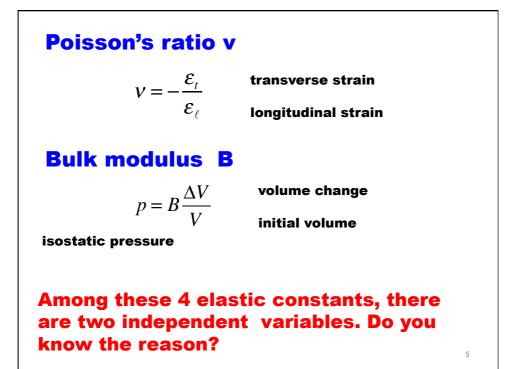
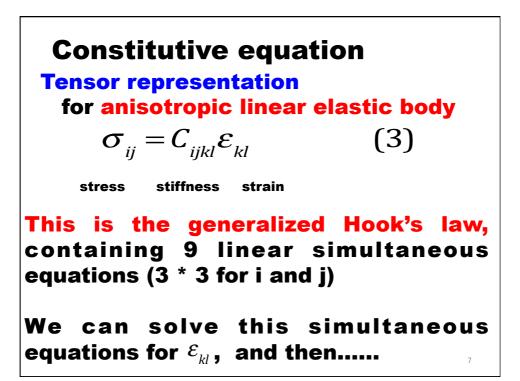
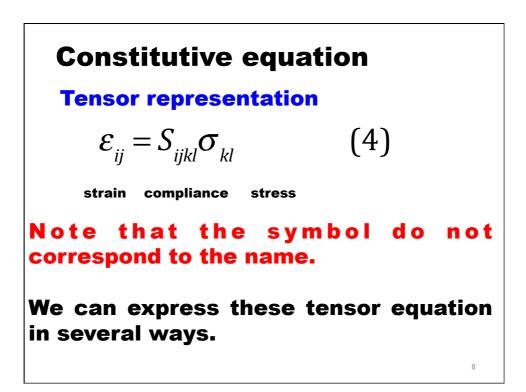


Deformation of linear elastic
isotropic uniform bodyAxial loading
: tensile and compression
 $\sigma = E\varepsilon$ (1)
normal stress Young's modulus normal strainShear loading
 $\tau = G\gamma$ (2)shear stress modulus of rigidity shear strain
(engineering strain)Do you know other two elastic constants?4

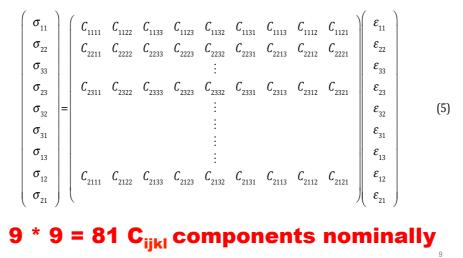


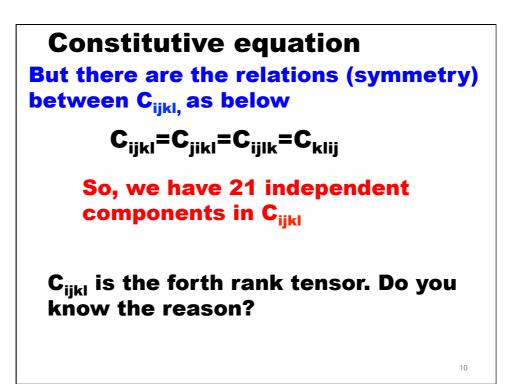
materials	E∕GPa
Mild steel	210
AI	70
diamond	~1000
SiC	420
Al ₂ O ₃	390
Si ₃ N ₄	300
ZrO ₂	200
glass	70
plolymer	1-3

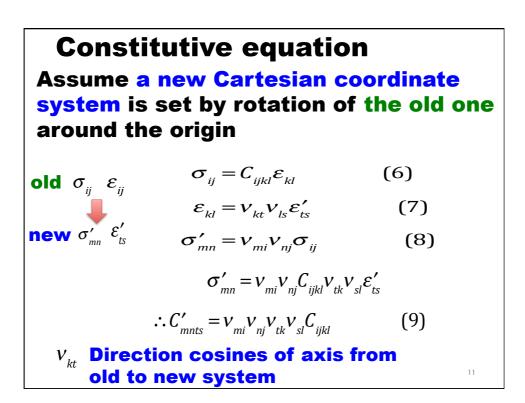












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Constitutive equation

Matrix representation (engineering strain form)

