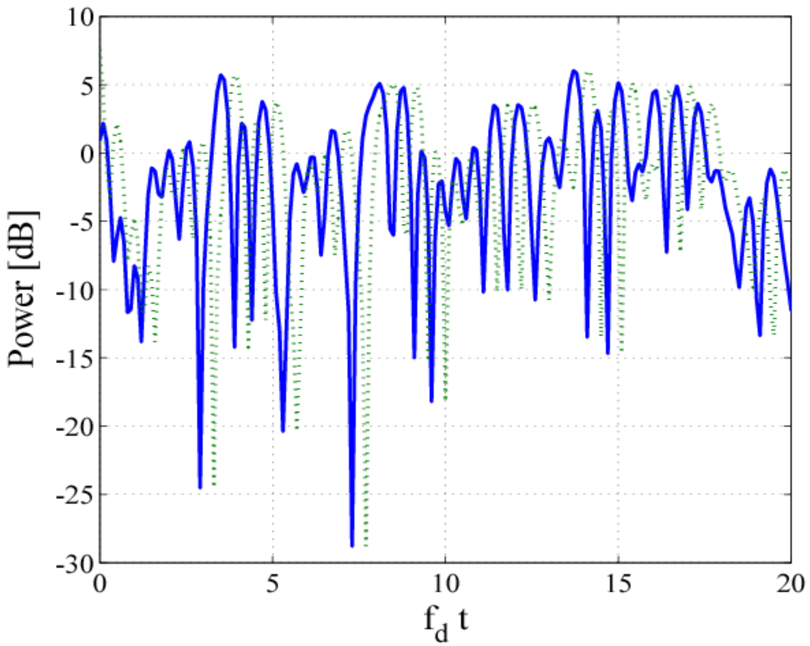
$$r=\sqrt{g}$$

$$f(g)=\frac{1}{2\sqrt{g}}f(r)=\frac{1}{\sigma_{\mathrm{h}}^2}\exp\left(-\frac{g}{\sigma_{\mathrm{h}}^2}\right)$$

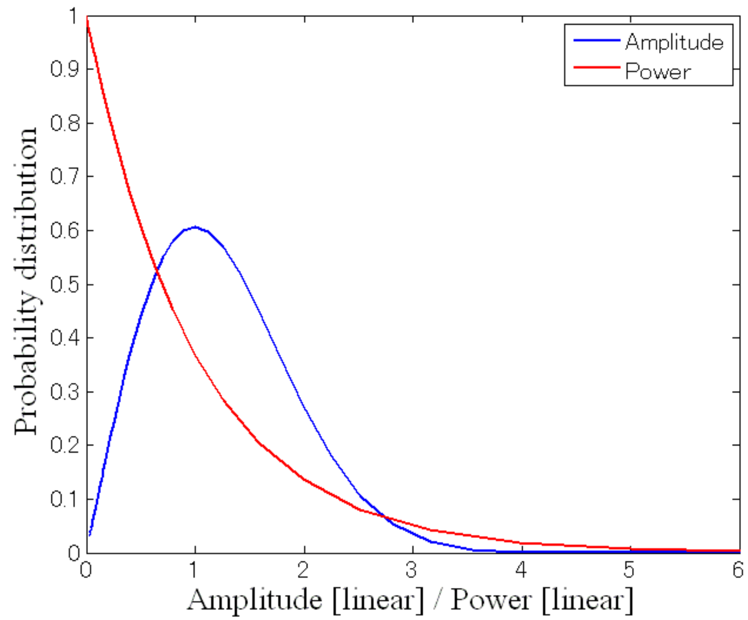
Rayleigh distribution

Rayleigh Distribution

Fading variation



Probability distribution



Cumulative Distribution

Rayleigh distribution

$$f(g) = \frac{1}{\sigma_{\mathrm{h}}^2} \exp\left(- \frac{g}{\sigma_{\mathrm{h}}^2} \right)$$

Cumulative probability distribution (CDF)

