13. Nonreciprocal circuits

14.1 Loss-less characteristics of circuits

isolators

circulators

14.2 Microwave response of magnetic material

14.3 Isolator

14.4 Edge-guided mode

14.5 Circulator

14.6 Application of circulators

Bi-directional transmission

Applicable to a phase modulator

Add Drop Multiplexer

Loss-less or lossy ?

Check whether S-matrix can be unitary or not.

(1) Isolator

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \mathbf{Sa} \quad \Rightarrow \quad \mathbf{SS}^{+} = \qquad \neq \mathbf{I}$$

Isolator is never a loss-less device.

(2) Circulator

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad \Rightarrow \quad \mathbf{SS}^{+} = \mathbf{I}$$

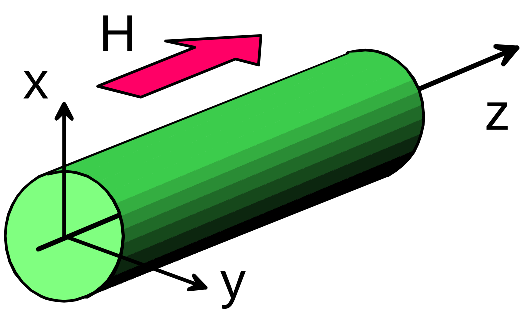
Circulator can be a loss-less device.

Microwave response of magnetic material

Apply a magnetic field along the z-direction, then

Poldar's tensor

$$[\mu] = \mu_0 \begin{bmatrix} \mu_r & -j\kappa & 0 \\ j\kappa & \mu_r & 0 \\ 0 & 0 & \mu_{rz} \end{bmatrix}$$



Faraday rotator

$$\mu_{rz} \approx 1$$

$$\omega_0 = -\gamma H_0 \qquad (\gamma = -2.8 MHz / Oe)$$

$$\omega_s = -\gamma M_s \qquad$$ (M_s: saturation magnetization)

ω: operating frequency

$$\mu_r = 1 + \frac{\omega_0 \omega_s}{\omega_0^2 - \omega^2}$$

$$\kappa = \frac{-\omega_s \omega}{\omega_0^2 - \omega^2}$$

Electromagnetic field in magnetic material

H and B of RF field : $\mathbf{h}e^{j\omega t}$ $\mathbf{b}e^{j\omega t}$

right-handed circularly polarized wave:

$$\mathbf{h}_{+} = \mathbf{i}h_x + \mathbf{j}h_y$$

$$h_y = -jh_x$$

$$h_x = \mathrm{Re}[e^{j\omega t}] = \cos\omega t$$

$$h_y = \mathrm{Re}[-je^{j\omega t}] = \cos(\omega t - \frac{\pi}{2})$$

$$\mathbf{b}_{+} = \mu_0 \begin{bmatrix} \mu_r & -j\kappa & 0 \\ j\kappa & \mu_r & 0 \\ 0 & 0 & \mu_{rz} \end{bmatrix} \begin{bmatrix} h_x \\ -jh_x \\ 0 \end{bmatrix}$$

$$= \mu_0 \begin{bmatrix} (\mu_r - \kappa)h_x \\ -j(\mu_r - \kappa)h_x \\ 0 \end{bmatrix} = \mu_0(\mu_r - \kappa) \begin{bmatrix} h_x \\ -jh_x \\ 0 \end{bmatrix} = \underbrace{\mu_0(\mu_r - \kappa)}_{\mu_{+}}\mathbf{h}_{+}$$

left-handed circularly polarized wave:

$$\mathbf{h}_{-} = \mathbf{i}h_x + \mathbf{j}h_y$$

$$h_y = +jh_x$$

$$h_x = \mathrm{Re}[e^{j\omega t}] = \cos\omega t$$

$$h_y = \mathrm{Re}[+je^{j\omega t}] = \cos(\omega t + \frac{\pi}{2})$$

$$\mathbf{b}_{-} = \mu_0 \begin{bmatrix} \mu_r & -j\kappa & 0 \\ j\kappa & \mu_r & 0 \\ 0 & 0 & \mu_{rz} \end{bmatrix} \begin{bmatrix} h_x \\ +jh_x \\ 0 \end{bmatrix}$$

