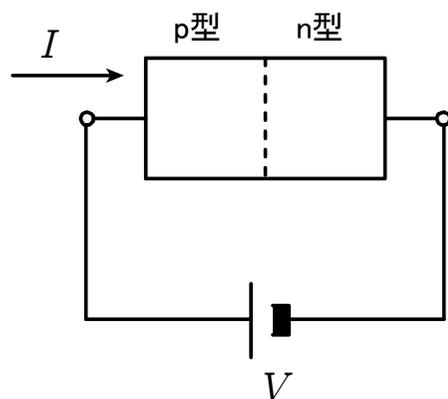
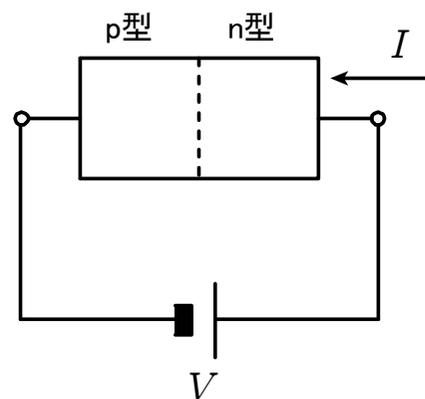


# バイポーラトランジスタの モデリング

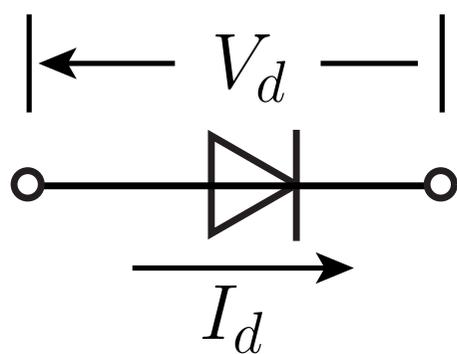
# バイポーラトランジスタのモデリング



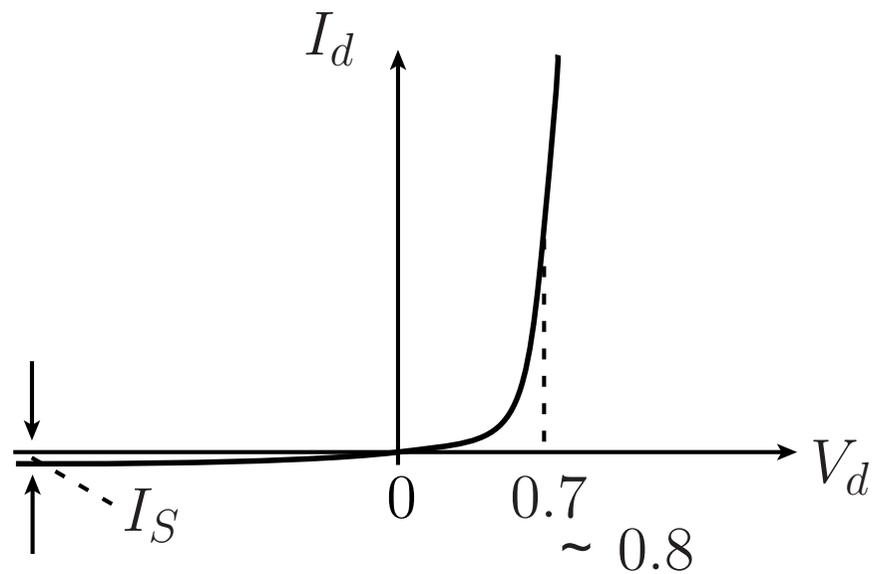
(a) 順方向バイアス

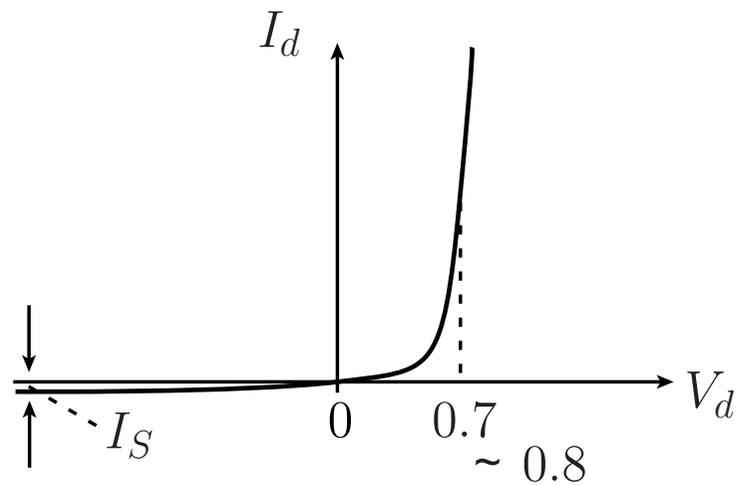


(b) 逆方向バイアス

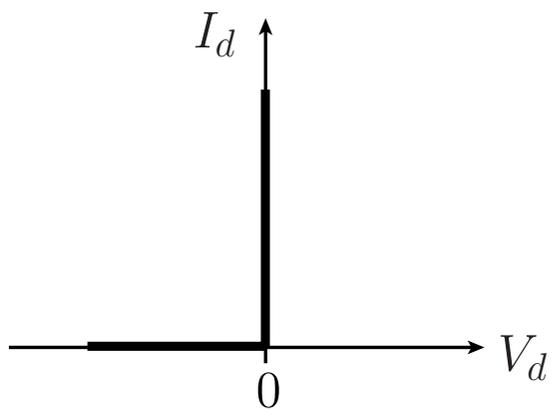


$$I_d = I_S \left\{ \exp\left(\frac{qV_d}{kT}\right) - 1 \right\}$$

