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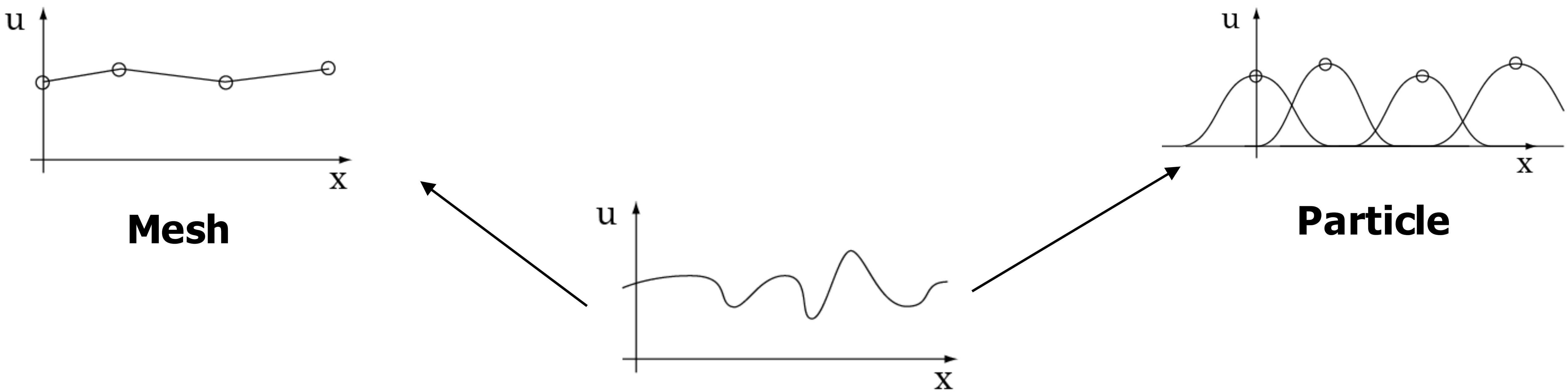
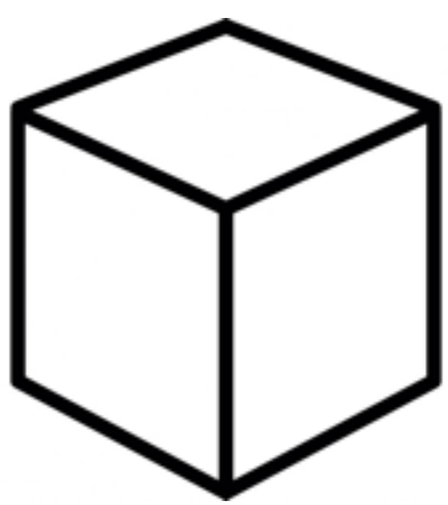
Particle-based discretization