$$= \mu_0 \begin{bmatrix} (\mu_r + \kappa)h_x \\ j(\mu_r + \kappa)h_x \\ 0 \end{bmatrix} = \mu_0(\mu_r + \kappa) \begin{bmatrix} h_x \\ jh_x \\ 0 \end{bmatrix} = \underbrace{\mu_0(\mu_r + \kappa)}_{\mu_{-}}\mathbf{h}_{-}$$

Faraday rotation

linearly polarized wave: $e_x = E_0 \cos \omega t,\ e_y = e_z = 0$

$$\mathbf{i} e_x = \mathbf{i} E_0 \cos \omega t$$

$$= \frac{E_0}{2}\{\mathbf{i}\cos\omega t + \mathbf{j}\cos(\omega t - \frac{\pi}{2})\} + \frac{E_0}{2}\{\mathbf{i}\cos\omega t + \mathbf{j}\cos(\omega t + \frac{\pi}{2})\}$$

r-circular polarized | l-circular polarized

$$\beta_+ = \omega\sqrt{\varepsilon\mu_0(\mu_r - \kappa)} \qquad \beta_- = \omega\sqrt{\varepsilon\mu_0(\mu_r + \kappa)}$$

$$\frac{E_0}{2}\{\mathbf{i}\cos(\omega t - \beta_+ z) + \mathbf{j}\cos(\omega t - \frac{\pi}{2} - \beta_+ z)\} \quad \frac{E_0}{2}\{\mathbf{i}\cos(\omega t - \beta_- z) - \mathbf{j}\sin(\omega t - \beta_- z)\}$$

$$= \frac{E_0}{2}\{\mathbf{i}\cos(\omega t - \beta_+ z) + \mathbf{j}\sin(\omega t - \beta_+ z)\}$$

$$e_x = \frac{E_0}{2}\{\cos(\omega t - \beta_+ z) + \cos(\omega t - \beta_- z)\} = E_0 \cos\left(\frac{\beta_- - \beta_+}{2} z\right)\cos\left(\omega t - \frac{\beta_- + \beta_+}{2} z\right)$$

$$e_y = \frac{E_0}{2}\{\sin(\omega t - \beta_+ z) - \sin(\omega t - \beta_- z)\} = E_0 \sin\left(\frac{\beta_- - \beta_+}{2} z\right)\cos\left(\omega t - \frac{\beta_- + \beta_+}{2} z\right)$$

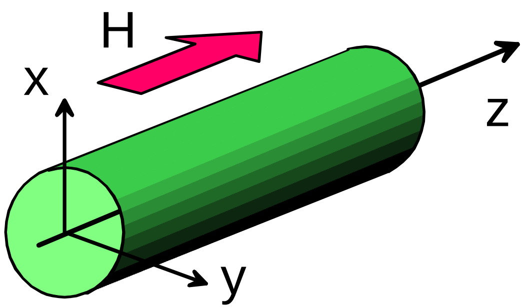
Angle of polarization rotation:

$$\theta_F = \frac{\beta_- - \beta_+}{2} z = \frac{\omega\sqrt{\varepsilon}}{2}(\sqrt{\mu_-} - \sqrt{\mu_+}) z$$

$$= \frac{\omega\sqrt{\varepsilon}}{2}(\sqrt{\mu_0(\mu_r + \kappa)} - \sqrt{\mu_0(\mu_r - \kappa)}) z$$