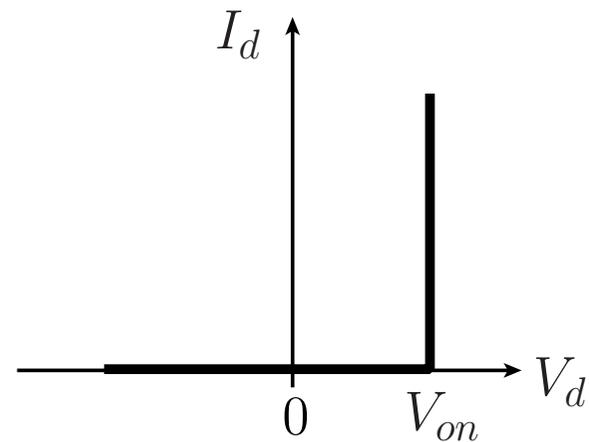




実際のpn接合ダイオードの特性

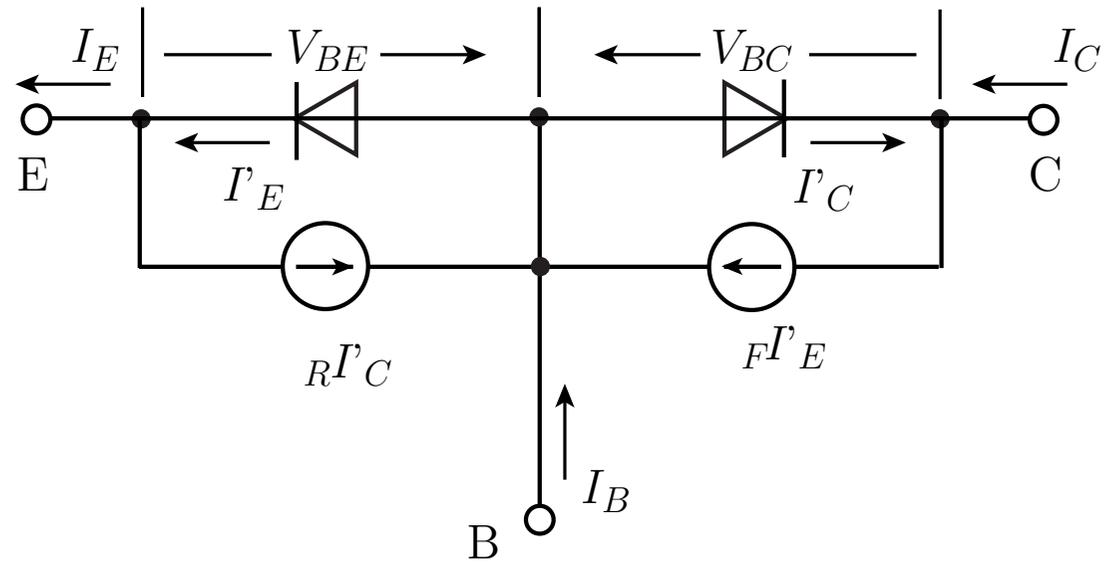


理想ダイオード特性



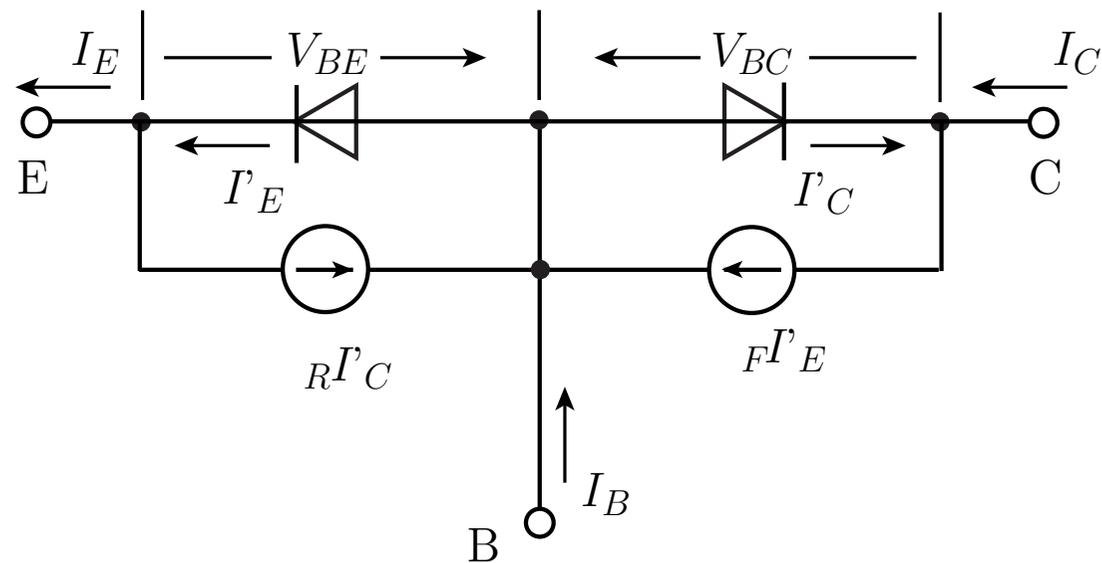
近似ダイオード特性

## Ebers-Mollモデル

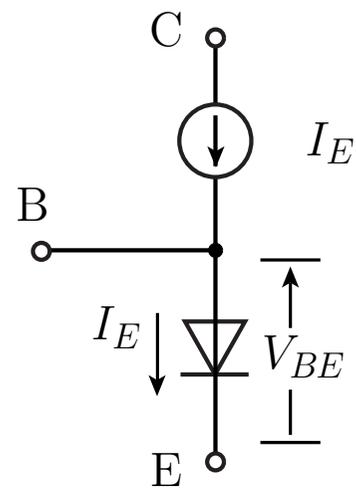


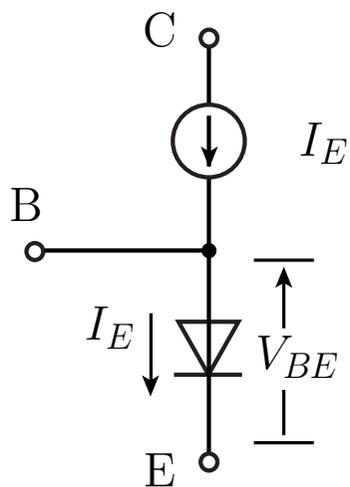
遮断領域 :  $V_{BE} < 0, V_{BC} < 0$   
能動活性領域 :  $V_{BE} > 0, V_{BC} < 0$   
飽和領域 :  $V_{BE} > 0, V_{BC} > 0$   
逆方向能動活性領域 :  $V_{BE} < 0, V_{BC} > 0$

