# Nuclear Reactor Physics Lecture Note (4)
## -Neutron Spectrum(2) Resonance Absorption-

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## 3.2 Resonance Absorption

### (1)Resonance Escape Probability

Resonance escape probability p

Fraction of fission neutrons that manage to slow down from fission to thermal energies without being absorbed

Resonance escape probability in hydrogen

Assumptions

Infinite medium of hydrogen

An infinitely massive absorber distributed uniformly

All neutron slowing down due to elastic scattering with hydrogen（ignore inelastic scattering）

Neutron source（neutron energy $E_S$）uniformly distributed in the medium

The resonance escape probability to energy E, p(E), is given by

$$p(E) = \exp\left[-\int_{E}^{E_s} \frac{dE'\Sigma_a(E')}{E'\Sigma_t(E')}\right] \quad \cdots (1)$$

where,

$\Sigma_a(E')$ : macroscopic absorption cross section at energy $E'$

$\Sigma_t(E')$ : macroscopic total cross section at energy $E'$

### (2)Breit-Wignar Single-Level Resonance Formula and Doppler Effect

(a) Breit-Wignar single-level resonance formula

（Energy dependence of absorption cross section for resonance which are spaced widely apart）

$$\sigma_\gamma(E_c) = \sigma_0 \frac{\Gamma_\gamma}{\Gamma}\left(\frac{E_0}{E_c}\right)^{\frac{1}{2}}\left[4\left(\frac{E_c - E_0}{\Gamma}\right)^2 + 1\right]^{-1} \quad \cdots (2)$$

where,

$E_0$ : the energy at which resonance occurs

$\Gamma$ : total line width of the resonance

（characterizing the width of the energy level and the full width at half-maximum

(FWHM) of the resonance)

$\Gamma_\gamma$ : radiation line width

(characterizing the probability that the compound nucleus will decay via gamma emission)

$\sigma_0$ : total cross section at the resonance energy $E_0$

$E_c$ : energy in center of mass system ($E_c = \frac{M}{m+M}E$)

・For low energy($E \ll E_0$) $\sigma_\gamma \propto \frac{1}{E^{\frac{1}{2}}}$ (or $\frac{1}{v}$)

・For high energy($E_0 \ll E$) $\sigma_\gamma \propto \frac{1}{E^{\frac{5}{2}}}$

(b)Doppler effect on cross section resonance

The nuclei are in thermal motion.

The relative speed between the nucleus and the neutron can be either greater or less than neutron speed.

→Doppler shift effect

as temperature increase, the resonance broadens, while its peak magnitude decreases.

Doppler-broadened Breit-Wigner resonance cross section (Bethe-Placzek approximation)

When $E \sim E_0$

$$\sigma_\gamma(E) = \sigma_0 \frac{\Gamma_\gamma}{\Gamma} \psi(\zeta, \chi) \quad \cdots (3)$$

$$\zeta = \Gamma \left(\frac{A}{4E_0 kT}\right)^{\frac{1}{2}}$$

$$\chi = 2\left(\frac{E - E_0}{\Gamma}\right)$$

T : absolute temperature, k : boltsmann constant

(3)Resonance escape probability in infinite dilution approximation $p^\infty$

Assumptions

Infinite hydrogen medium

Ignoring absorption by hydrogen and scattering by absorbing nucleus

Absorbing nucleus are infinitely diluted

$$p^\infty = \exp\left[-\frac{\pi N_A \sigma_0 \Gamma_\gamma}{2 N_H \sigma_s^H E_0}\right] \quad \cdots (4)$$

where,

$N_A$ : number density of absorbing nucleus

$N_H$ : number density of hydrogen

$\sigma_s^H$ : microscopic scattering cross section of hydrogen (assumed as constant)

Feature of $p^{\infty}$

No temperature dependence

Increases with increasing moderator（hydrogen） density

Ex：Moderator void in light water reactor

→low moderator density

→low p

→negative reactivity

Decreases in low $E_0$

→lower energy resonances give a lot of effect on reactivity.

Ex：6.67eV resonance in $^{238}U$