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Fourier transform

$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x)\, e^{-2\pi i x \xi}\, dx,$$

$$f(x) = \int_{-\infty}^{\infty} \hat{f}(\xi)\, e^{2\pi i \xi x}\, d\xi,$$

https://en.wikipedia.org/wiki/Fourier_transform

Discrete Fourier transform

$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x)\ e^{-2\pi i x \xi}\,dx,$$

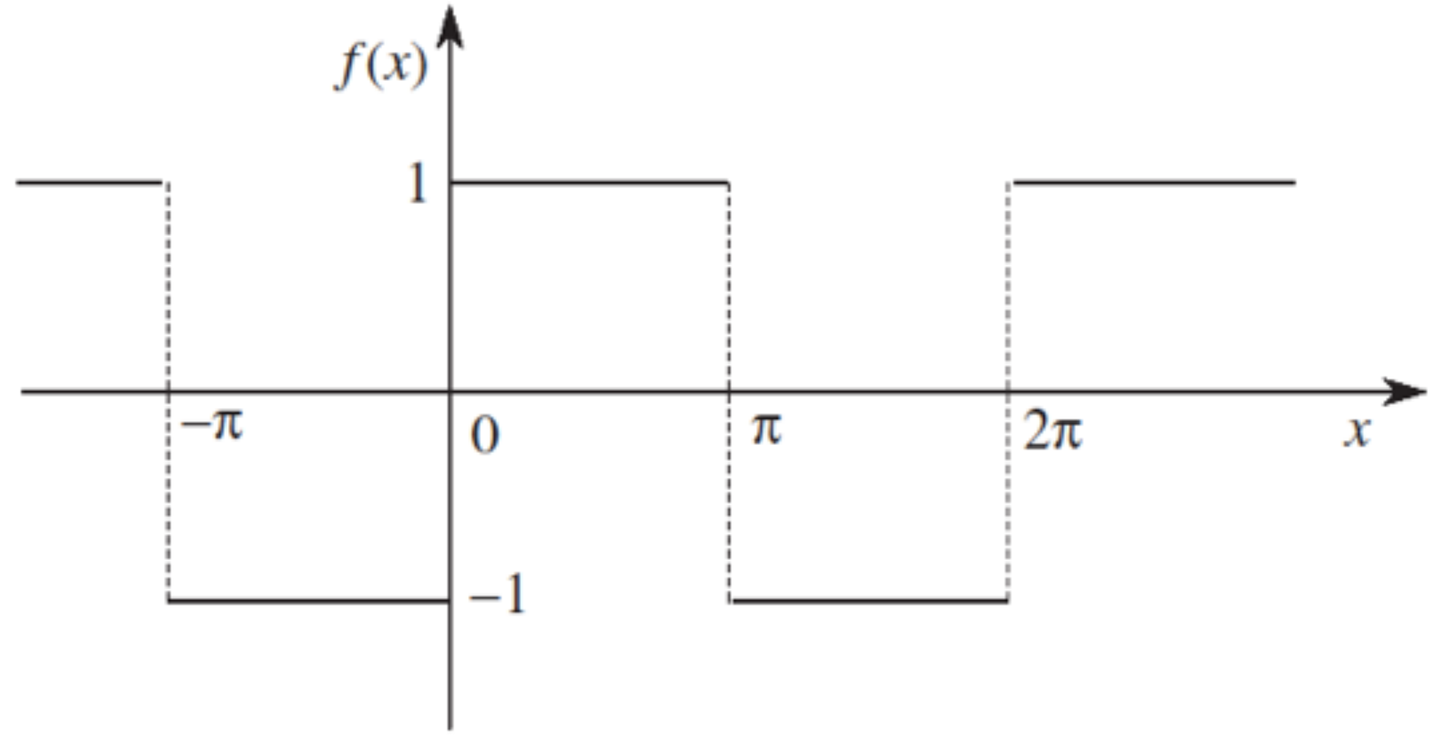
$$f(x) = \int_{-\infty}^{\infty} \hat{f}(\xi)\ e^{2\pi i \xi x}\,d\xi,$$

