

Nuclear Reactor Physics Lecture Note (1)  
-Perturbation Theory-

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## 1. Perturbation theory

Perturbation theory : Effective method to calculate the change of multiplication factor by a small change in the core geometry or composition, etc.

### 1.1 Criticality eigenvalue problem and adjoint equation

Criticality eigenvalue problem

$$-\nabla D \nabla \phi + \Sigma_a \phi(\mathbf{r}) = \frac{1}{k} v \Sigma_f \phi(\mathbf{r}) \quad \dots (1)$$

The equation can be expressed by operator notation.

$$M\phi = \frac{1}{k} F\phi \quad \dots (2)$$

where,  $M \equiv -\nabla D(\mathbf{r}) \nabla + \Sigma_a(\mathbf{r}) \equiv$  Destruction operator (leakage plus absorption)

$F \equiv v \Sigma_f(\mathbf{r}) \equiv$  Production operator

Boundary conditions at the core surface

$$\phi(\mathbf{r}_s) = 0 \quad \dots (3)$$

We define the inner product  $(f, g)$  between any two functions  $f(\mathbf{r})$  and  $g(\mathbf{r})$  as

$$(f, g) \equiv \int_V d^3r f^*(\mathbf{r}) g(\mathbf{r}) \quad \dots (4)$$

where  $f^*(\mathbf{r})$  denotes the complex conjugate of  $f(\mathbf{r})$ , and  $V$  is the core volume.

Definition of the adjoint operator  $M^\dagger$  :

$$(M^\dagger f, g) = (f, Mg) \quad \dots (5)$$

for every  $f(\mathbf{r})$  and  $g(\mathbf{r})$  satisfying the boundary conditions  $f(\mathbf{r}_s) = 0 = g(\mathbf{r}_s)$ .

We define the adjoint flux  $\phi^\dagger$  as the solution of adjoint equation for Eq.(2), i.e.

$$M^\dagger \phi^\dagger = \frac{1}{k} F^\dagger \phi^\dagger \quad \dots (6)$$

## 1.2 First order perturbation theory

We suppose perturbation of the macroscopic absorption cross section, for example, by adding a localized absorber, to a new value

$$\Sigma_a'(\mathbf{r}) = \Sigma_a(\mathbf{r}) + \delta\Sigma_a(\mathbf{r}) \quad \dots (7)$$

We assume that the perturbation in the core absorption is small.

The corresponding change in  $k$  is governed by the perturbed criticality problem

$$M'\phi' = \frac{1}{k'} F\phi' \quad \dots (8)$$

The perturbation in the core absorption appears as a perturbation  $\delta M$  in the destruction operator

$$M' = M + \delta M, \quad \delta M \equiv \delta\Sigma_a(\mathbf{r}) \quad \dots (9)$$

The scalar product of Eq.(8) with adjoint flux  $\phi^\dagger$  characterizing the unperturbed core

$$(\phi^\dagger, M'\phi') = \frac{1}{k'} (\phi^\dagger, F\phi') \quad \dots (10)$$

From the definition of adjoint operator (Eq.(5))

$$(\phi^\dagger, M\phi') = (M^\dagger\phi^\dagger, \phi') = \left(\frac{1}{k} F^\dagger\phi^\dagger, \phi'\right) = \frac{1}{k} (\phi^\dagger, F\phi') \quad \dots (11)$$

From Eq.(9), Eq.(10), Eq.(11), we find

$$\left(\frac{1}{k'} - \frac{1}{k}\right) = \frac{(\phi^\dagger, \delta M\phi')}{(\phi^\dagger, F\phi')} \quad \dots (12)$$

we can calculate  $\delta k = k' - k$  from Eq.(12).

Definition of core reactivity  $\rho$

$$\rho \equiv \frac{k - 1}{k} \quad \dots (13)$$

The perturbation in reactivity

$$\Delta\rho = \rho' - \rho = \frac{k' - 1}{k'} - \frac{k - 1}{k} = \frac{1}{k} - \frac{1}{k'} \quad \dots (14)$$

From Eq.(9), Eq.(13), Eq.(14), we find

$$\Delta\rho = -\frac{(\phi^\dagger, \delta\Sigma_a\phi')}{(\phi^\dagger, F\phi')} \quad \dots (15)$$

If the perturbation  $\delta\Sigma_a$  is small, the corresponding perturbation in the flux  $\delta\phi \equiv \phi' - \phi$  is small, so

$$\Delta\rho = -\left\{ \frac{(\phi^\dagger, \delta\Sigma_a\phi)}{(\phi^\dagger, F\phi)} + \frac{(\phi^\dagger, \delta\Sigma_a\delta\phi)}{(\phi^\dagger, F\phi)} - \frac{(\phi^\dagger, \delta\Sigma_a\phi)(\phi^\dagger, F\delta\phi)}{(\phi^\dagger, F\phi)^2} + \dots \right\} \quad \dots (16)$$

Neglecting second and higher order quantities (first order perturbation theory)

$$\Delta\rho \cong -\frac{(\phi^\dagger, \delta\Sigma_a\phi)}{(\phi^\dagger, F\phi)} \quad \dots (17)$$

### 1.3 Perturbation theory in one-speed diffusion model

In one-speed diffusion model  $\phi^\dagger = \phi$ , so

$$\Delta\rho \cong \frac{\int_V d^3r \phi(\mathbf{r}) \delta\Sigma_a(\mathbf{r}) \phi(\mathbf{r})}{\int_V d^3r \phi(\mathbf{r}) v \Sigma_f(\mathbf{r}) \phi(\mathbf{r})} \quad \dots (18)$$

### 1.4 Neutron importance

We imagine an absorber inserted into the reactor core at a point  $\mathbf{r}_0$  such that

$$\delta\Sigma_a(\mathbf{r}) = \alpha \delta(\mathbf{r} - \mathbf{r}_0) \quad \dots (19)$$

where  $\alpha$  : effective strength of the absorber,  $\delta(\mathbf{r} - \mathbf{r}_0)$  :  $\delta$ -function

then

$$\begin{aligned} \Delta\rho &= -\frac{\int_V d^3r \phi^\dagger(\mathbf{r}) \delta\Sigma_a(\mathbf{r}) \phi(\mathbf{r})}{\int_V d^3r \phi^\dagger(\mathbf{r}) v \Sigma_f(\mathbf{r}) \phi(\mathbf{r})} \\ &= -\frac{\alpha \phi^\dagger(\mathbf{r}_0) \phi(\mathbf{r}_0)}{C} \quad C : \text{constant} \end{aligned}$$

then

$$\phi^\dagger(\mathbf{r}_0) \propto \frac{\Delta\rho}{\alpha \phi(\mathbf{r}_0)}$$

$\phi^\dagger(\mathbf{r}_0)$  is proportional to the change in reactivity per neutron absorbed at  $\mathbf{r}_0$  per unit time, so

$\phi^\dagger(\mathbf{r}_0)$  is referred to as the neutron importance or importance function.