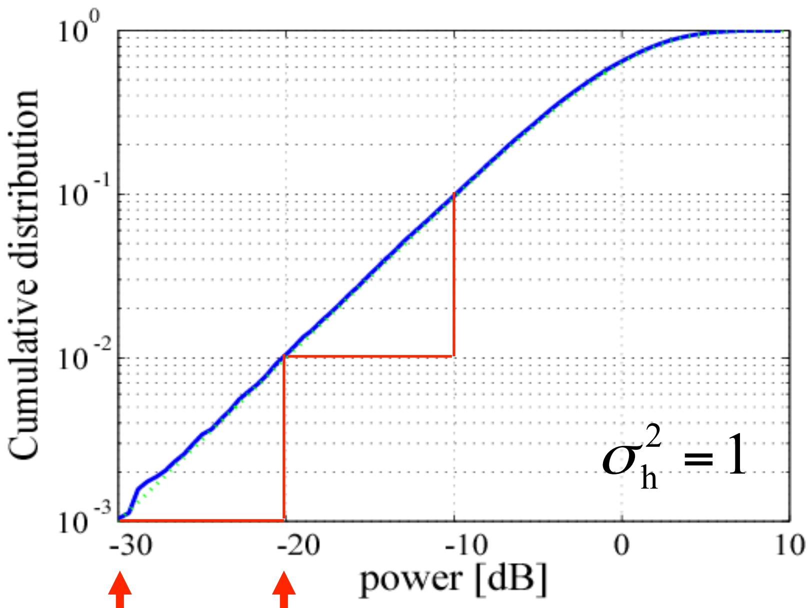
$$\tilde{f}(g) = \int_0^{\tilde{g}} f(g) \mathrm{d}g = 1 - \exp\left(- \frac{\tilde{g}}{\sigma_{\mathrm{h}}^2} \right)$$

Taylor expansion

$$\tilde{f}(p) = \tilde{f}(0) + g\tilde{f}'(0) + \frac{g^2}{2}\tilde{f}''(0) + \cdots$$

$$= \frac{g}{\sigma_{\mathrm{h}}^2} + \frac{g^2}{2\left(\sigma_{\mathrm{h}}^2\right)^2} + \cdots$$

CDF



Power attenuation more than 20dB
with probability 1/100

Power attenuation more than 30dB
with probability 1/1000

CDF of SNR

Signal-to-Noise Ratio (SNR)

$$\gamma = \frac{P\left|h_B(t)\right|^2}{\sigma^2} = \frac{Pg(t)}{\sigma^2}$$

Rayleigh fading

$$f(g) = \frac{1}{\sigma_h^2}\exp\left(-\frac{g}{\sigma_h^2}\right)$$

$$\sigma_h^2 = E\left[\left|h_B(t)\right|^2\right] = \bar{g}$$

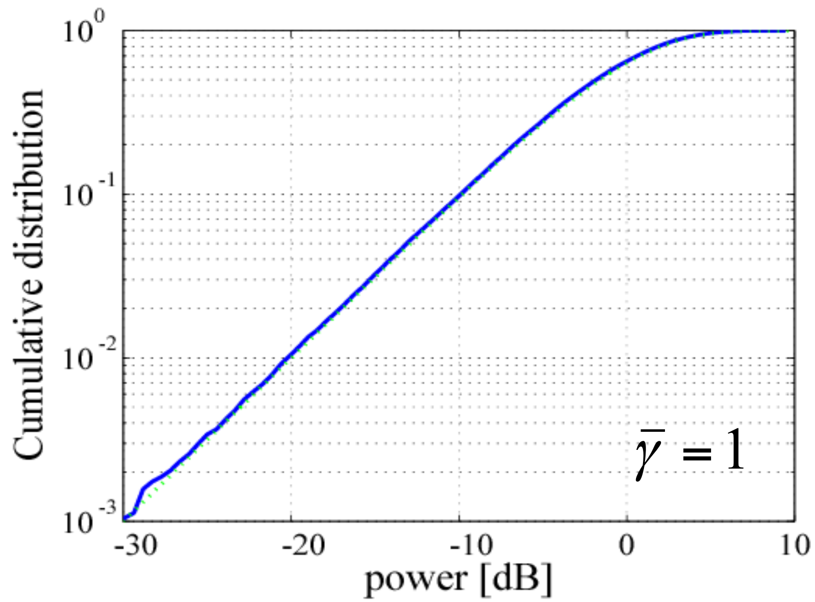
PDF of SNR

$$f(\gamma) = \frac{1}{\bar{\gamma}}\exp\left(-\frac{\gamma}{\bar{\gamma}}\right)$$

