2 Short test (Solution) (40)

Solve the linear systems of equations and state the rank of the coefficient matrix of each system using Gaussian elimination or Gauss-Jordan elimination.

(1) (20)

$$\begin{aligned} 3x + 3y + 2z &= 1 \\ 2x + y + z &= 1 \\ 2x - 2y &= 2 \end{aligned}$$

over $\mathbb{R}$.

Solution. Augmented matrix is

$$\left[\begin{array}{rrr|r} 3 & 3 & 2 & 1 \\ 2 & 1 & 1 & 1 \\ 2 & -2 & 0 & 2 \end{array}\right]$$

and by Gaussian elimination we obtain

$$\left[\begin{array}{rrr|r} 3 & 0 & 1 & 2 \\ 0 & 3 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

And hence the system has infinitely many solutions of the form $[\frac{2}{3}, -\frac{1}{3}, 0] + t[-\frac{1}{3}, -\frac{1}{3}, 1]$ for $t \in \mathbb{R}$. The rank of the coefficient matrix is 2.

□

(2) (20)

$$3x + 2y = 1$$

$$x + 4y = 1$$

over $\mathbb{Z}_7$.

Solution. Augmented matrix is

$$\left[\begin{array}{rr|r} 3 & 2 & 1 \\ 1 & 4 & 1 \end{array}\right]$$

by $R_2 + 4R_1$, $2R_2$ and $R_1 + 3R_2$ applied in a sequence we obtain

$$\left[\begin{array}{rr|r} 1 & 0 & 3 \\ 0 & 1 & 3 \end{array}\right]$$

And hence the system has a unique solution $[3, 3]$. The rank of the coefficient matrix is 2. □