Stress and strain tensor are **expressed** by 6-dimensional vector as below,

 $\begin{array}{ll} \sigma_{11} \rightarrow \sigma_{1} & \varepsilon_{11} \rightarrow \varepsilon_{1} \\ \sigma_{22} \rightarrow \sigma_{2} & \varepsilon_{22} \rightarrow \varepsilon_{2} \\ \sigma_{33} \rightarrow \sigma_{3} & \varepsilon_{23} = \varepsilon_{32} \rightarrow \frac{1}{2} \varepsilon_{4} & (\varepsilon_{4} = 2\varepsilon_{23}) \\ \sigma_{23} = \sigma_{32} \rightarrow \sigma_{4} & \varepsilon_{31} = \varepsilon_{13} \rightarrow \frac{1}{2} \varepsilon_{5} & (\varepsilon_{5} = 2\varepsilon_{31}) \\ \sigma_{12} = \sigma_{21} \rightarrow \sigma_{6} & \varepsilon_{12} = \varepsilon_{21} \rightarrow \frac{1}{2} \varepsilon_{6} & (\varepsilon_{6} = 2\varepsilon_{12}) \end{array}$

Constitutive equation

Matrix representation (engineering strain form)

In tensor representation, \mathcal{E}_{23} and \mathcal{E}_{32} are added separately, but matrix representation, \mathcal{E}_4 is added once, so, the value of \mathcal{E}_4 should be double of \mathcal{E}_{23} , and \mathcal{E}_4 is the same as engineering shear strain γ_{22}

$$\varepsilon_{23} = \varepsilon_{32} \rightarrow \frac{1}{2}\varepsilon_4 \qquad (\varepsilon_4 = 2\varepsilon_{23} = \gamma_{23})$$

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Constitutive equation Matrix representation (engineering strain form)

~ ~ ~ ~

$$\sigma_{m} = C_{mn} \mathcal{E}_{n} \qquad (10)$$

$$\begin{pmatrix} \sigma_{1} \\ \sigma_{2} \\ \sigma_{3} \\ \sigma_{4} \\ \sigma_{5} \\ \sigma_{6} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{pmatrix} \begin{pmatrix} \varepsilon_{1} \\ \varepsilon_{2} \\ \varepsilon_{3} \\ \varepsilon_{4} \\ \varepsilon_{5} \\ \varepsilon_{6} \end{pmatrix} \qquad (11)$$
6 * 6 = 36 C_{mn} components nominally¹⁵

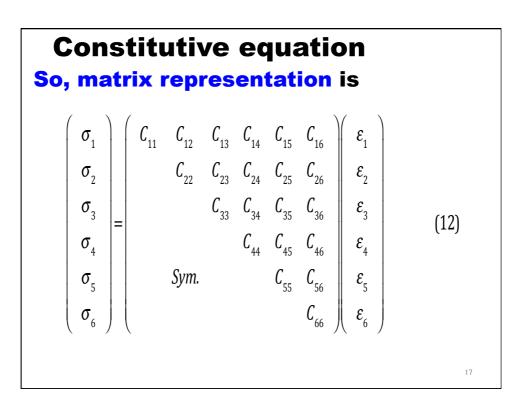
Constitutive equation But there are the relations (symmetry) between C_{mn} , also $C_{mn}=C_{nm}$ So, we can undersntand why 21 independent components

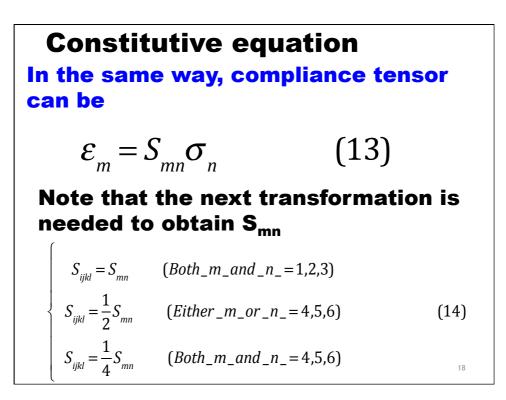
in C_{iikl} (21=6+5+4+3+2+1)

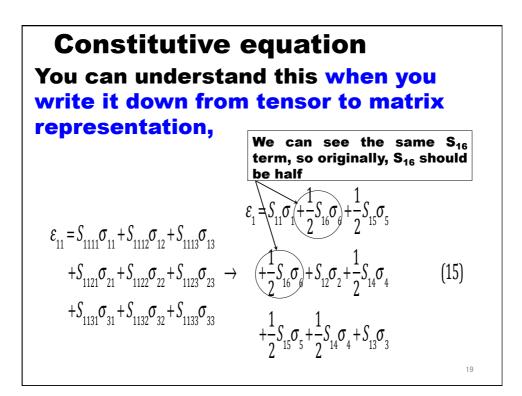
This comes from elastic strain energy can be expressed by positive quadratic function of strain

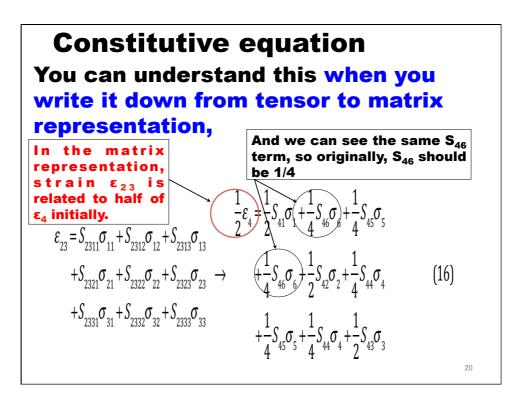
$$W = \frac{1}{2} C_{ijkl} \varepsilon_{ij} \varepsilon_{kl}$$

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Origin of elasticity

Elasticity means that shape and dimension of a body return back to the original state after unloading.