$$\bar{\gamma} = \frac{P\bar{g}}{\sigma^2}$$

Average SNR

CDF



Error Rate in Fading Channel

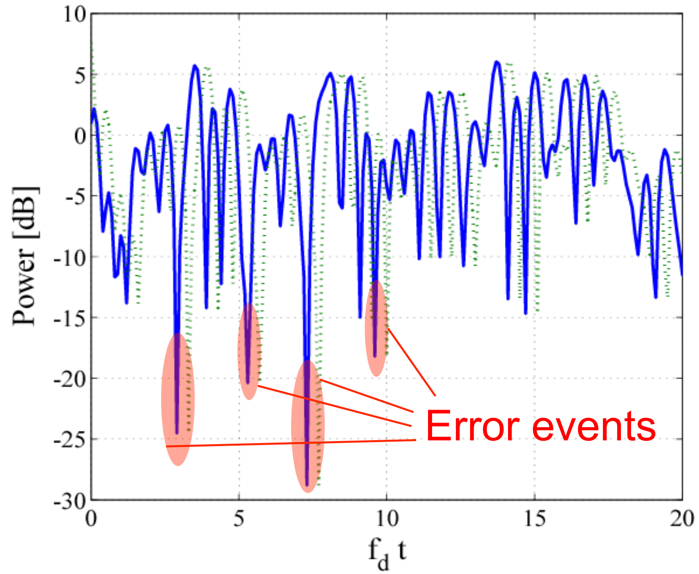
BER of BPSK

$$p_{\mathrm{eb}}(\gamma) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\gamma}\right) \qquad \gamma = \frac{P|h|^2}{\sigma^2}$$

Rayleigh fading channel

$$f(\gamma) = \frac{1}{\bar{\gamma}}\exp\left(-\frac{\gamma}{\bar{\gamma}}\right) \qquad \bar{\gamma} = \mathrm{E}\left[\frac{P|h(t)|^2}{\sigma^2}\right]$$

Fading channel



Average BER

$$\bar{p}_{\mathrm{eb}}(\bar{\gamma}) = \int p_{\mathrm{eb}}(\gamma) f(\gamma)\mathrm{d}\gamma$$

$$= \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right)$$

Mathematical formulas

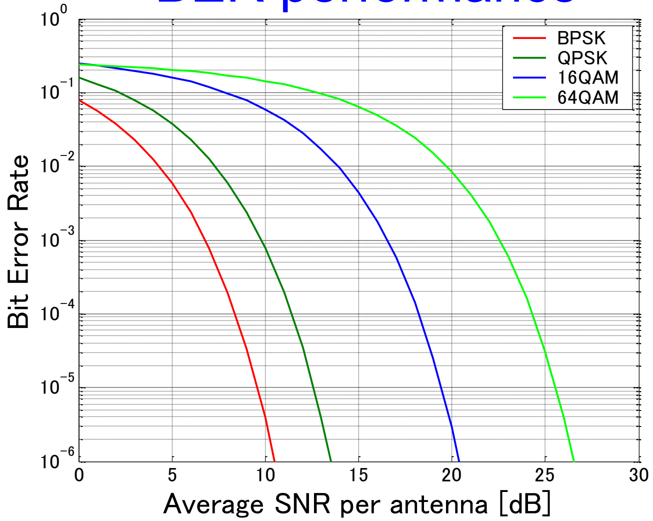
$$\int f(x)g'(x)\mathrm{d}x = f(x)g(x) - \int f'(x)g(x)\mathrm{d}x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{erfc}(x) = -\frac{2}{\sqrt{\pi}}\exp\left(-x^2\right)$$

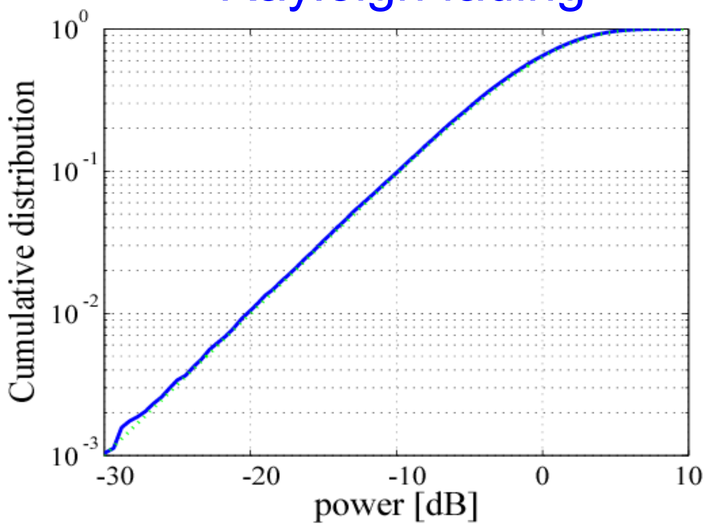
$$\int_0^\infty \exp\left(-ax^2\right)\mathrm{d}x = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