Faraday rotation

+z propagation



Faraday rotator

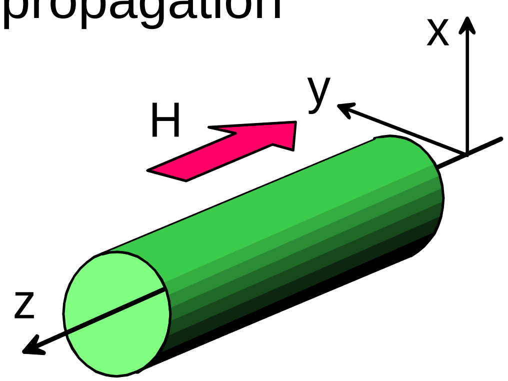
$$\mu_+ = \mu_0(\mu_r - \kappa)$$

$$\mu_- = \mu_0(\mu_r + \kappa)$$

$$\theta_F = \frac{\omega\sqrt{\varepsilon}}{2}(\sqrt{\mu_-} - \sqrt{\mu_+})L$$

$$= \frac{\omega\sqrt{\varepsilon}}{2}(\sqrt{\mu_0(\mu_r + \kappa)} - \sqrt{\mu_0(\mu_r - \kappa)})L$$

-z propagation



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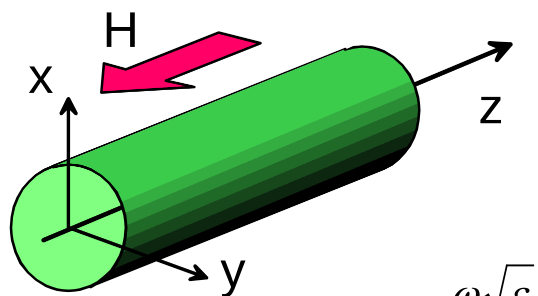
H-field reversed



$$H_0 \rightarrow -H_0 \quad M_s \rightarrow -M_s$$

$$\kappa \rightarrow -\kappa$$

$$\mu_+ = \mu_0(\mu_r + \kappa)$$

$$\mu_- = \mu_0(\mu_r - \kappa)$$

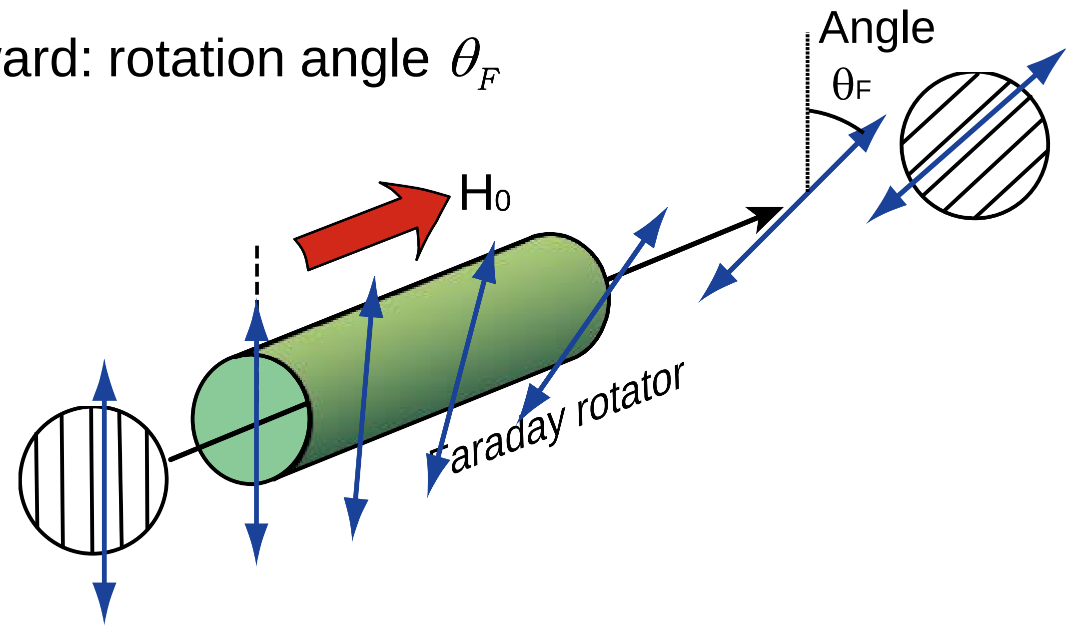
$$\theta = \frac{\omega\sqrt{\varepsilon}}{2}(\sqrt{\mu_-} - \sqrt{\mu_+})L$$

$$= \frac{\omega\sqrt{\varepsilon}}{2}(\sqrt{\mu_0(\mu_r - \kappa)} - \sqrt{\mu_0(\mu_r + \kappa)})L = -\theta_F$$

