Neutron Transport Theory Lecture Note (1)
-Nuclear Reactions and Nuclear Cross Section

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1. Nuclear Reactions and Nuclear Cross Sections

1.1 Reactions between Neutrons and nuclei

- Nuclear fission (n, fission):

$$^{1}_{0}\mathrm{n} + {}^{A_1}_{Z_1}\mathrm{X} \to {}^{A_2}_{Z_2}\mathrm{X} + {}^{A_3}_{Z_3}\mathrm{X} + \text{neutrons} + 200\mathrm{MeV}$$

- Radioactive capture (n, γ): excited state

$$^{1}_{0}\mathrm{n} + {}^{A}_{Z}\mathrm{X} \to \left({}^{A+1}_{Z}\mathrm{X}\right)^{*} \to {}^{A+1}_{Z}\mathrm{X} + \gamma$$

- Scattering (n,n) or (n,n'):

$$^{1}_{0}\mathrm{n} + {}^{A}_{Z}\mathrm{X} \to {}^{1}_{0}\mathrm{n} + {}^{A}_{Z}\mathrm{X} \qquad \text{elastic scattering (n, n)}$$

$$^{1}_{0}\mathrm{n} + {}^{A}_{Z}\mathrm{X} \to {}^{1}_{0}\mathrm{n} + \left({}^{A}_{Z}\mathrm{X}\right)^{*}$$

$$\to {}^{1}_{0}\mathrm{n} + {}^{A}_{Z}\mathrm{X} + \gamma \qquad \text{inelastic scattering (n, n')}$$

1.2 Microscopic Cross Sections

The probability that a neutron-nuclear reaction will occur is characterized by a quantity called a nuclear cross section.

(1) Definition

Consider a beam of neutrons travelling with the same speed and direction and a sufficiently thin target (one atomic layer thick).

The rate R at which reactions occur per unit area on the target:

$$\mathrm{R} = \sigma \cdot \mathrm{I} \cdot \mathrm{N_A} \quad \left[\mathrm{cm^{-2}} \cdot \mathrm{S^{-1}}\right]$$

The microscopic cross section σ is defined as

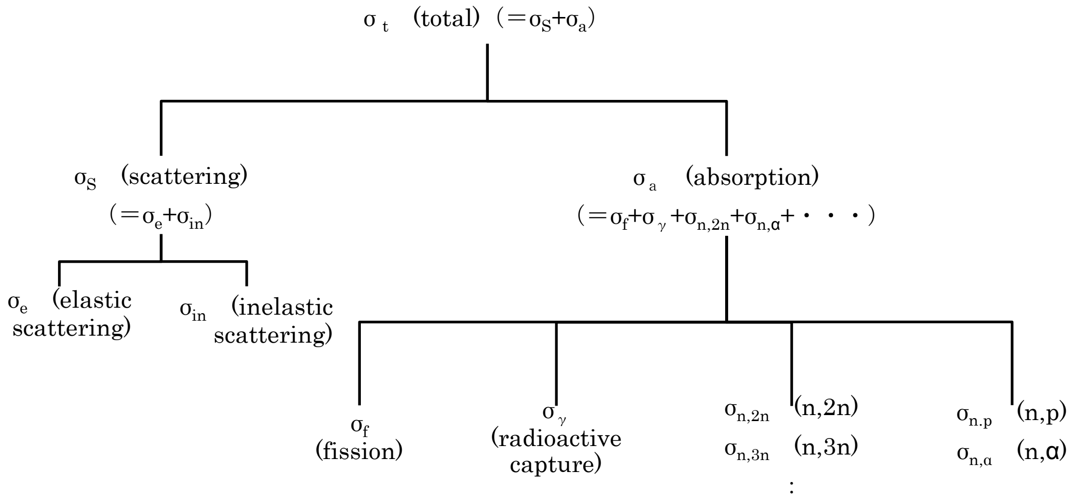
$$\sigma = \frac{\left(\mathrm{R}/\mathrm{N_A}\right)}{\mathrm{I}} \left[\mathrm{cm^2}\right]$$

$$= \frac{(\text{Number of reactions/nucleus/sec})}{(\text{Number of incident neutrons/cm}^2\text{/sec})}$$

The unit : b (barn)

$1\mathrm{b} = 10^{-24}\mathrm{cm}^2$

(2) Type of neutron cross sections



Microscopic cross sections depend on the nucleus, reactions and incident neutron speed (energy)

1.3 Macroscopic Cross Sections

(1) Definition

Consider a monoenergetic neutron beam incident on the surface of a target of arbitrary thickness.

The total reaction rate per unit area in dx:

$$dR = \sigma_t I N dx$$

The dR can be equal to the decrease in beam intensity between x and x+dx (with the prescription that any type of interactions will remove an incident neutron)

$$-dI(x) = \sigma_t I N dx$$

$$\therefore \frac{dI}{dx} = -N\sigma_t I(x)$$

$$\therefore I(x) = I_0 \exp(-\sigma_t N x)$$

The definition of total macroscopic cross section

$$\Sigma_t \equiv \sigma_t N[\mathrm{cm}^{-1}]$$

(2) Neutron mean free path and collision frequency

(Probability that a neutron moves a distance x without any interaction)

$$= \frac{I_0 \exp(-\Sigma_t x)}{I_0} = \exp(-\Sigma_t x)$$

So

(probability that a neutron has its first interaction in dx)

$$= \exp(-\Sigma_t x) \cdot \Sigma_t dx \equiv p(x)dx$$

We can calculate the average distance a neutron travels before interacting with a nucleus in the sample.

$$\bar{x} \equiv \int_0^\infty x p(x) dx = \Sigma_t \int_0^\infty x \exp(-\Sigma_t x) dx$$

$$= \frac{1}{\Sigma_t} \qquad \text{(neutron mean free path)}$$

Σ_t is the probability per unit path length that a neutron will undergo a reaction, so

$$(\text{The collision frequency}) = v\Sigma_t[s^{-1}]$$

where v is the neutron speed.

(3) Macroscopic cross sections for specific reaction

ex. macroscopic fission cross section

$$\Sigma_f = \sigma_f N$$

macroscopic absorption cross section

$$\Sigma_a = \sigma_a N$$

(4) Macroscopic cross sections for mixture

ex. The total cross section of homogenized mixture of three different species of nuclide X, Y, and Z:

$$\Sigma_t = N_X \sigma_t^X + N_Y \sigma_t^Y + N_Z \sigma_t^Z$$