BER Performance in Fading Channel

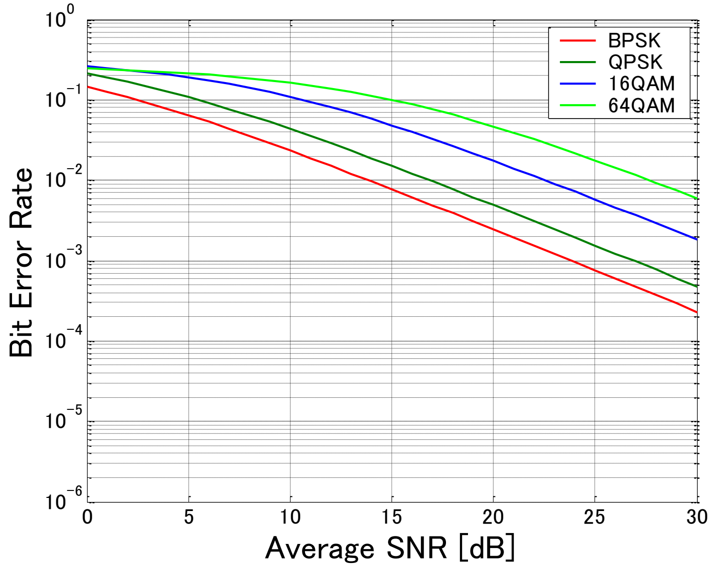
BER performance



Rayleigh fading



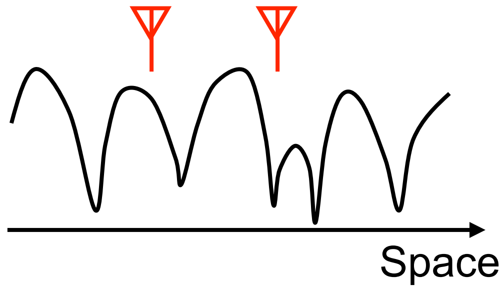
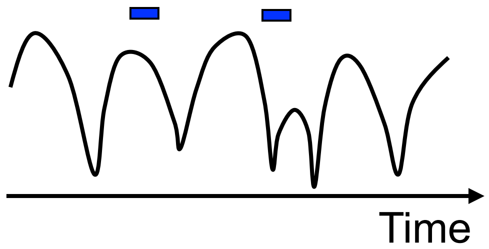
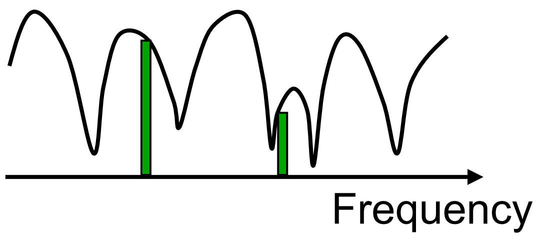
Average BER performance



One of biggest problem
in wireless communications

Diversity Technologies

Fading combatting schemes using more than 2 {Antenna, Symbol, Subcarrier}

Antenna	Array antenna In multi-path env.	
Time	Repetition, Interleaver In Doppler spread env.	
Frequency	OFDM In delay spread env.	

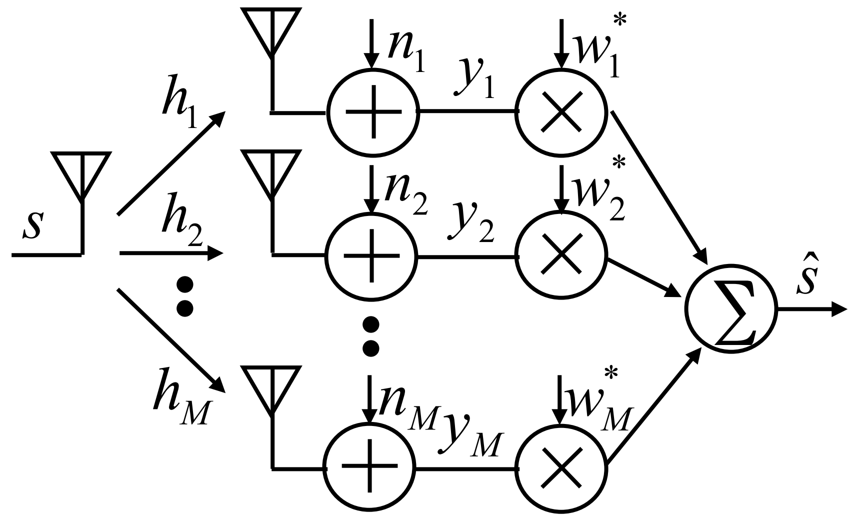
Antenna Signal Processing

Receive signal model

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_M \end{bmatrix} s + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_M \end{bmatrix}$$

$$\mathbf{y} = \mathbf{h}s + \mathbf{n}$$

Array antenna receiver



Weighted combining

$$\hat{s} = \mathbf{w}^H \mathbf{y} = \mathbf{w}^H \mathbf{h} s + \mathbf{w}^H \mathbf{n}$$

$$\mathbf{w} = \begin{bmatrix} w_1 & w_2 & \cdots & w_M \end{bmatrix}^T$$

Maximum Ratio Combining

SNR after weighted combining

Vector inner product

$$\gamma = \frac{\mathrm{E}[|\mathbf{w}^H \mathbf{h} s|^2]}{\mathrm{E}[|\mathbf{w}^H \mathbf{n}|^2]} = \frac{|\mathbf{w}^H \mathbf{h}|^2 \, \mathrm{E}[|s|^2]}{\mathbf{w}^H \, \mathrm{E}[\mathbf{n}\mathbf{n}^H]\mathbf{w}} = \frac{|\mathbf{w}^H \mathbf{h}|^2 P}{|\mathbf{w}|^2 \sigma^2}$$

