

Practice Problems for the Section on Potential Games

1. Let $G = (N, (S_i, u_i)_{i \in N})$ be a potential game where P is a potential function. Consider the game $G' = (N, (S_i)_{i \in N}, P)$ where the payoff function for each player $i \in N$ in game G' is the function P , common to all players. Consider the following statement: "A strategy combination $s^* = (s_1^*, s_2^*, \dots, s_n^*)$ is a Nash equilibrium of G if and only if it is a Nash equilibrium of game G' ." Prove the statement if it is correct or provide a counterexample if it is not correct.
2. Show that the congestion game in Problem Set 2 (Question 3) is a potential game by explicitly finding a function P that satisfies the definition of a potential game.
3. Show that the chicken game in Lecture 1 is a potential game.
4. Show that the matching coins game below is not a potential game.

$A \setminus B$	C	D
C	$-1, 1$	$1, -1$
D	$1, -1$	$-1, 1$

Also, show by using the same logic that the rock-paper-scissors game in Lecture 1 is not a potential game.

5. Finish the proof of Theorem 2 by showing that for a given x^0 , for any strategy profile x , the function P defined below is a potential function for the game G ,

$$P(x) = I(\gamma),$$

where γ is a simple path connecting x^0 and x .