Advanced Noncooperative Game Theory Problem Set 4 (due July 20)

1. Consider the following game with two players: player 1 and player 2. Player 1 moves first and has three choices: a, b, c. If player 1 chooses a, the game ends, and player 1 receives a payoff of 2, while player 2 receives a payoff of 2. If player 1 chooses b or c, it is then player 2's turn to move. Player 2 knows whether player 1 has chosen b or c and two choices b' and c'.

   - If player 1 chooses b and player 2 chooses b', then player 1 and player 2 each receive 3 and 3 respectively.
   - If player 1 chooses b and player 2 chooses c', then player 1 and player 2 each receive 0 and 1 respectively.
   - If player 1 chooses c and player 2 chooses b', then player 1 and player 2 each receive 1 and 4 respectively.
   - If player 1 chooses c and player 2 chooses c', then player 1 and player 2 each receive 1 and 0 respectively.

   (a) Draw the game tree associated with this game.

   (b) Define the strategic form game associated with this game and find all Nash equilibria.

   (c) Find all subgame-perfect equilibria of this game.

2. Consider the strategic form game below and consider only pure strategies:

$1 \setminus 2$	X	Y
X	$4, 2$	$0, 0$
Y	$0, 0$	$2, 4$

   (a) Find all Nash equilibria of the above game.

   Before the game above is played, let player 1 have the option of whether to "burn" (B) or to "not burn" (NB). By choosing B, player 1's payoff is reduced by 1, while by choosing NB player 1's payoff is unchanged. Suppose that player 2 can observe whether player 1 has chosen B or NB.

   (b) Draw the game tree associated with this modified game.

   (c) Find all subgame-perfect equilibria.

3. Consider the infinitely repeated version of the prisoner's dilemma, whose component game is given by the following matrix.

$1 \setminus 2$	C	D
C	$6, 6$	$0, 8$
D	$8, 0$	$2, 2$

Suppose for simplicity that $\delta_1 = \delta_2 = \delta$. Consider the following **modified trigger** strategy of player i:

- Choose C in the first repetition.
- Choose D in the t-th repetition with $t \geq t^* + 1$, where t^* is the first time that player $j \neq i$ has chosen D. Otherwise, choose C.

The modified trigger strategy is the same as the trigger strategy, except player i chooses D in the modified trigger *only when* player $j \neq i$ has chosen D.[1]

(a) Give a formal description of the modified trigger strategy.

(b) Find a $\bar{\delta}$ with $0 < \bar{\delta} < 1$ such that for all $\delta \geq \bar{\delta}$, both players choosing the modified trigger is a Nash equilibrium of the infinitely repeated game.

(c) Is there a δ with $0 < \delta < 1$ such that both players choosing the modified trigger is a subgame-perfect equilibrium of the infinitely repeated game? If so, find one and prove that it is a subgame-perfect equilibrium. If not, prove that there is no such δ.

[1] Some texts call this strategy the "(grim) trigger" strategy.