Vector outer product

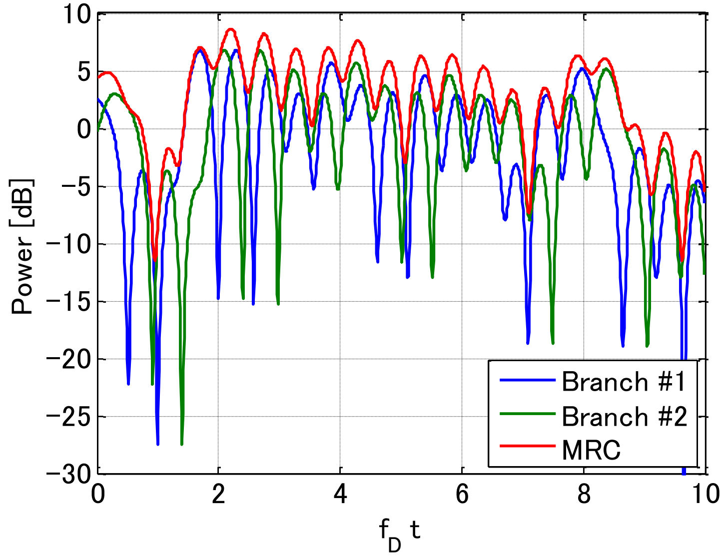
$$\mathrm{E}[\mathbf{n}\mathbf{n}^H] = \sigma^2 \mathbf{I}_M$$ ← Uncorrelated noise

Maximum Ratio Combining (MRC)

$$\mathbf{w}_{\mathrm{opt}} = \arg\max \gamma = \alpha \mathbf{h}$$

$$\gamma_{\mathrm{opt}} = \frac{|\mathbf{h}|^2 P}{\sigma^2} = \frac{\sum_{i=1}^{M} |h_i|^2 P}{\sigma^2} = \sum_{i=1}^{M} \gamma_i$$ ← Sum of branch (antenna) SNR

Performance of MRC



PDF & Characteristic Function

PDF of sum of random variables

Independent random variables x y

$$f(x) \quad f(y) \quad f(x,y) = f(x)f(y)$$

$z = x + y$ ← Sum of random variables

$f(z) = \int f(x) f(z-x)\mathrm{d}x$ ← Convolution

Characteristic function of PDF

$$f(\gamma) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \varphi(t)\exp(-j\gamma t)\mathrm{d}\gamma \longleftrightarrow \varphi(t) = \int_0^{\infty} f(\gamma)\exp(j\gamma t)\mathrm{d}\gamma$$

Theorem on Fourier transformation

$$\gamma = \sum_i \gamma_i \longleftrightarrow \varphi(t) = \prod_i \varphi_i(t)$$

