Nuclear Reactor Physics Lecture Note (5)
-Neutron Spectrum(3) Heterogeneity Effect, Neutron Slowing Down and Diffusion -

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3.3 Resonance absorption in finite dilution and core heterogeneous effect

(1) Resonance absorption in finite dilution

Resonance escape probability in finite dilution

In the general case of finite dilution, the neutron flux is depressed for the energies in the neighborhood of the resonance. (Energy self-shielding effect)

Doppler effect on resonance escape probability

As temperature increases

- The resonance peak decreases and the width becomes wider
- Decreasing self-shielding and flux depression
- Increasing resonance absorption
- Decreasing of resonance escape probability
    - Negative reactivity effect by fuel temperature rise

(2) Core heterogeneous effect

(a) Resonance escape probability in heterogeneous lattice

If the fuel is lamped into a heterogeneous lattice, the resonance escape probability increases dramatically.

Reason：

Neutrons that are slowed down to resonance energies in the moderator are primarily absorbed in the outer regions of the fuel element.

- Spatial self-shielding effect

3.4 Neutron slowing down and diffusion

During slowing down, neutrons diffuse in the medium.

(1)The Fermi age

Assumptions

Neutrons gain ξ in lethargy by a scattering

Number of scatterings is continuous and neutron lethargy is also continuous

① Neutron current and neutron slowing down density

$$\mathrm{div}\mathbf{J}(\mathbf{r},u)+\frac{\partial q(\mathbf{r},u)}{\partial u}=0 \qquad \cdots(1)$$

where,

$\mathbf{J}(\mathbf{r},u)$: neutron current

$q(\mathbf{r},u)$: neutron slowing down density

（Number of slowing down neutrons at lethargy u per unit time）

Definition $\quad q(\mathbf{r},u)\equiv\int_0^u du'\int_u^\infty du''\,\Sigma_s(u'\rightarrow u'')\phi(\mathbf{r},u')$

② Neutron current and neutron flux

In diffusion approximation

$$\mathbf{J}(\mathbf{r},u)=-D(u)\mathrm{grad}\phi(\mathbf{r},u) \qquad \cdots(2)$$

where,

$D(u)$: diffusion constant at lethargy u

③ Neutron slowing down density and neutron flux

$$q(\mathbf{r},u)=\xi\Sigma_s(u)\phi(\mathbf{r},u) \qquad \cdots(3)$$

From Eq. (1)～(3),

$$\frac{D(u)}{\xi\Sigma_s(u)}\nabla^2 q(\mathbf{r},u)=\frac{\partial q(\mathbf{r},u)}{\partial u} \qquad \cdots(4)$$

（Notice : By the assumptions, it is exactly true only in heavy moderators like graphite.）

we define the Fermi age $\tau(u)$

$$\tau(u)=\int_0^u\frac{D(u')}{\xi\Sigma_s(u')}du' \qquad \cdots(5)$$

From Eq. (4) and (5), we obtain

$$\nabla^2 q(\mathbf{r},\tau) = \frac{\partial q(\mathbf{r},t)}{d\tau} \quad \cdots(6)$$

(The age equation)

Physical meaning of Fermi age

$\tau = \frac{1}{6}\langle r^2 \rangle = \frac{1}{6}$ of the average $(\text{crow} - \text{flight distance})^2$ from the point where a neutron enters a system with E_0 to the point at which it slows down to an age τ.

τ_{th}: the age to thermal energies (or simply τ)

(can be used as an index of neutron diffusion when fission neutrons slow down to thermal energies)

(2)The neutron migration area

Neutron migration area M^2

Definition $M^2 = L^2 + \tau$

where,

$$L = \sqrt{\frac{D}{\Sigma_a}} \quad \text{(neutron diffusion length)}$$

Migration length M

$\frac{1}{\sqrt{6}}$ of the root-mean-square distance a neutron travels from its appearance as a fast fission neutron to its capture as a thermal neutron.

3.5 Thermal spectrum

(1) Equilibrium thermal spectrum

Assumption

No absorption, no neutron source, no leakage.

The neutron spectrum is expressed by the Maxwell-Boltzmann distribution function characterizing the energies of the particles of an ideal gas at temperature T.

（The neutrons are in thermal equilibrium at the same temperature T as the scattering medium.）

$$\phi(E) = \frac{2\pi n_0}{(\pi kT)^{\frac{3}{2}}}\left(\frac{2}{m}\right)^{\frac{1}{2}} E \; \exp\left(-\frac{E}{kT}\right)$$

n_0 ：neutron density in the medium

k：Boltzmann constant

m：neutron mass

The most probable neutron energy: $E_T = kT$

Corresponding speed: $v_T = \sqrt{\frac{2kT}{m}}$

(2) Nonequilibrium thermal spectra

The presence of absorption or leakage or a slowing down source can act to distort the thermal spectrum.