能動活性領域： $V_{BE} > 0, V_{BC} < 0$

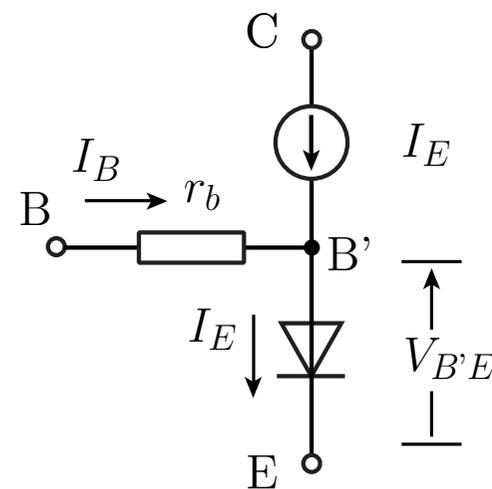


$V_{BC} < 0$  より  $I'_C \approx 0$





ベース広がり抵抗



精度が劣る

精度の向上

簡単

複雑



見通しの良さ

# モデルの簡単化＝見通しの良い解析



$$V_{BE} = 0.70V$$

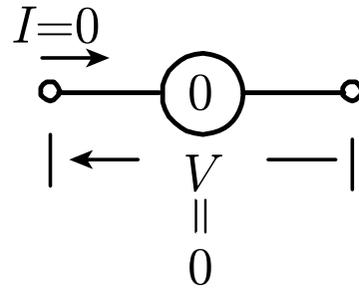
さらなる簡単化

$$\alpha \rightarrow 1$$

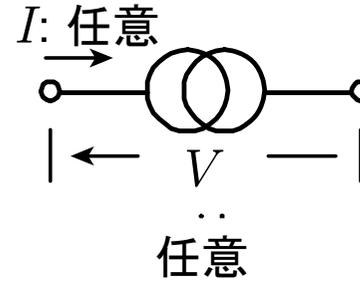
$$I_B = I_E - I_C = (1 - \alpha)I_E$$

$$I_B = 0 \text{ かつ } V_{BB'} = r_b I_B = 0$$

$$I_B=0 \text{ のとき } V_{BB'} = r_b I_B = 0 \quad (\alpha=1)$$



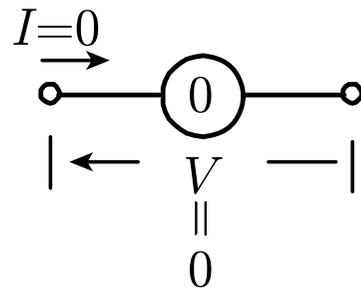
(a) ナレータ



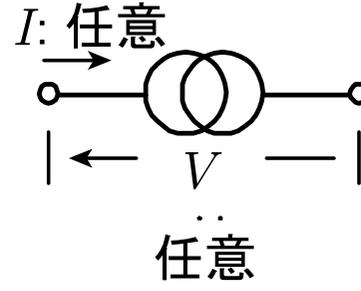
(b) ノレータ



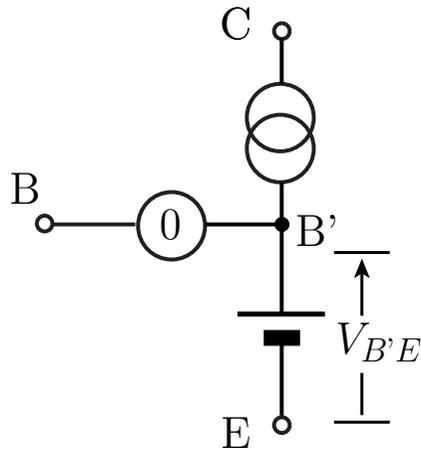
# ナレータノ・レータによるトランジスタのモデリング



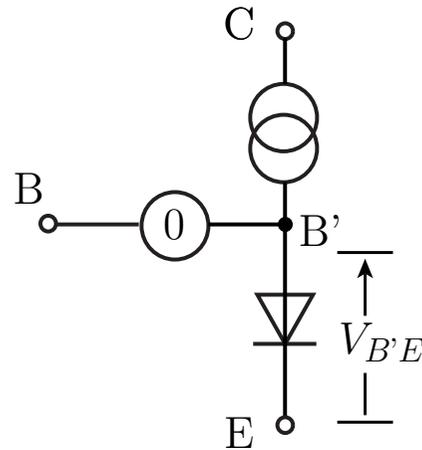
(a) ナレータ



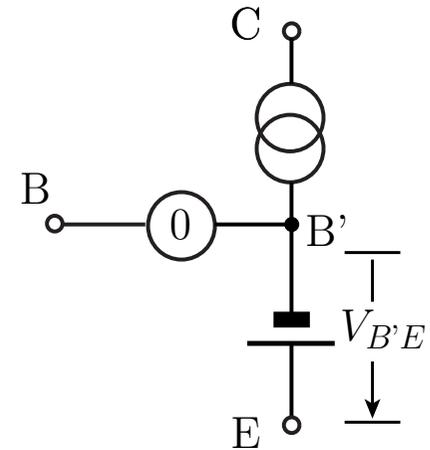
(b) ノレータ



npnトランジスタ  
モデル(1)

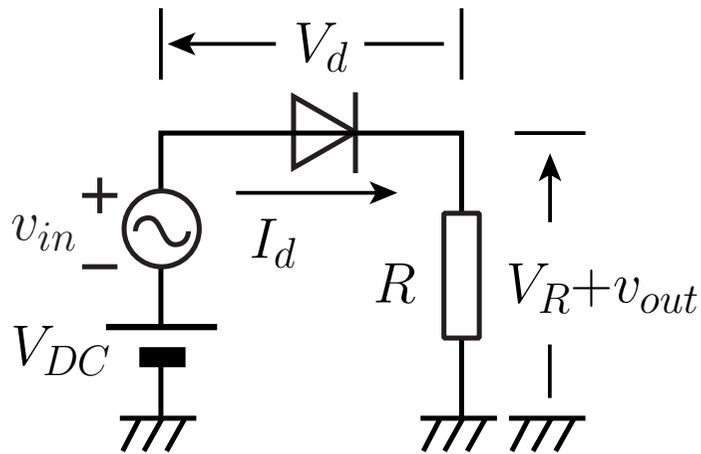


npnトランジスタ  
モデル(2)