$$f(x_n) = \sum_{k=-N/2}^{N/2} \hat{f}_k e^{ikx_n}$$

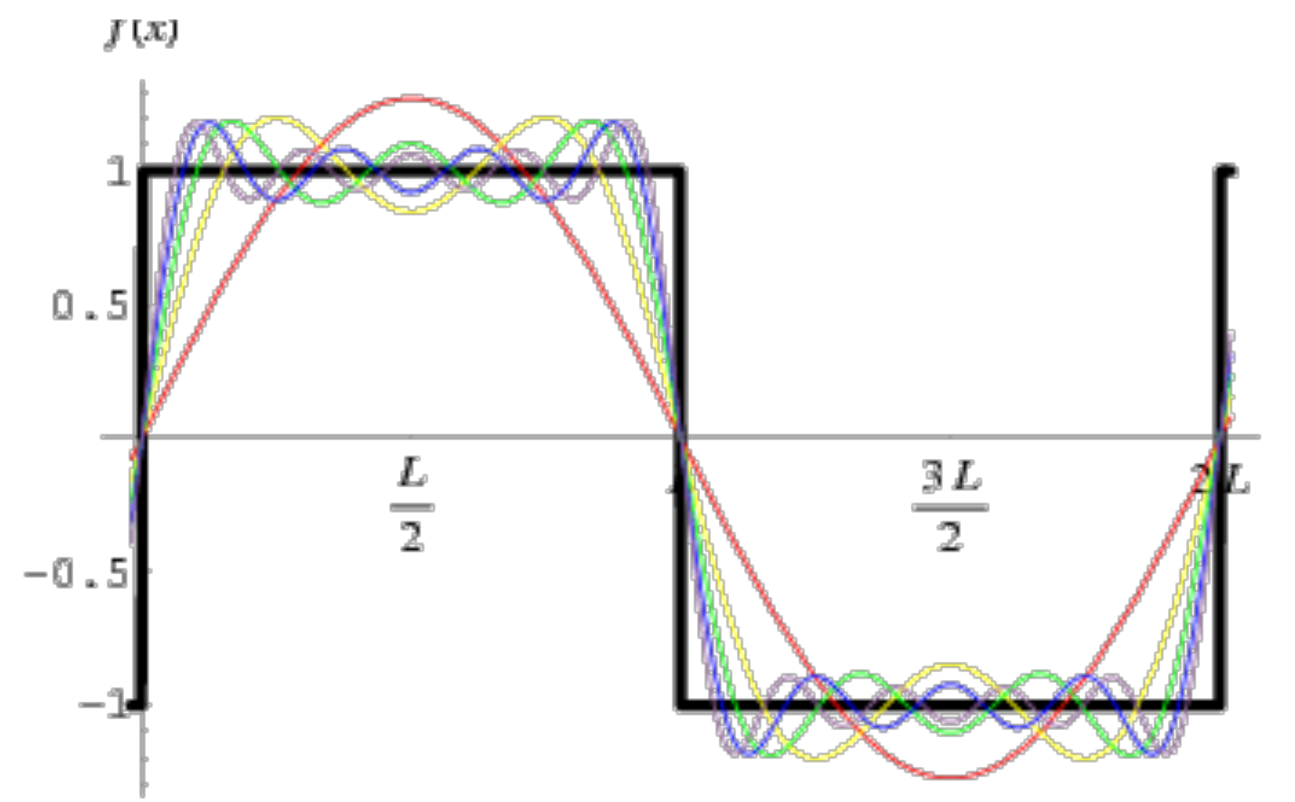
$$\hat{f}_k = \sum_{n=0}^{N} f(x_n) e^{-ikx_n}$$

Periodic square function

$$f(x) = \begin{cases} 1 & \text{if } 0 \le x < \pi \\ -1 & \text{if } \pi \le x < 2\pi \end{cases}$$



k	$\mathrm{Re}(\hat{f}_k)$	$\mathrm{Im}(\hat{f}_k)$	$\lvert\hat{f}_k\rvert$
0	0	0	0
1	0.125	−0.628	0.641
2	0	0	0
3	0.125	−0.187	0.225
4	0	0	0
5	0.125	−0.084	0.150
6	0	0	0
7	0.125	−0.025	0.127
8	0	0	0