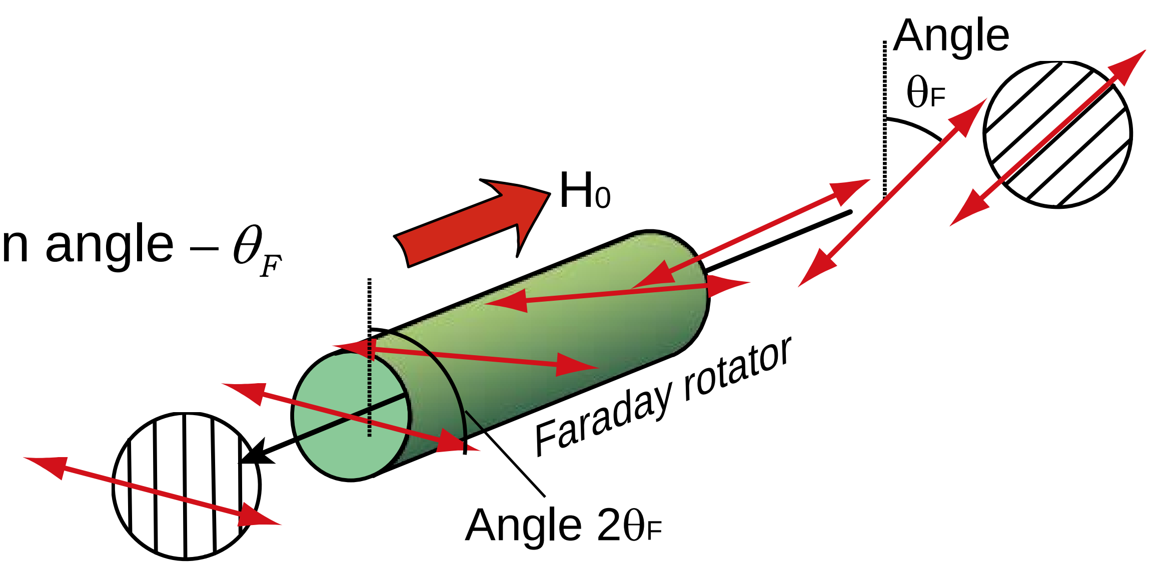
Faraday Isolator

polarizer + Faraday rotator + polarizer

Forward: rotation angle θ_F



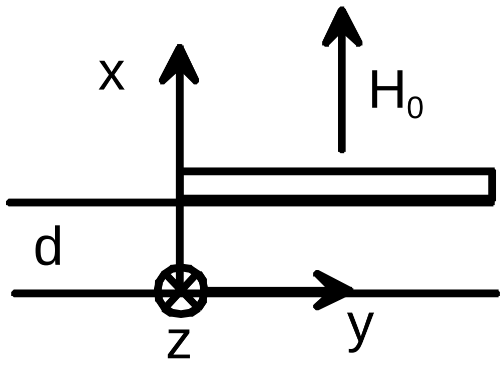
Backward: rotation angle $-\theta_F$



Edge-guided mode

$$\nabla \times \mathbf{E} = -j\omega\mu_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mu_r & -j\kappa \\ 0 & j\kappa & \mu_r \end{pmatrix} \mathbf{H}$$