CDF of SNR after MRC

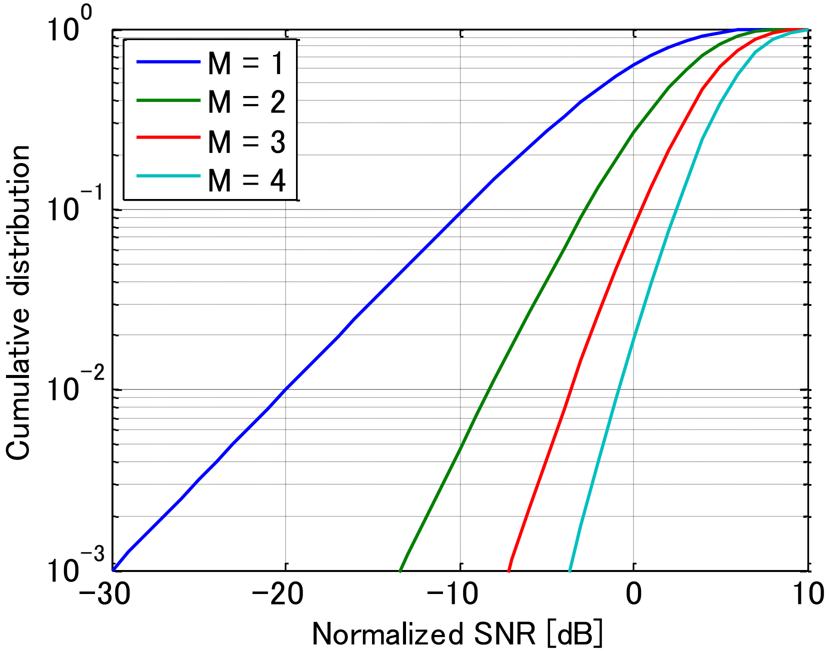
SNR after MRC

$$\gamma = \sum_{i=1}^{M} \gamma_i$$

Characteristic function of each antenna

$$f(\gamma_i) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma_i}{\bar{\gamma}} \right) \leftrightarrow \varphi_i(t) = \frac{1}{1 - j\bar{\gamma}t}$$

CDF



PDF of SNR after MRC

$$\varphi(t) = \prod_{i=1}^{M} \varphi_i(t) = \left(\frac{1}{1 - j\bar{\gamma}t} \right)^M$$

$$f(\gamma) = \frac{1}{(M-1)!\bar{\gamma}^M} \gamma^{M-1} \exp\left(-\frac{\gamma}{\bar{\gamma}} \right)$$

Gamma dist. (χ square dist.)

BER of MRC Diversity

Average BER

$$\bar{p}_{\mathrm{e}}(\bar{\gamma}) = \int p_{\mathrm{e}}(\gamma) f(\gamma)\,\mathrm{d}\gamma$$

BER in AWGN (Gaussian Noise)

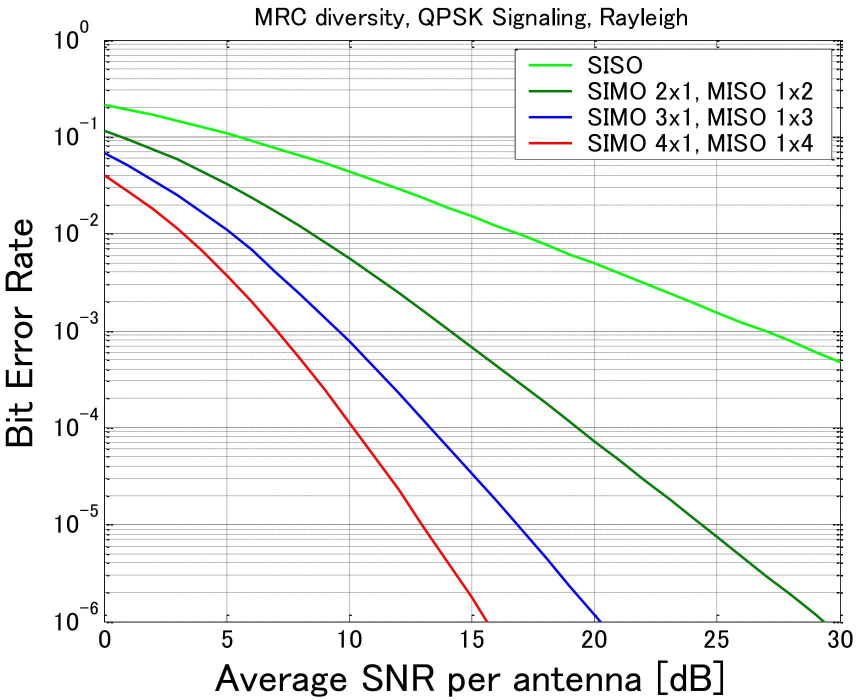
$$P_{\mathrm{e}} = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\gamma}\right)$$

PDF of SNR after MRC

$$f(\gamma) = \frac{1}{(M-1)!\,\bar{\gamma}^{M}} \gamma^{M-1} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right)$$

$$\bar{p}_{\mathrm{e}}(\bar{\gamma}) = \frac{1}{2}\left(1-\sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right)^{M} \sum_{i=0}^{M-1} \binom{M-1+i}{i} \left(\frac{1}{2}\left(1-\sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right)\right)^{i}$$

BER performance



Summary

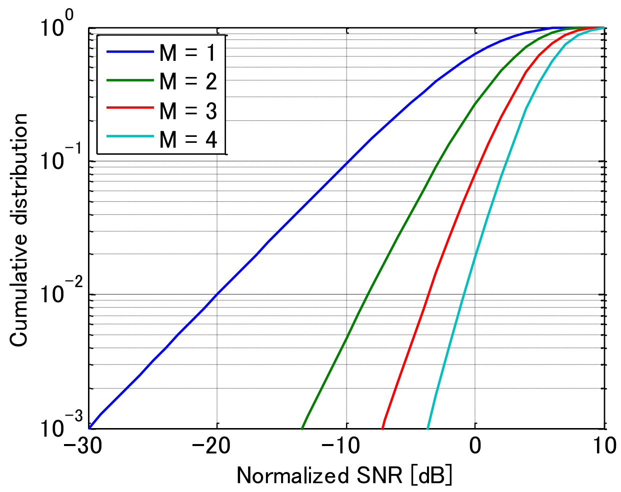
- Error rate in fading channel

$$p_{\mathrm{eb}}(\gamma) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\gamma}\right) \Rightarrow \bar{p}_{\mathrm{eb}}(\bar{\gamma}) = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right)$$

