Complexity 2017.9.28

Homework exercise from Lecture 2

Homework rule: Solve one of the following problems and hand your answer in at the next class. (If you cannot attend the next class, you can submit your answer via email before the class.) You do not have to write a long answer. Usually one page is enough. I will decide OK or NG, and you can get one point by each OK answer.

* For writing an answer, you may use Japanese.

Basic problems

1. Explain the meaning of

$$\mathrm{TIME}(\, t_1(\ell)\,) \subsetneq \mathrm{TIME}(\, t_2(\ell)\,).$$

   To be concrete, use one problem L witnessing this relation and explain how difficult/easy is it computationally. More specifically, explain what should be shown for L in terms of algorithms (or, more precisely, Turing machines) to solve L to prove that L is the witness of the theorem.

2. For the RAM model, we can prove the following theorem.

Theorem 2.3' (for RAM) There exists a universal RAM program $\mathtt{P}_{\mathrm{univ}}$ such that for any RAM program $\mathtt{P}$ and for any $x \in \{0,1\}^*$, it simulates $\mathtt{P}(x)$ with the following efficiency for a constant $c_{\mathtt{P}}$ determined by $\mathtt{P}$:

$$\mathrm{time}_{\mathtt{P}_{\mathrm{univ}}}(\langle \overline{\mathtt{P}}, x\rangle) \;\leq\; c_{\mathtt{P}}\mathrm{time}_{\mathtt{P}}(x)^2.$$

It seems difficult to get a linear bound like the Turing machine case. Explain why.