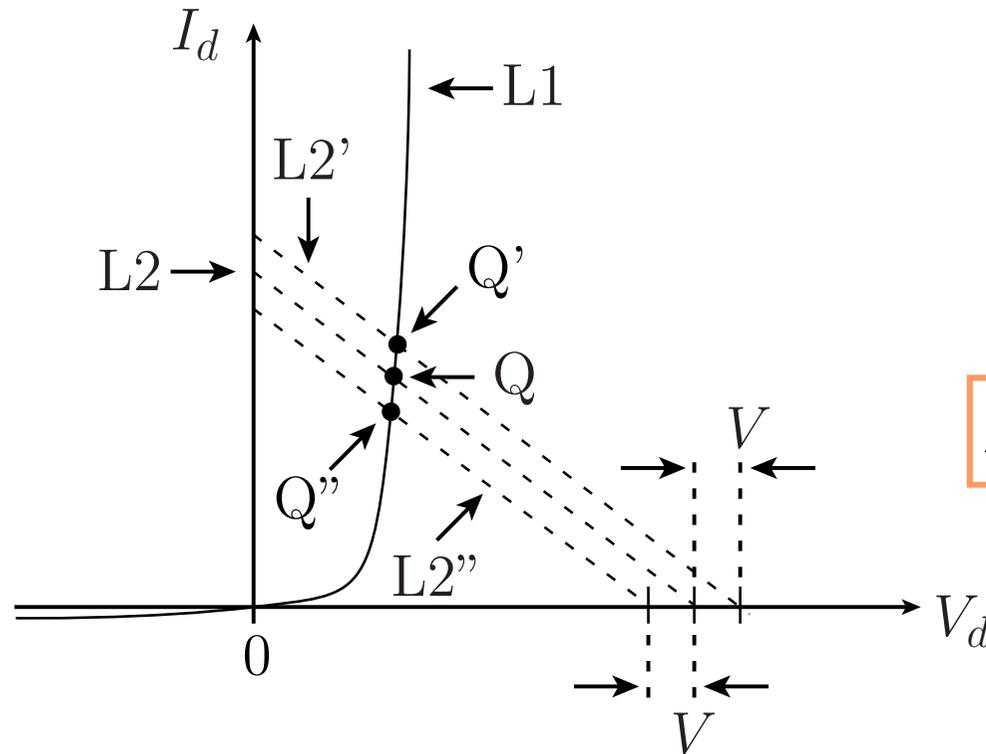


pnpトランジスタ  
モデル

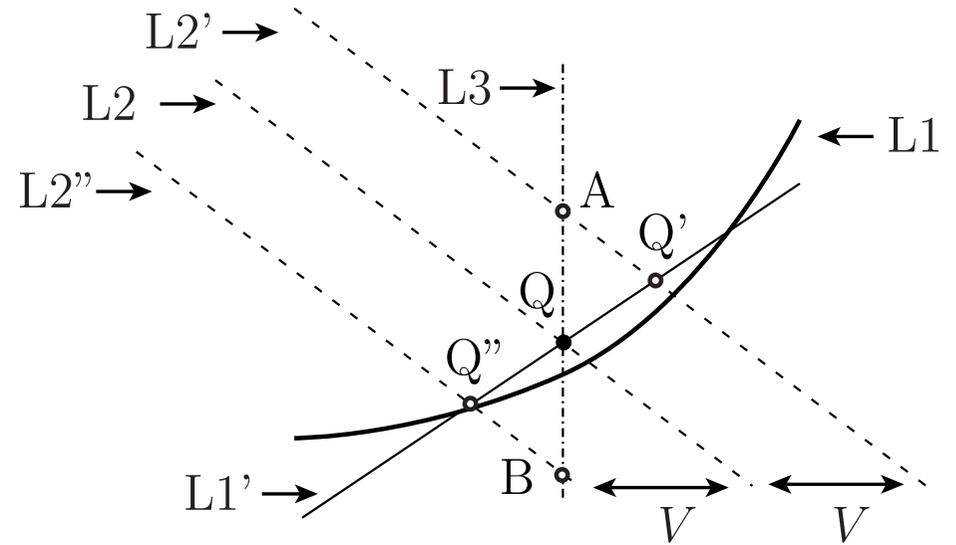
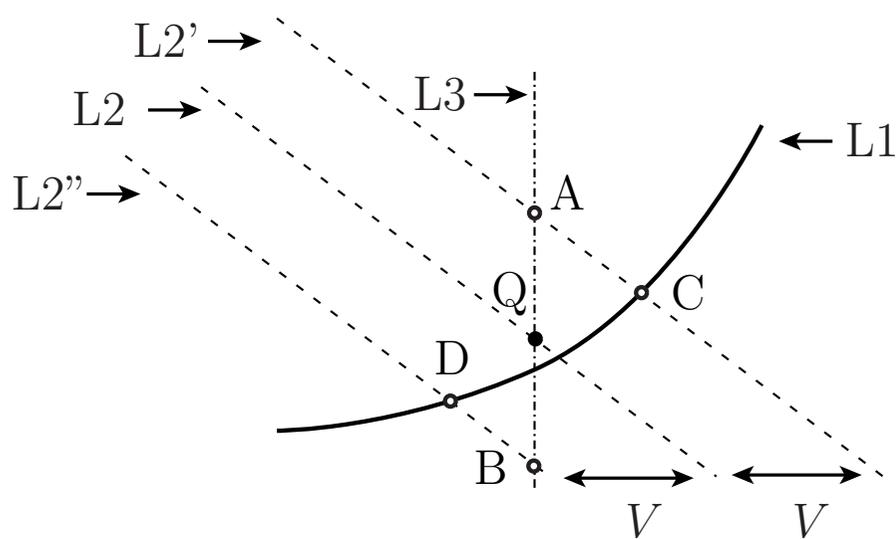
# 信号を加えた場合の解析



$$I_d = \frac{V_{DC} + v_{in} - V_d}{R} = -\frac{1}{R}V_d + \frac{V_{DC} + v_{in}}{R}$$



$\Delta V : v_{in}$  の振幅

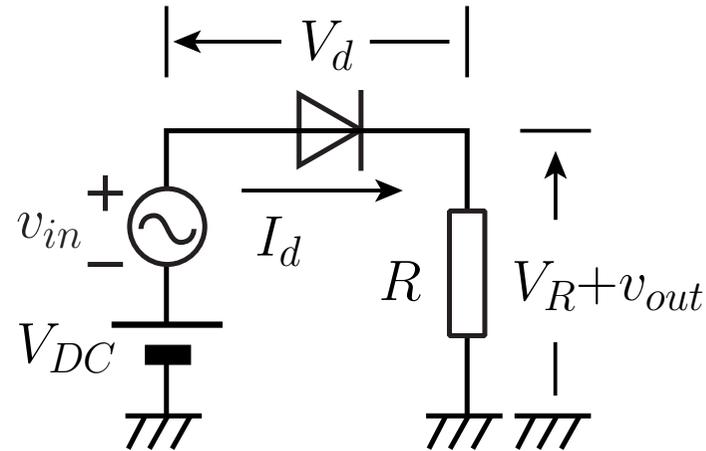


$$I_d = I_S \left\{ \exp\left(\frac{qV_d}{kT}\right) - 1 \right\}$$

$$G = \left. \frac{\partial I_d}{\partial V_d} \right|_{V_d = V_Q} = \frac{qI_S}{kT} \exp\left(\frac{qV_d}{kT}\right) \doteq \frac{qI_Q}{kT}$$

$$I_Q \doteq I_S \exp\left(\frac{qV_Q}{kT}\right)$$

## 直流分と信号分を分けた解析



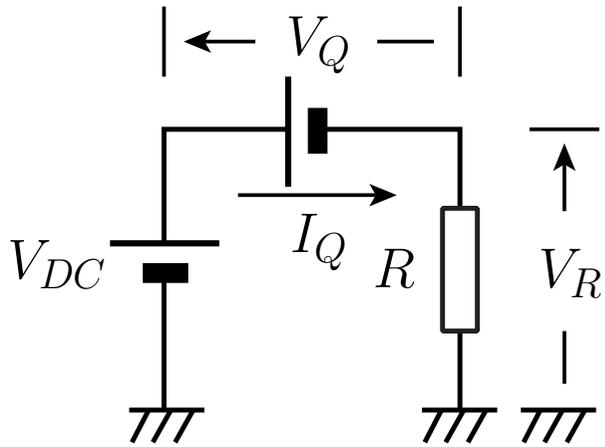
$$V_{DC} + v_{in} = V_Q + v_d + V_R + v_{out} \quad (\text{全体}) \quad V_d = V_Q + v_d$$