$$u(\mathbf{x}) = \int u_j W(\mathbf{x} - \mathbf{x}_j)$$



$$m = \rho V$$

$$u(\mathbf{x}) = \sum_j u_j W(\mathbf{x} - \mathbf{x}_j) V_j$$

$$u(\mathbf{x}) = \sum_j u_j \frac{m_j}{\rho_j} W(\mathbf{x} - \mathbf{x}_j)$$

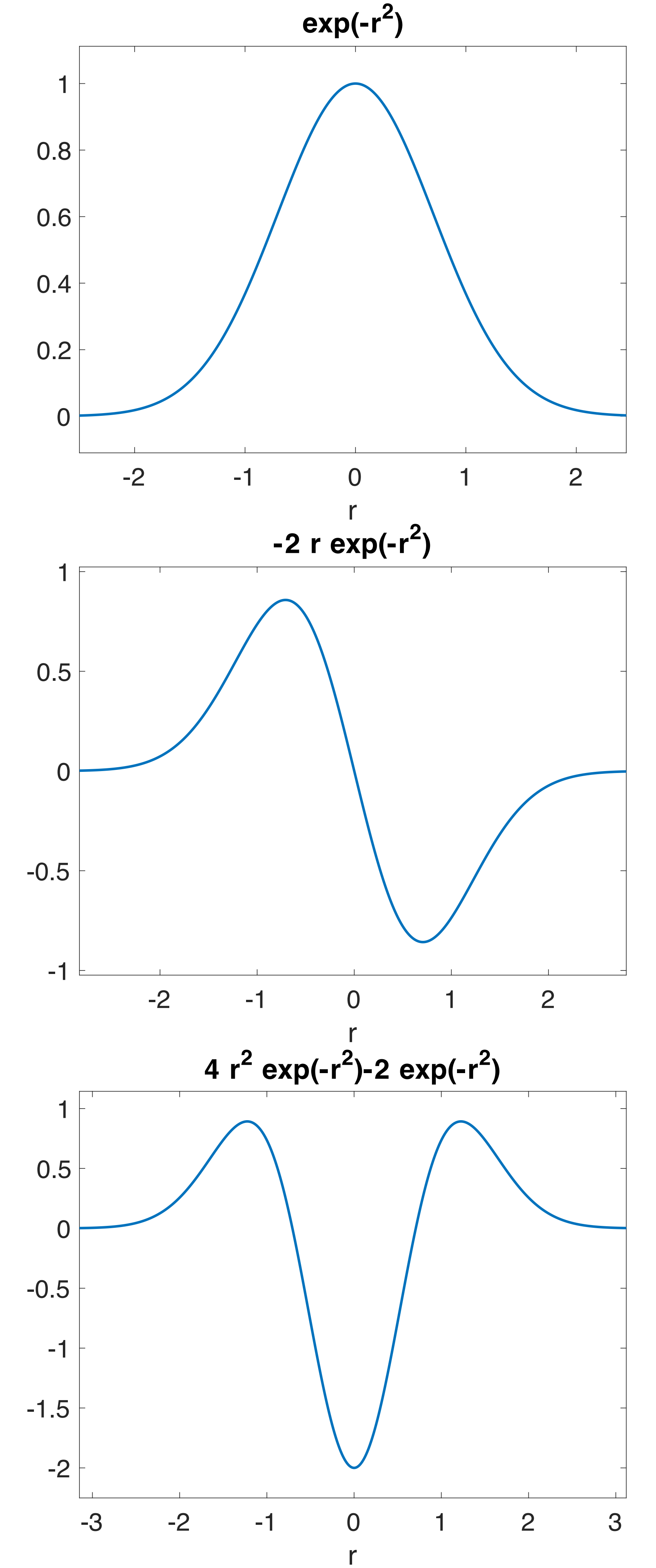
Gradients

$$\nabla u(\mathbf{x}) = \nabla \sum_j u_j \frac{m_j}{\rho_j} W(\mathbf{x} - \mathbf{x}_j)$$

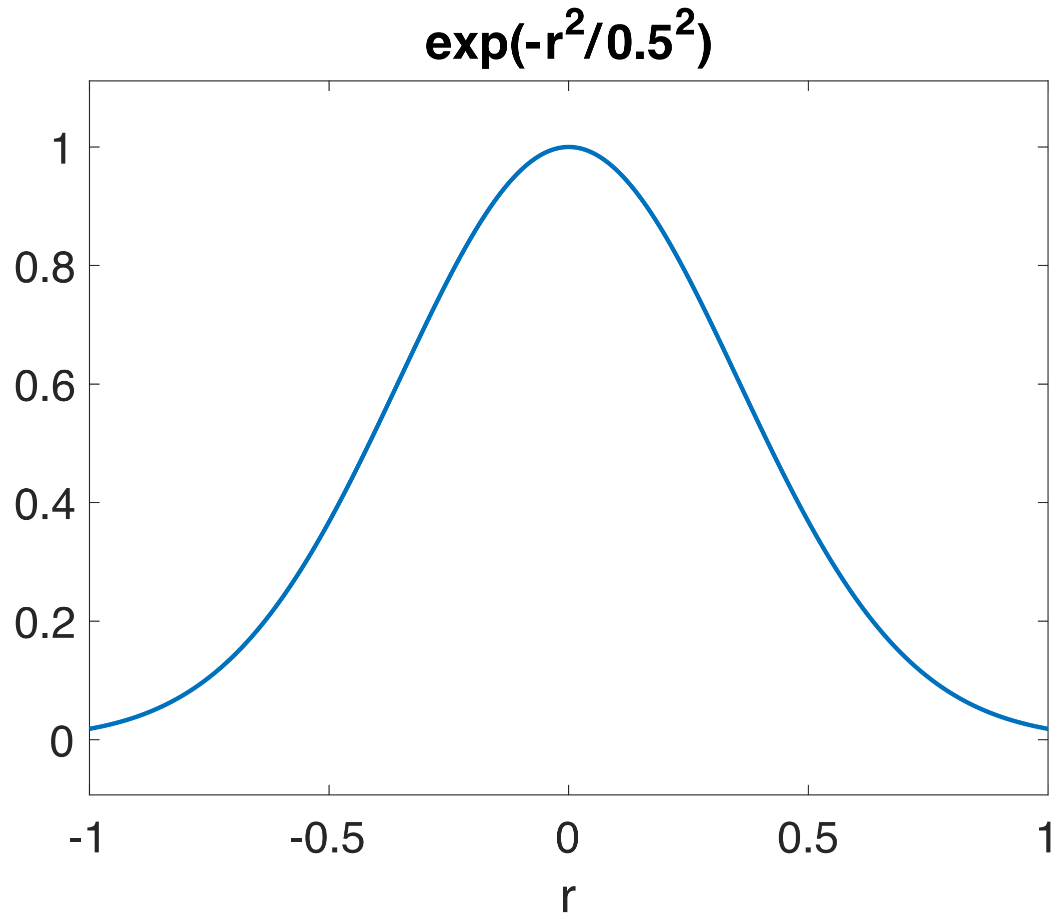
$$= \sum_j u_j \frac{m_j}{\rho_j} \nabla W(\mathbf{x} - \mathbf{x}_j)$$