Energy elasticity

metal and ceramics stiffness decreases when it is heated.

Entropy elasticity

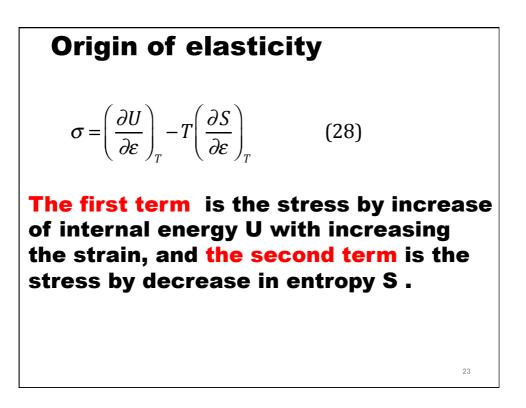
polymer stiffness increases when it is heated.

Origin of elasticity Thermodynamics 1st law $dU = \delta Q + \sigma d\varepsilon$ (26) From the 2nd law, in the reversible

process, entropy change per unit volume can be expressed by

$$dS = \frac{\delta Q}{T} \tag{27}$$

By combining these two equations, we obtain for the isothermal reversible process, 22



Origin of elasticity On the other hand, Helmholz energy F per unit volume is, F = U - TS (29) The infinitesimal change of F is dF = dU - TdS - SdT $= \delta Q + \sigma d\varepsilon - TdS - SdT$ (30)

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Origin of elasticity

Considering isothermal condition dT=0, Eq.(30) gives,

$$\left(\frac{\partial F}{\partial \varepsilon}\right)_{T} = \sigma$$
 (31)

And if we consider temperature changes in quasi-static by keeping strain constant,

$$\left(\frac{\partial F}{\partial T}\right)_{\varepsilon} = -S \tag{32}$$

Origin of elasticity

From Eq.(31) and (32) we have,

$$\left(\frac{\partial\sigma}{\partial T}\right)_{\varepsilon} = \frac{\partial}{\partial T} \left(\frac{\partial F}{\partial \varepsilon}\right) = \frac{\partial^2 F}{\partial T \partial \varepsilon} = \frac{\partial}{\partial \varepsilon} \left(\frac{\partial F}{\partial T}\right) = -\left(\frac{\partial S}{\partial \varepsilon}\right)_T$$
(33)

Substitute this relation into Eq.(28), so,

$$\sigma = \left(\frac{\partial U}{\partial \varepsilon}\right)_T + T \left(\frac{\partial \sigma}{\partial T}\right)_{\varepsilon}$$
(34)

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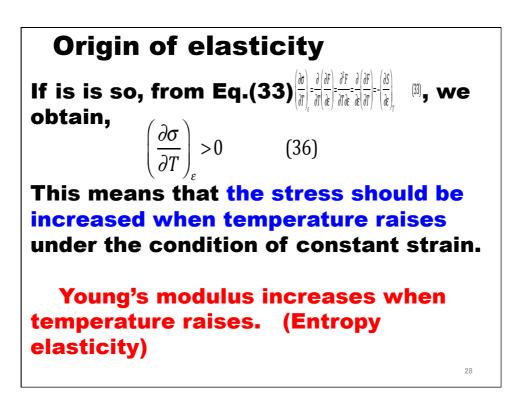
Origin of elasticity

Experimental results showed that 8% vulcanized gum(sulfur added) had a relation $\sigma = cT$ when it was heated.

Look back Eq.(28), Eq.(28) should be,

$$\sigma = 0 - T \left(\frac{\partial S}{\partial \varepsilon}\right)_{T}$$
(35)

And corresponding to $\sigma = cT$ results, the 2nd term $(\partial S / \partial \varepsilon)_T$ in Eq.(35) should be negative.



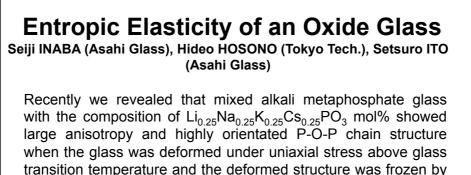
Origin of elasticity

For energy elasticity, from Eq.(28), we can see,

 $\sigma = \left(\frac{\partial U}{\partial \varepsilon}\right)_T - 0 \qquad (37)$

The work done by external load is stored as the increase in internal energy by the change in distance and angle between atoms.

Young's modulus decreases when temperature raises.



transition temperature and the deformed under unlaxial stress above glass transition temperature and the deformed structure was frozen by cooling under stress. In the present study we found the anisotropic glass showed a huge thermal shrinkage of 35% in length of glass when heated at deformation point (η =10 9.7 Pa.s). We will discuss the shrinkage mechanism in terms of viscoelastic properties.

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