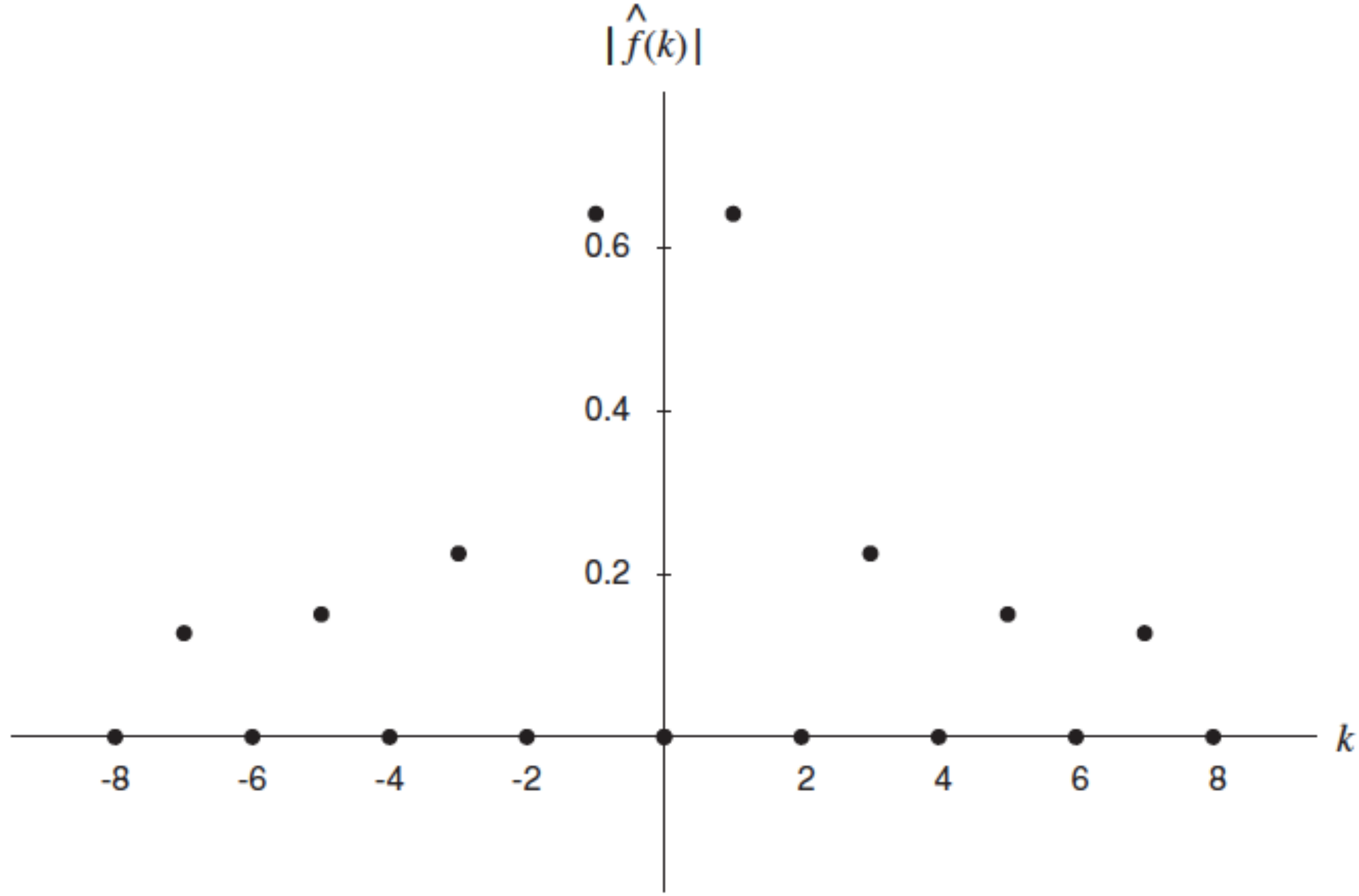


Fourier transform of a real function

$$\hat{f}_{-k} = \frac{1}{N} \sum_{j=0}^{N-1} f_j e^{ikx_j}$$

$$\hat{f}^*_{-k} = \frac{1}{N} \sum_{j=0}^{N-1} f_j e^{-ikx_j}$$

$$\hat{f}_{-k} = \hat{f}^*_k$$



Product of functions in Fourier space

$$H(x) = f(x)g(x)$$

$$\hat{H}_m = (\widehat{fg})_m = \frac{1}{N}\sum_{j=0}^{N-1} f_j g_j e^{-imx_j}$$

$$\hat{H}_m = \frac{1}{N}\sum_{j=0}^{N-1}\sum_{k}\sum_{k'} \hat{f}_k \hat{g}_{k'} e^{ikx_j} e^{ik'x_j} e^{-imx_j}$$

$$\hat{H}_m = \sum_{k=-N/2}^{N/2-1} \hat{f}_k \hat{g}_{m-k}$$

Product of two functions

$$f(x) = \sin 2x \qquad g(x) = \sin 3x$$

$$\hat{f}_k = \begin{cases} \mp i/2 & \text{if } k = \pm 2 \\ 0 & \text{otherwise,} \end{cases} \qquad \hat{g}_k = \begin{cases} \mp i/2 & \text{if } k = \pm 3 \\ 0 & \text{otherwise.} \end{cases}$$