$$\nabla \times \mathbf{H} = j\omega\varepsilon \mathbf{E}$$



$$\frac{\partial}{\partial x} = 0 \quad \text{in ferrite plate}$$

$$\begin{cases} \dfrac{\partial E_x}{\partial z} = -j\omega\mu_0(\mu_r H_y - j\kappa H_z) \\ -\dfrac{\partial E_x}{\partial y} = -j\omega\mu_0(j\kappa H_y + \mu_r H_z) \\ \dfrac{\partial H_z}{\partial y} - \dfrac{\partial H_y}{\partial z} = j\omega\varepsilon E_x \end{cases}$$

$$H_y = \frac{\mu_r}{\omega\mu_0(\mu_r{}^2 - \kappa^2)}\left(j\frac{\partial E_x}{\partial z} + \frac{\kappa}{\mu_r}\frac{\partial E_x}{\partial y}\right)$$

$$H_z = \frac{\mu_r}{\omega\mu_0(\mu_r{}^2 - \kappa^2)}\left(\frac{\kappa}{\mu_r}\frac{\partial E_x}{\partial z} - j\frac{\partial E_x}{\partial y}\right)$$

$$\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} + \omega^2\mu_0\mu_{eff}\varepsilon E_x = 0$$

$$\frac{(\mu_r{}^2 - \kappa^2)}{\mu_r} = \mu_{eff}$$

$$\frac{\partial^2 E_x}{\partial y^2} = (\beta^2 - \omega^2\mu_0\mu_{eff}\varepsilon)E_x \qquad \left(\frac{\partial}{\partial z} = -j\beta\right)$$

Edge-guided mode

$$\frac{\partial^2 E_x}{\partial y^2} = (\beta^2 - \omega^2 \mu_0 \mu_{eff} \varepsilon) E_x \qquad (\frac{\partial}{\partial z} = -j\beta)$$

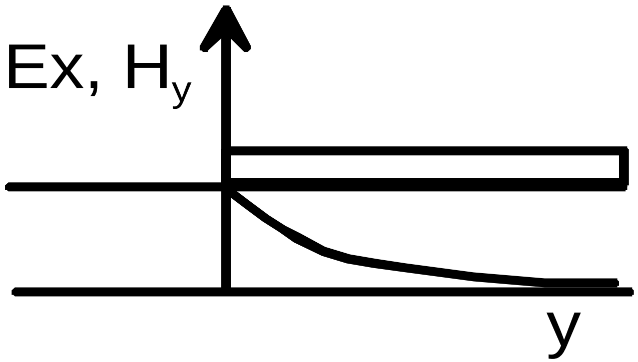
$$E_x = (Ae^{-\alpha y} + Be^{\alpha y})e^{-j\beta z} \quad \alpha = \sqrt{\beta^2 - \omega^2 \mu_0 \mu_{eff} \varepsilon}$$

$$E_x = 0 \text{ at } y = \infty$$

$$\therefore E_x = Ae^{-\alpha y} e^{-j\beta z}$$

$$H_y = \frac{1}{\omega \mu_0 \mu_{eff}} \left(\beta - \frac{\kappa}{\mu_r} \alpha \right) E_x = \frac{1}{\omega \mu_0 \mu_{eff}} \left(\beta - \frac{\kappa}{\mu_r} \alpha \right) Ae^{-\alpha y} e^{-j\beta z}$$

$$H_z = \frac{j}{\omega \mu_0 \mu_{eff}} \left(\alpha - \frac{\kappa}{\mu_r} \beta \right) Ae^{-\alpha y} e^{-j\beta z}$$



$$H_z = 0 \text{ on } y = 0$$

$$\alpha = \frac{\kappa}{\mu_r} \beta \qquad \alpha = \sqrt{\beta^2 - \omega^2 \mu_0 \mu_{eff} \varepsilon}$$

