Advanced Noncooperative Game Theory Problem Set 3 (due July 13)

1. Let x and θ be taken from $[0, 10]$, and define the function $f : [0, 10]^2 \to \mathbb{R}$ by

$$f(x, \theta) = -x^2 + 2x\theta$$

   (a) Show that f satisfies increasing differences. Determine whether f satisfies strictly increasing differences or not.

   (b) Treating θ as a parameter, let $x^*(\theta)$ denote the solution to the maximization problem:

$$\max_{x\in[0,10]} f(x, \theta)$$

   Find an explicit form of $x^*(\theta)$ and show that it is nondecreasing in θ.

   Now, let $g : [0, 10]^2 \to \mathbb{R}$ be given by

$$g(x, \theta) = (x - 5 + \theta)^2$$

   (c) Show that g satisfies increasing differences.

   (d) Treating θ as a parameter, let $\bar{x}(\theta)$ denote the maximum solution to the maximization problem:

$$\max_{x\in[0,10]} g(x, \theta)$$

   Find an explicit form of $\bar{x}(\theta)$ and show that it is nondecreasing in θ.

   (e) Consider the minimum solution to the above maximization problem $\underline{x}(\theta)$. Find an explicit form and show that it is nondecreasing in θ.

2. Consider the following game where $N = \{1, 2\}$, $S_1 = S_2 = [0, a]$, and define the payoff function of each player by

$$u_1(s_1, s_2) = (a - (s_1 + s_2))s_1 - cs_1$$
$$u_2(s_1, s_2) = (a - (s_1 + s_2))s_2 - cs_2$$

   where $a > c > 0$. Consider only pure strategies.

   (a) For each player, find all strategies that are strictly dominated.

   (b) Consider an infinite version of the iterated elimination of strictly dominated strategies where all strategies that are strictly dominated in each state are eliminated. Which strategies of each player remain?

(c) Show that u_1 does not have increasing differences in (s_1, s_2). (Hence, it is not a supermodular game.)

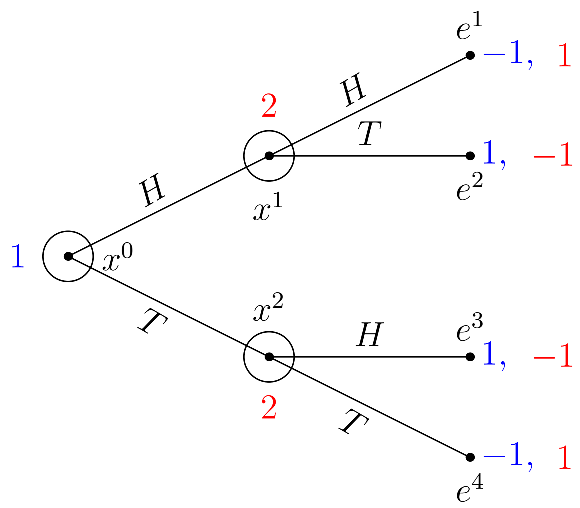
Now, by change of variables, introduce a new variable for player 2, t_2, such that $t_2 = -s_2$. Now, player 2's strategy set is given by $T_2 = [-a, 0]$.

(f) Write down the payoffs of each player in terms of s_1 and t_2.

(g) Show that u_1 has increasing differences in (s_1, t_2). (Hence with strategy set T_2, the game is now a supermodular game.)

3. Construct two games in game tree form which have the same strategic form representation.

4. Consider the following game in game-tree form below from page 1 of the lecture notes, also given below.



(a) Consider the behavioral stategies for player 1 given by $b_1(\{x_0\}) = (0.8, 0.2)$ where the first component (0.8) is the probability that H is chosen, and the second component (0.2) is the probability that T is chosen. For player 2, consider $b_2(\{x^1\}) = (0.6, 0.4)$ and $b_2(\{x^2\}) = (0.3, 0.7)$ where the first component is the probability that H is chosen, and the second component is the probability that T is chosen. For each terminal node, calculate the probability that the node is reached under these behavioral strategies.

(b) Consider the mixed strategies given by $\sigma_1(H) = 0.8$, $\sigma_1(T) = 0.2$, $\sigma_2(H-H) = 0.21$, $\sigma_2(H-T) = 0.39$, $\sigma_2(T-H) = 0.09$, $\sigma_2(T-T) = 0.31$. For each terminal node, calculate the probability that the node is reached under these mixed strategies and show that the answer is the same as part (a).

(c) Find a mixed strategy for player 2 other than the one given in (b) that also leads to the same probabilities that the terminal nodes are reached as in the behavioral strategies given in (a).

(d) Consider the mixed strategies given by $\sigma_1(H) = 0.6$, $\sigma_1(T) = 0.4$, $\sigma_2(H-H) = 0.4$, $\sigma_2(H-T) = 0.3$, $\sigma_2(T-H) = 0.2$, $\sigma_2(T-T) = 0.1$. For each terminal node, calculate the probability that the node is reached under these mixed strategies.

(e) Given $b_1(\{x^0\}) = (0.6, 0.4)$, find a behavioral strategy for player 2 that yields the same probability that the terminal nodes are reached as in the mixed strategies in part (d), and show that there is only one such behavioral strategy.