- Antenna signal processing

$$\mathbf{y} = \mathbf{h}s + \mathbf{n}$$

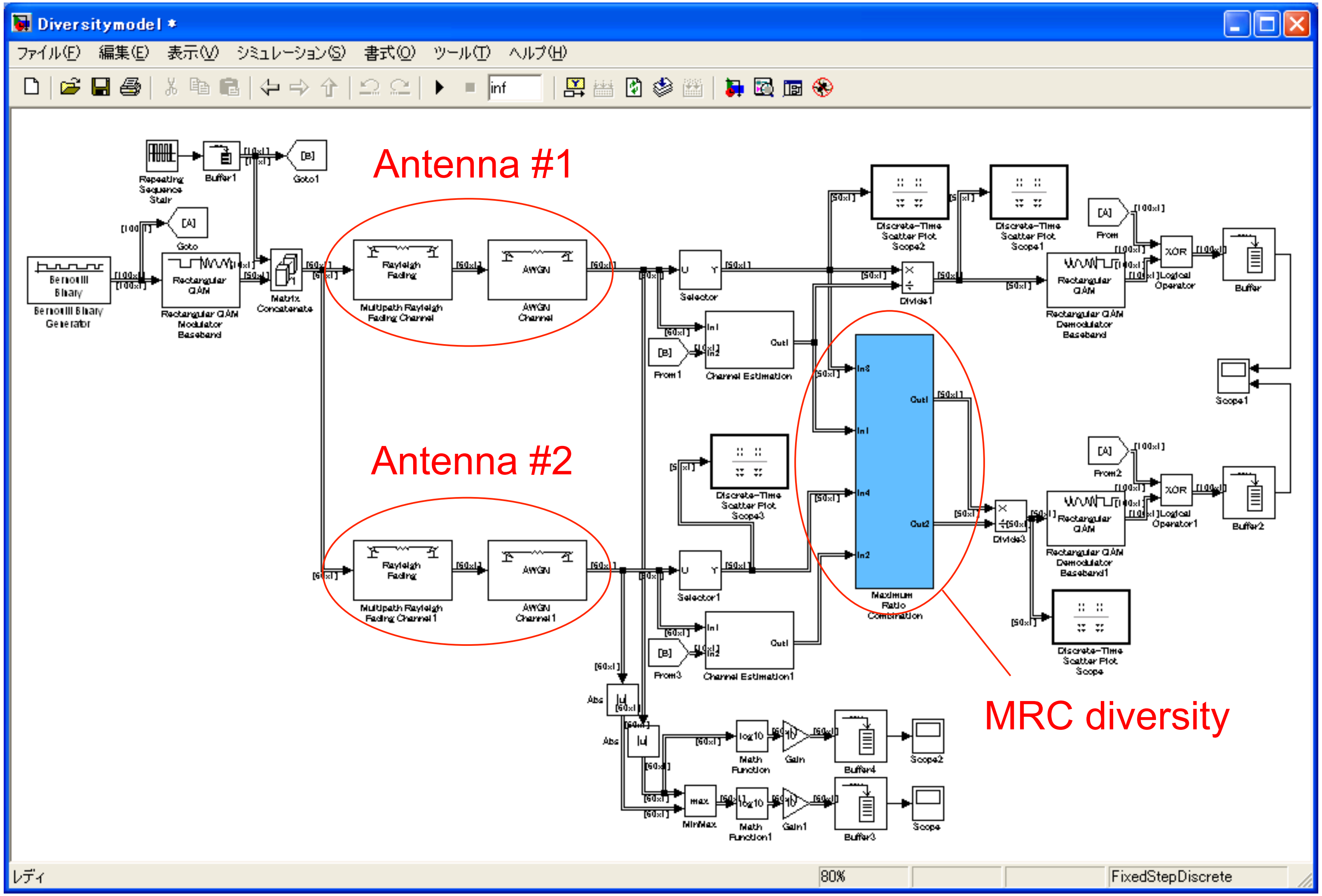
$$\hat{s} = \mathbf{w}^H\mathbf{y} = \mathbf{w}^H\mathbf{h}s + \mathbf{w}^H\mathbf{n}$$