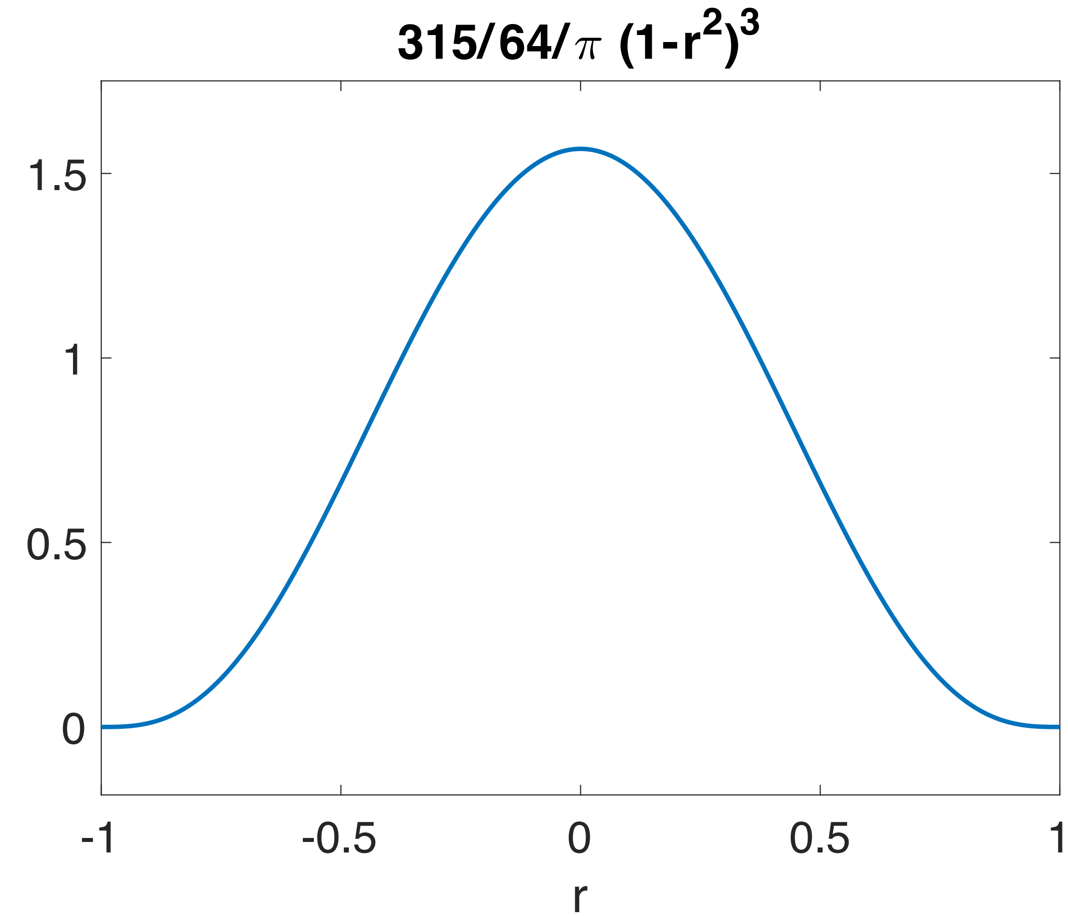
$$\nabla^2 u(\mathbf{x}) = \nabla^2 \sum_j u_j \frac{m_j}{\rho_j} W(\mathbf{x} - \mathbf{x}_j)$$