$$\text{一) } V_{DC} = V_Q + V_R \quad (\text{直流分})$$

---

$$v_{in} = v_d + v_{out} \quad (\text{信号分})$$

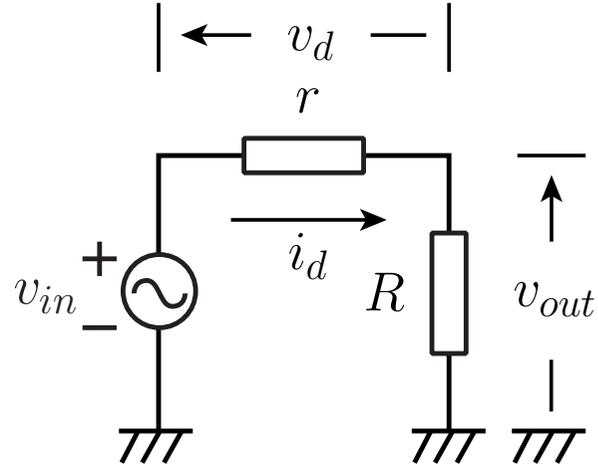
$$V_{DC} = V_Q + V_R$$



直流成分

$$V_{DC} = V_Q + RI_Q$$

$$v_{in} = v_d + v_{out}$$



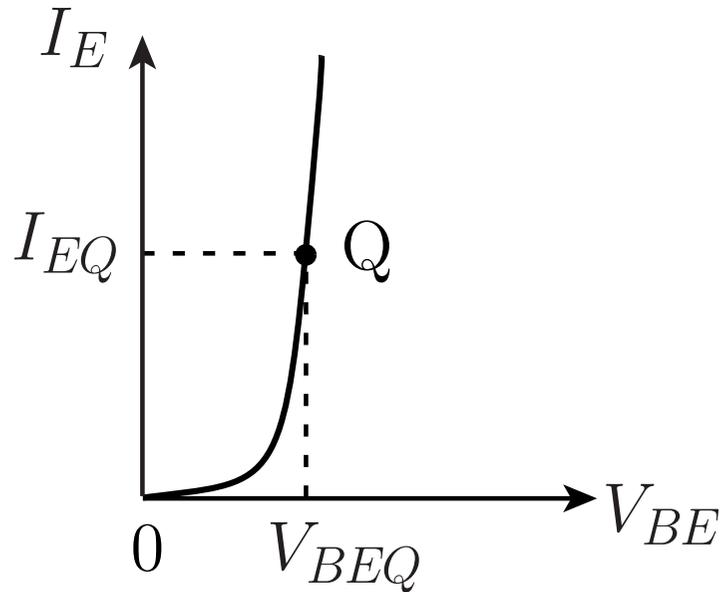
小信号成分

$$v_{in} = v_d + Ri_d$$

$$i_d = Gv_d$$

$$r = \frac{1}{G} = \frac{kT}{qI_Q}$$

# バイポーラトランジスタの小信号モデル



$$I_E = I_{ES} \left\{ \exp\left(\frac{qV_{BE}}{kT}\right) - 1 \right\}$$



$$I_E = \left. \frac{\partial I_E}{\partial V_{BE}} \right|_{V_{BE} = V_{BEQ}} \cdot \Delta V_{BE} + I_{EQ}$$

$$r_e = \left[ \left. \frac{\partial I_E}{\partial V_{BE}} \right|_{V_{BE} = V_{BEQ}} \right]^{-1} \approx \frac{kT}{qI_{EQ}} = \frac{0.026[\text{V}]}{I_{EQ}[\text{A}]}$$



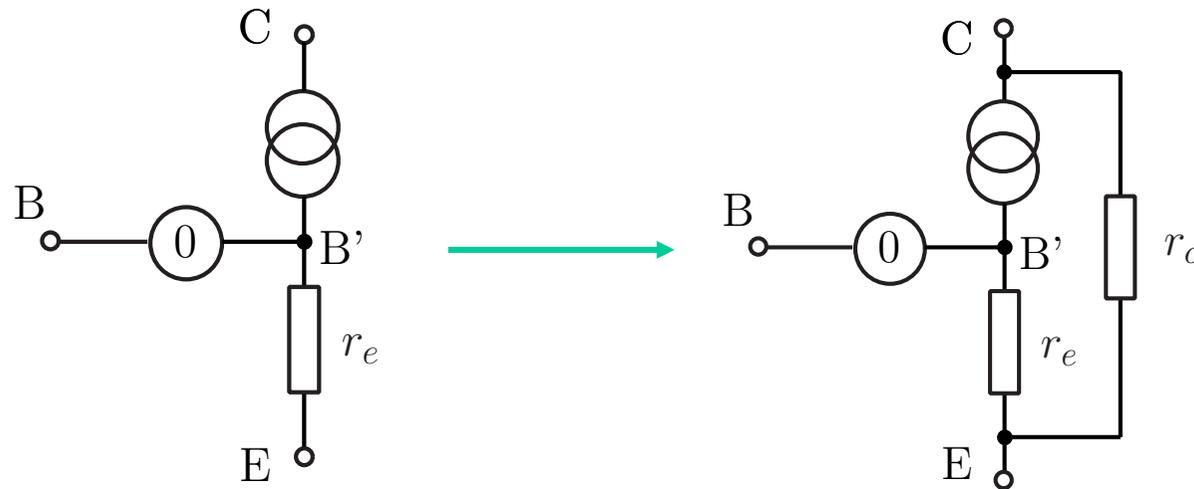
$$r_e = \left[ \frac{\partial I_E}{\partial V_{BE}} \bigg|_{V_{BE} = V_{BEQ}} \right]^{-1} \approx \frac{kT}{qI_{EQ}} = \frac{0.026[\text{V}]}{I_{EQ}[\text{A}]}$$