- Performance of MRC diversity

$$\gamma_{\mathrm{opt}} = \frac{|\mathbf{h}|^2 P}{\sigma^2} = \frac{\sum_{i=1}^{M} |h_i|^2 P}{\sigma^2} = \sum_{i=1}^{M} \gamma_i \Rightarrow f(\gamma) = \frac{1}{(M-1)!\bar{\gamma}^M}\gamma^{M-1}\exp\left(-\frac{\gamma}{\bar{\gamma}}\right)$$

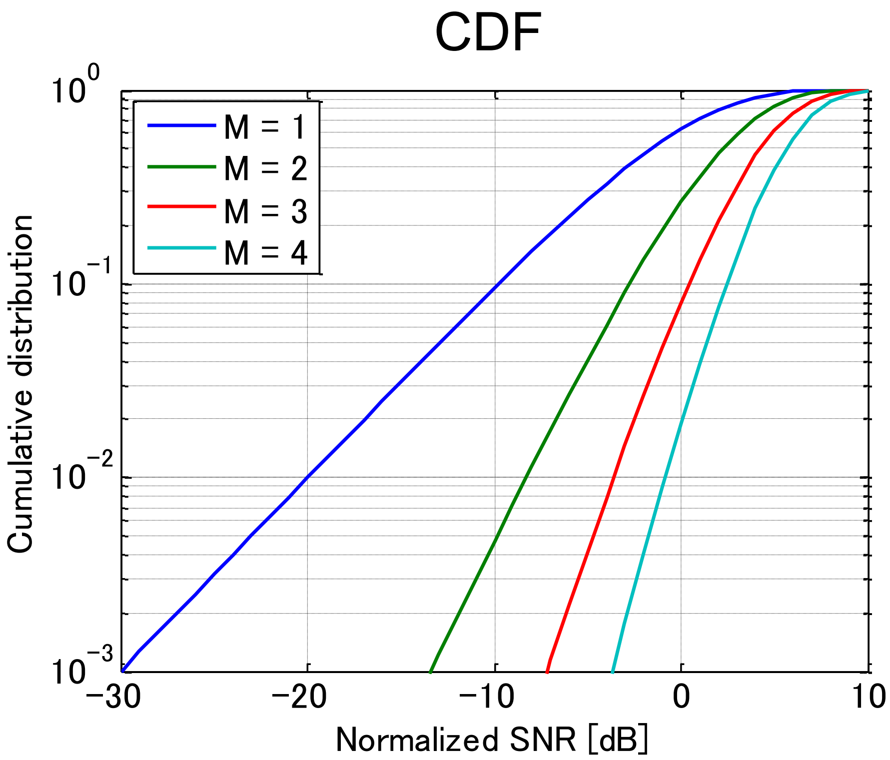
Demo


Diversitymodel *
ファイル(F) 編集(E) 表示(V) シミュレーション(S) 書式(O) ツール(T) ヘルプ(H)
Antenna #1
Antenna #2
MRC diversity
Bernoulli Binary Generator
Rectangular QAM Modulator Baseband
Matrix Concatenate
Multipath Rayleigh Fading Channel
AWGN Channel
Multipath Rayleigh Fading Channel1
AWGN Channel1
Selector
Selector1
Channel Estimation
Channel Estimation1
Maximum Ratio Combination
Discrete-Time Scatter Plot Scope
Rectangular QAM Demodulator Baseband
Logical Operator
Buffer
Scope
Math Function
Gain
MinMax
レディ
80%
FixedStepDiscrete

Diversity Order

PDF of SNR after MRC

$$f(\gamma) = \frac{1}{(M-1)!\bar{\gamma}^M}\gamma^{M-1}\exp\left(-\frac{\gamma}{\bar{\gamma}}\right)$$



CDF of SNR after MRC

$$\tilde{f}(\gamma) = 1 - \exp\left(-\frac{\gamma}{\bar{\gamma}}\right)\sum_{i=1}^{M}\frac{\left(\gamma/\bar{\gamma}\right)^{i-1}}{(i-1)!}$$

1 antenna

$$\tilde{f}(\gamma) = 1 - \exp\left(-\frac{\gamma}{\bar{\gamma}}\right) \cong \frac{\gamma}{\bar{\gamma}} + O\left(\gamma^2\right)$$

2 antenna MRC

Diversity order

$$\tilde{f}(\gamma) = 1 - \exp\left(-\frac{\gamma}{\bar{\gamma}}\right) - \frac{\gamma}{\bar{\gamma}}\exp\left(-\frac{\gamma}{\bar{\gamma}}\right) \cong \frac{1}{2}\left(\frac{\gamma}{\bar{\gamma}}\right)^{2} + O\left(\gamma^3\right)$$