$$\beta = \omega \sqrt{\mu_0 \mu_r \varepsilon}$$

$$\alpha = \frac{\kappa}{\mu_r} \omega \sqrt{\mu_0 \mu_r \varepsilon}$$

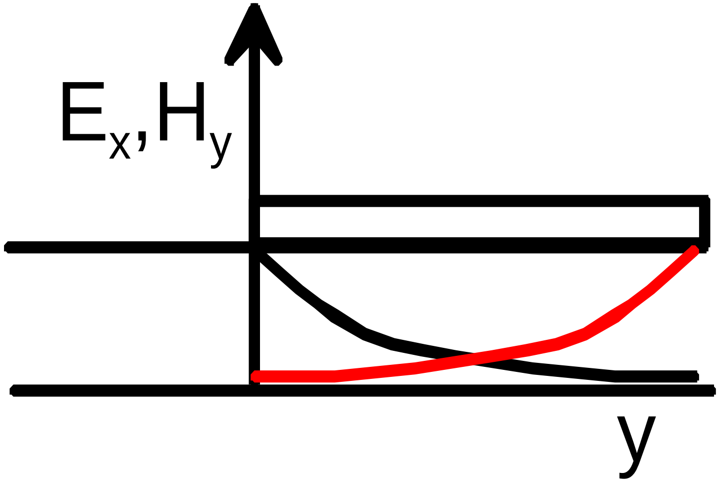
$$H_y = \sqrt{\frac{\varepsilon}{\mu_0 \mu_r}} Ae^{-\alpha y} e^{-j\beta z}$$

Edge-guided mode

Reversing the propagation direction (β-->-β)

$$\alpha = \frac{\kappa}{\mu_r}\beta \quad \Rightarrow \quad \alpha_{back} = \frac{\kappa}{\mu_r}(-\beta) = -\alpha_{forward}$$

$$H_y = \sqrt{\frac{\varepsilon}{\mu_0 \mu_r}} A e^{\alpha y} e^{j\beta z}$$



Junction type circulator

eigen excitation of 3 port rotationally symmetric circuit

$$\mathbf{u}_0 = \frac{1}{3}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad \mathbf{u}_+ = \frac{1}{3}\begin{pmatrix} 1 \\ e^{-j2\pi/3} \\ e^{j2\pi/3} \end{pmatrix} \qquad \mathbf{u}_- = \frac{1}{3}\begin{pmatrix} 1 \\ e^{j2\pi/3} \\ e^{-j2\pi/3} \end{pmatrix}$$

$$\begin{bmatrix} S_{11} & S_{31} & S_{21} \\ S_{21} & S_{11} & S_{31} \\ S_{31} & S_{21} & S_{11} \end{bmatrix} \mathbf{U}_i = S_i \mathbf{U}_i \quad \Rightarrow$$

$$S_{11} = \frac{1}{3}(S_0 + S_+ + S_-)$$

$$S_{21} = \frac{1}{3}\left(S_0 + e^{-j\frac{2}{3}\pi} S_+ + e^{j\frac{2}{3}\pi} S_-\right)$$

$$S_{31} = \frac{1}{3}\left(S_0 + e^{j\frac{2}{3}\pi} S_+ + e^{-j\frac{2}{3}\pi} S_-\right)$$

Junction type circulator -- design

basic configuration: $\frac{2\pi}{\lambda} R \sqrt{\varepsilon_r \mu_{rz}} \approx 1.84$

U_0 excitation: μ_e ⇒ Disk center is replaced with open circuit.

$S_0 = e^{j\phi_0}$ ϕ_0 : disk center - port electrical length

U_+ excitation: μ_+ ⇒ $\mu_+ L_0 // C$

$$S_+ = \frac{\dfrac{1}{j\omega C + \dfrac{1}{j\omega\mu_+ L_0}} - Z_c}{\dfrac{1}{j\omega C + \dfrac{1}{j\omega\mu_+ L_0}} + Z_c} = \frac{-Z_c + j\dfrac{\omega\mu_+ L_0}{1-\omega^2\mu_+ L_0 C}}{Z_c + j\dfrac{\omega\mu_+ L_0}{1-\omega^2\mu_+ L_0 C}} = e^{j\phi_+}$$