## アーリー効果

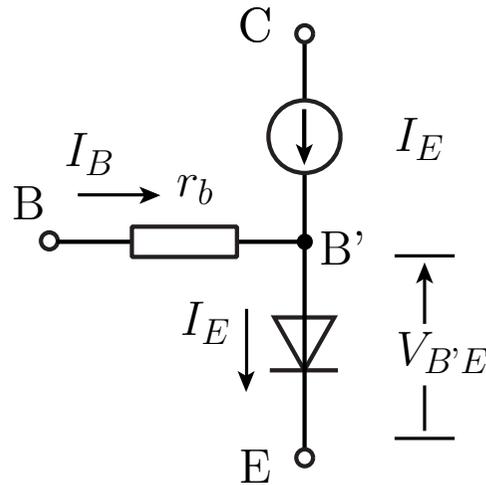
コレクタ電流がコレクタ・エミッタ間電圧によって変化する現象

$$I_C = \alpha_F I_E \left( 1 + \frac{V_{CE}}{V_A} \right)$$

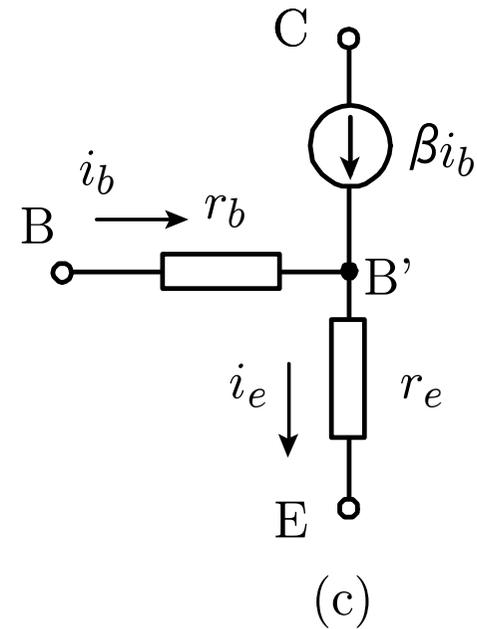
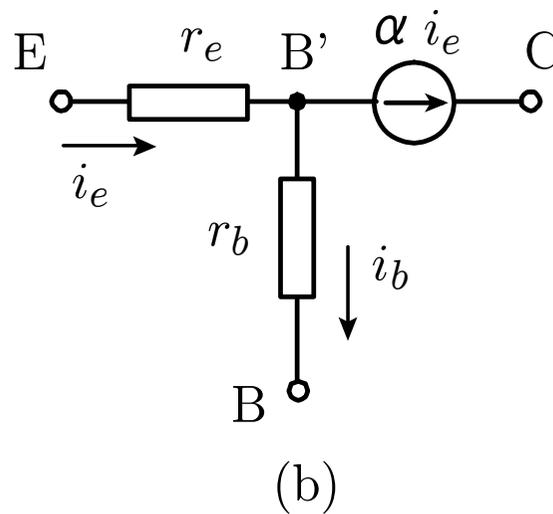
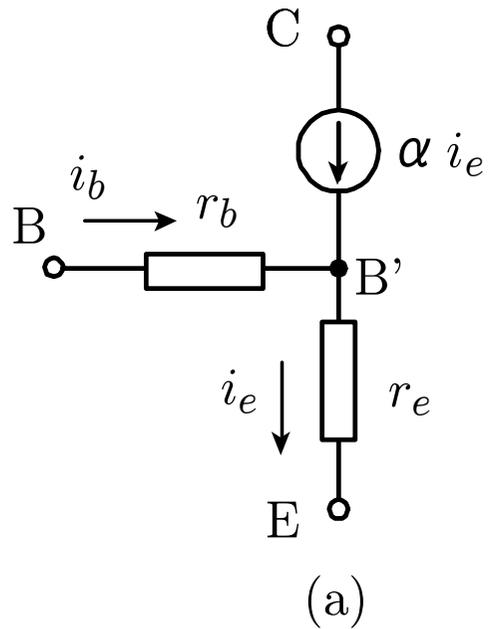
$$r_o = \frac{1}{\left. \frac{\partial I_C}{\partial V_{CE}} \right|_{V_{CE} = V_{CEQ}}} = \frac{V_A}{\alpha_F I_{EQ}}$$



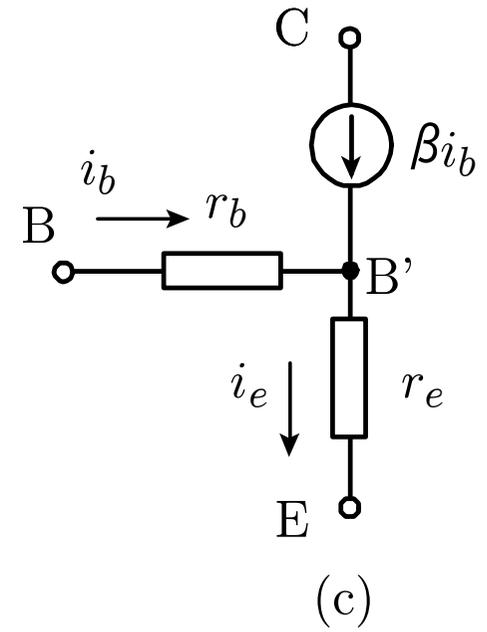
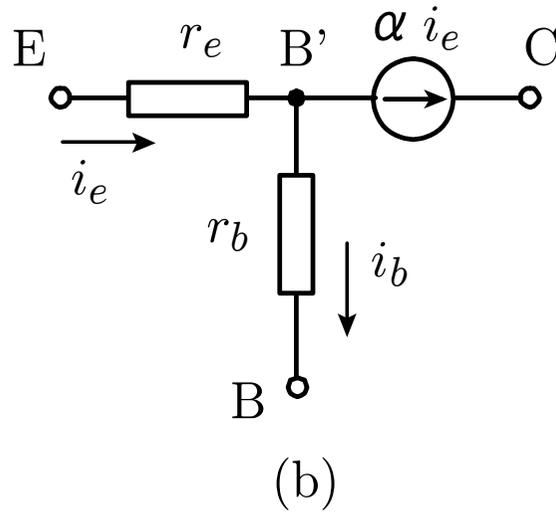
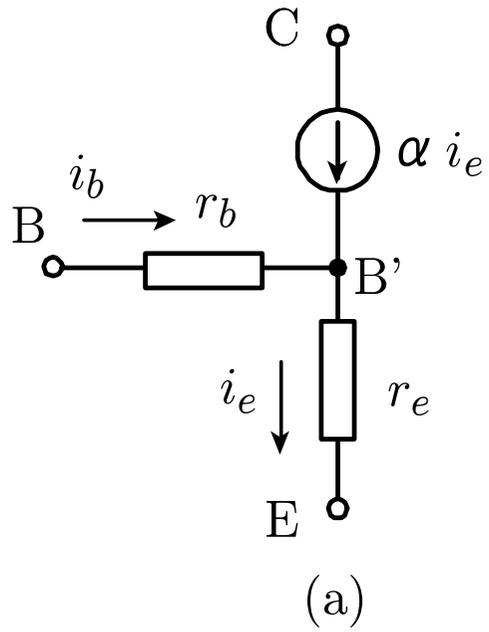
# その他のバイポーラトランジスタの小信号モデル