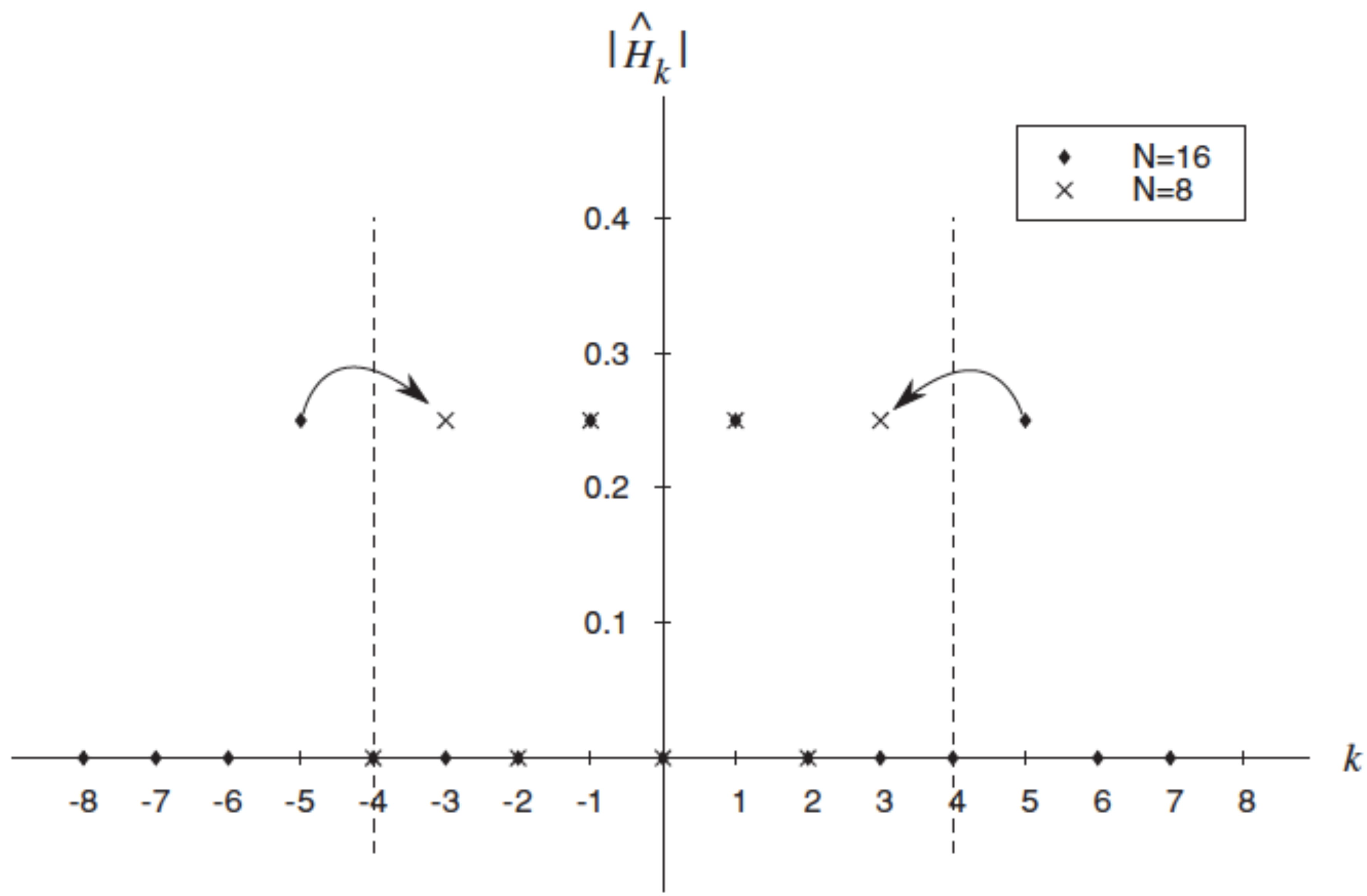
$$H(x) = f(x)g(x)$$

$$\hat{H}_m = \sum_{k=-N/2}^{N/2-1} \hat{f}_k \hat{g}_{m-k} \qquad H(x) = 0.5(\cos x - \cos 5x)$$

$$\hat{H}_k = \begin{cases} 1/4 & \text{if } k = \pm 1 \\ -1/4 & \text{if } k = \pm 5 \\ 0 & \text{otherwise} \end{cases}$$

Aliasing

$$\hat{H}_m = \sum_{k=-N/2}^{N/2-1} \hat{f}_k \hat{g}_{m-k}$$



Derivative in Fourier space

$$f(x_n) = \sum_{k=-N/2}^{N/2} \hat{f}_k e^{ikx_n}$$

$$f'(x_n) = \sum_{k=-N/2}^{N/2} ik\hat{f}_k e^{ikx_n}$$

Derivative in Fourier space

$$f(x) = \cos 3x \qquad f'(x) = -3\sin 3x$$

$$\widehat{Df}_k = \begin{cases} -(3/2)i & \text{if } k = -3 \\ (3/2)i & \text{if } k = 3 \\ 0 & \text{otherwise.} \end{cases}$$

$$f(x) = 2\pi x - x^2 \qquad f'(x) = 2\pi - 2x$$

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