

### Homework exercise from Lecture 3

**Homework rule:** Choose two of the basic problems or one advanced problem, and hand your answer in at the next class, i.e., the next week. (If you cannot attend the next class, you can submit your answer via email before the class.) Since we have one week this time, you need to solve either two basic problems or one advanced problem. (You cannot try both basic and advanced problems for each homework.)

\* For writing an answer, you may use Japanese.

#### Basic problems

1. By using Theorem 3.2, we can give a better time bound for  $L_{25}$ , improving Lemma 3.3. Show this time bound and give a reasonably detail explanation (no need to be a formal proof) justifying it.
2. Give a proof of Lemma 3.4.
3. Give an outline of the proof of Space Hierarchy Theorem. For simplicity we may use  $t_1(\ell) = \ell^2$ .

#### An advanced problem

1. Consider the computation of converting a given positive integer  $n$  in the binary representation to the same one in the unary representation. Explain a one-tape Turing machine that achieves this computation in  $O(n \log n)$  moves. You do not have to give a detail description of this Turing machine; the explanation need to be detailed enough to see that the computation can be done in  $O(n \log n)$  moves.