Homework exercise from Lecture 1

Homework rule: Choose one of the basic problems (in case that there are multiple problems) and hand your answer in at the next class. (If you cannot attend the next class, you can submit your answer via email before the class.) You do not have to write a long answer. Usually one page is enough. I will decide OK or NG, and you can get one point by each OK answer. You can try one of the advanced problems. In this case, you can spend one week to solve the problem, and you need to hand your answer in at the next next class. You can get two points by each OK answer to the advanced problem. (You cannot try both basic and advanced problems for each homework.)

* For writing an answer, you may use Japanese.

Basic problems

1. Give an outline of the proof of Theorem 1.1. Explain $O(\mathrm{time}_{\mathtt{M}}(x)^2)$ moves are necessary and sufficient for your explained one-tape Turing machine $\mathtt{M}'$ to simulate a given multi-tape Turing machine $\mathtt{M}$ on any input x.
2. Consider any functions $f, g : \mathcal{Z}^+ \to \mathcal{Z}^+$.
   (1) We write $f = \Omega(g)$ if $g = O(f)$ holds. State the condition for $f = \Omega(g)$ like Definition 1.2.
   (2) We write $f = \overset{\infty}{\omega}(g)$ if $f \neq O(g)$. State the condition for $f = \overset{\infty}{\omega}(g)$ like Definition 1.2.

An advanced problem

1. Give an outline of the proof of Theorem 1.2. Explain $O(\mathrm{time}_{\mathtt{M}}(x) \log(\mathrm{time}_{\mathtt{M}}(x)))$ moves are necessary and sufficient for your explained two-tape Turing machine to simulate a given multi-tape Turing machine $\mathtt{M}$ on any input x.