問： $\alpha$ と $\beta$ との  
関係を求めよ。

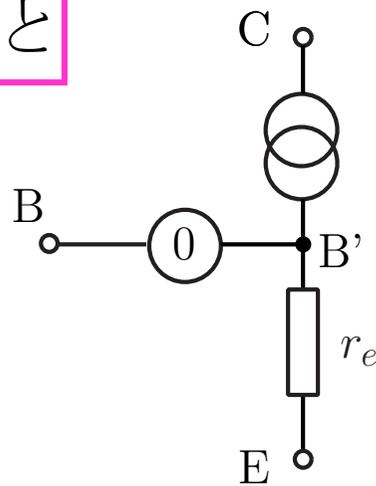


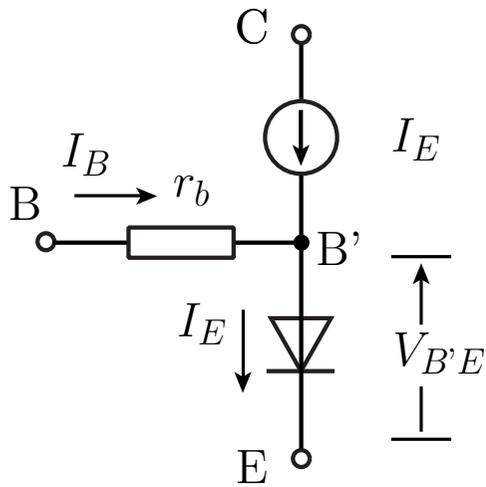
$$r_e = \frac{0.026V}{I_{EQ}[A]}$$



$$r_e = \frac{0.026\text{V}}{I_{EQ}[\text{A}]}$$

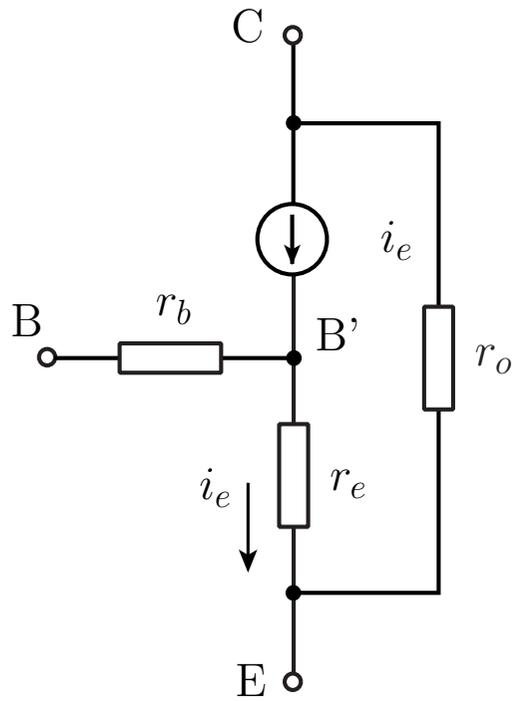
$\alpha \rightarrow 1 (\beta \rightarrow \infty)$  とすると



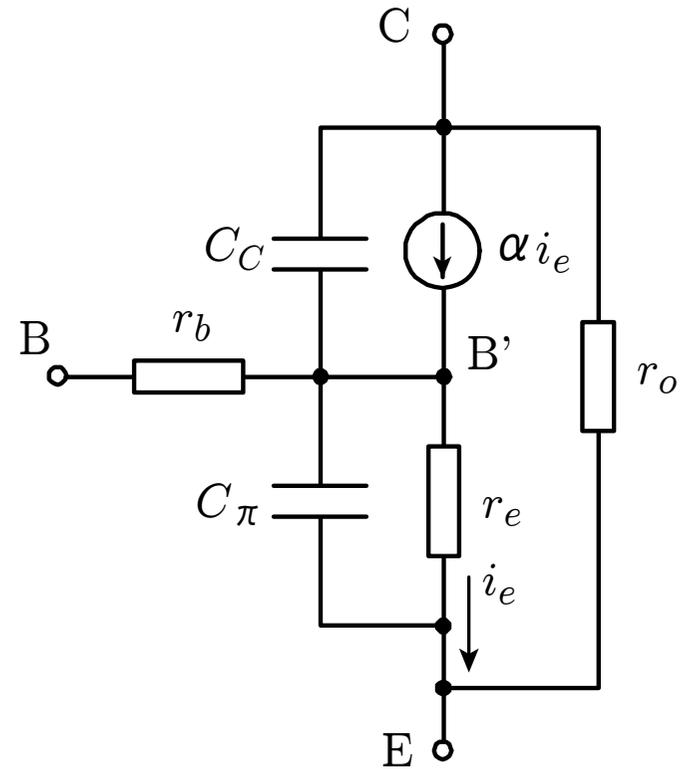


問：左図に相当するpnpトランジスタの大信号モデルを示せ．また，求めた大信号モデルから小信号モデルを求めよ．

# バイポーラトランジスタの高周波小信号モデル



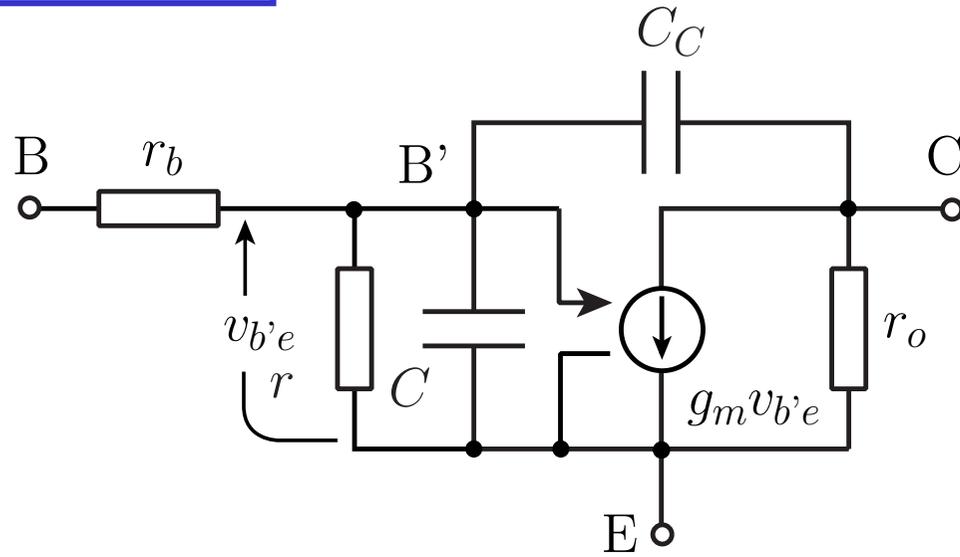
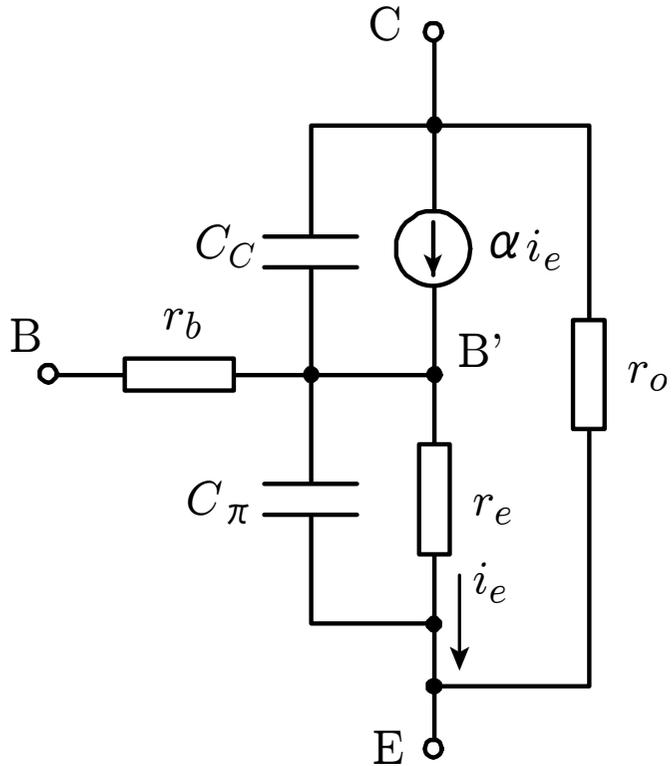
寄生容量を考慮