U_- excitation: μ_- ⇒ $\mu_- L_0 // C$

$$S_- = \frac{-Z_c + j\dfrac{\omega\mu_- L_0}{1-\omega^2\mu_- L_0 C}}{Z_c + j\dfrac{\omega\mu_- L_0}{1-\omega^2\mu_- L_0 C}} = e^{j\phi_-}$$

Junction type circulator -- design

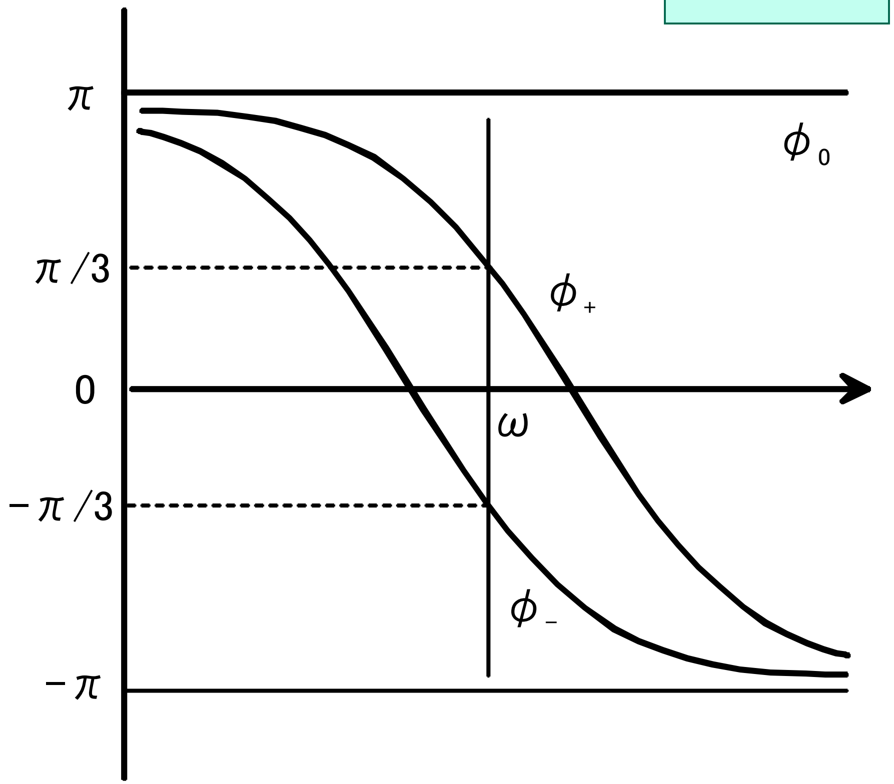
M_s, H_0 --> $\mu_e, \ \mu_+, \ \mu_-$

disk and line structure --> L_0, C

$$\phi_0 = \pi$$

$$\phi_+ = \frac{\pi}{3}$$

$$\phi_- = -\frac{\pi}{3}$$



Junction type circulator -- design

$$S_0 = e^{j\pi}, S_+ = e^{j\frac{\pi}{3}}, S_- = e^{-j\frac{\pi}{3}}$$

⇩

$$S_{11} = \frac{1}{3}(S_0 + S_+ + S_-) = 0$$

$$S_{21} = \frac{1}{3}(S_0 + e^{-j\frac{2}{3}\pi} S_+ + e^{j\frac{2}{3}\pi} S_-) = 0$$

$$S_{31} = \frac{1}{3}(S_0 + e^{j\frac{2}{3}\pi} S_+ + e^{-j\frac{2}{3}\pi} S_-) = e^{j\pi}$$

$$S = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

Circulator !

Application of circulator

(1) Bi-directional transmission

(2) Isolator is constructed in combination with an anti-reflection terminator.

(3) Applicable to a phase modulator

(4) Add Drop Multiplexer (ADM)