$$\alpha = \frac{\alpha_0}{1 + j \frac{\omega}{\omega_\alpha}}$$

$$\omega_\alpha = \frac{1}{C_\pi r_e} : \text{電流増幅率遮断角周波数}$$

# 等価変換

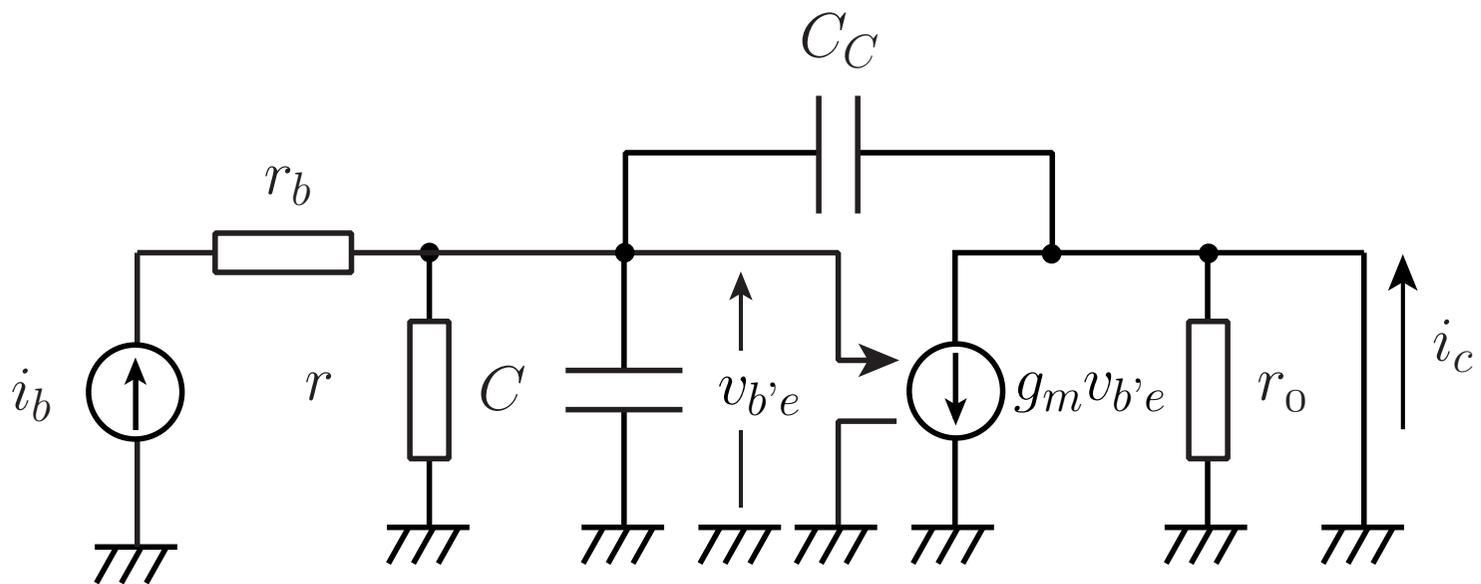


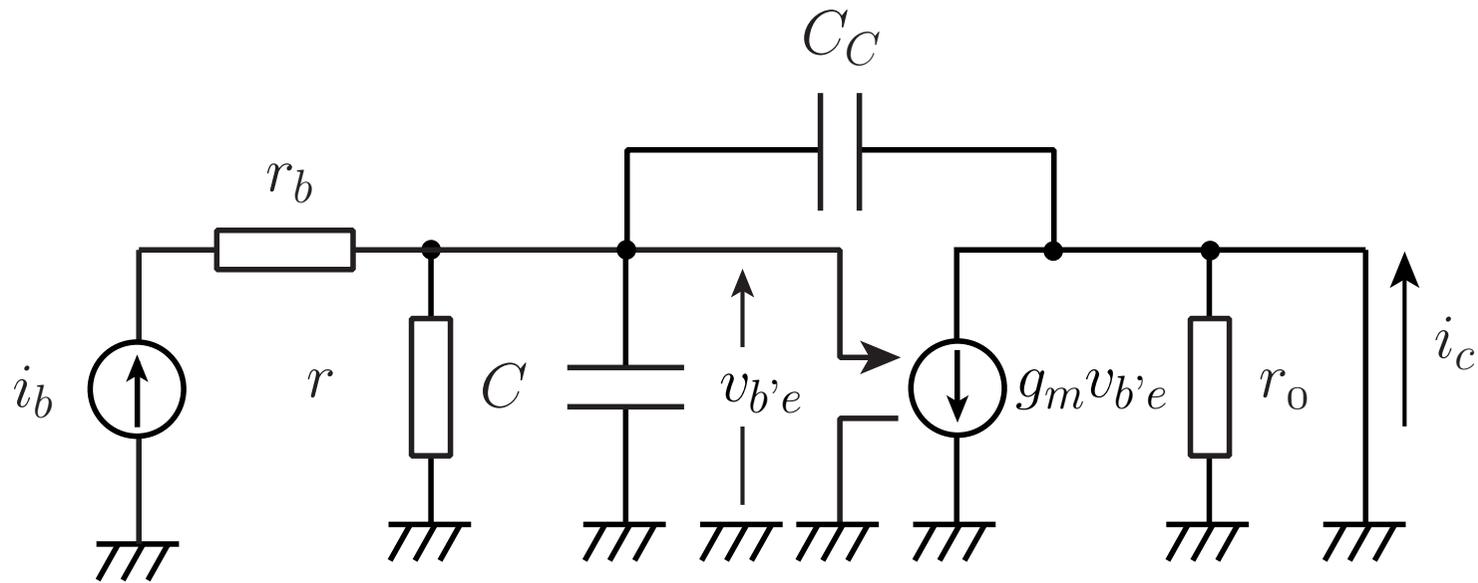
$$r_{\pi} = \frac{r_e}{1 - \alpha_0}$$

$$g_m = \frac{\alpha_0}{r_e}$$

バイポーラトランジスタの $f_T$

$f_T$ :  $|i_c|=|i_b|$ となる周波数





$$v_{b'e} = \frac{r_{\pi}}{1 + j\omega(C_{\pi} + C_C)r_{\pi}} i_b$$

$$i_c = g_m v_{b'e} - j\omega C_C v_{b'e}$$

$$\frac{i_c}{i_b} = \frac{(g_m - j\omega C_C)r_{\pi}}{1 + j\omega(C_{\pi} + C_C)r_{\pi}}$$

$g_m \gg \omega C_C$ ,  $\omega(C_{\pi} + C_C)r_{\pi} \gg 1$  と仮定

$$\frac{i_c}{i_b} \approx \frac{g_m r_{\pi}}{j\omega(C_{\pi} + C_C)r_{\pi}}$$

$$f_T = \frac{g_m}{2\pi(C_{\pi} + C_C)}$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_C)}$$

$$C_\pi = \frac{\Delta Q_b}{v_{b'e}} \quad C_\pi \gg C_C$$

$$f_T \approx \frac{g_m}{2\pi C_\pi} = \frac{g_m v_{b'e}}{2\pi \Delta Q_b} = \frac{i_c}{2\pi \Delta Q_b}$$

$$\frac{i_c}{\Delta Q_b} = \frac{I_C}{Q_b} = \frac{qAD_n \frac{n_p(0)}{W_B}}{\frac{1}{2}n_p(0)W_BqA} = \frac{2D_n}{W_B^2}$$

$$\frac{D_n}{\mu} = \frac{kT}{q} \quad (\text{アインシュタインの関係式})$$

$$f_T = \frac{\mu}{\pi W_B^2} \cdot \frac{kT}{q}$$