$$= \sum_j u_j \frac{m_j}{\rho_j} \nabla^2 W(\mathbf{x} - \mathbf{x}_j)$$

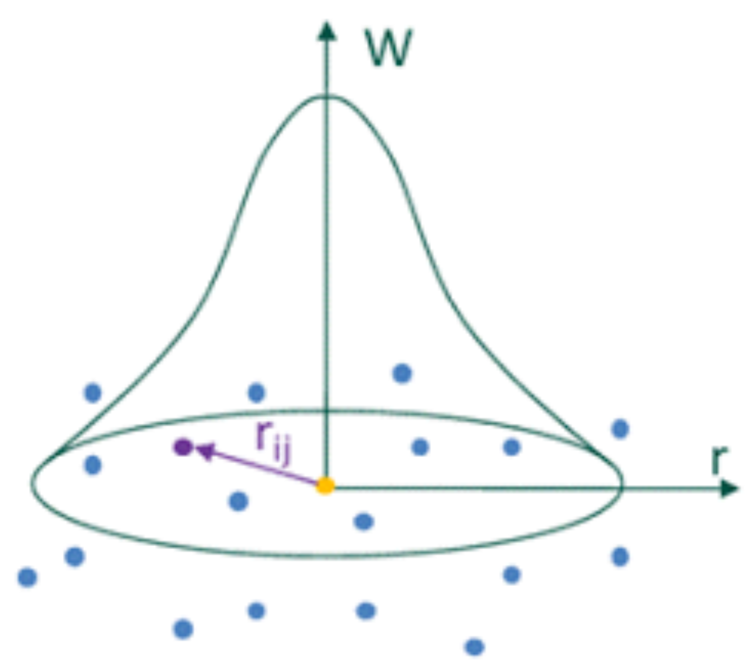


Compact support functions



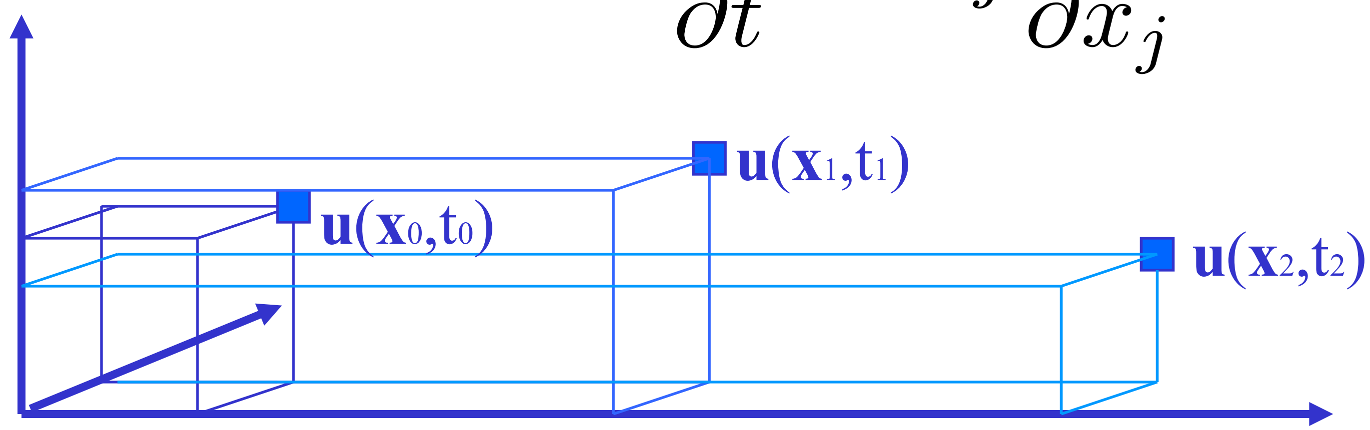


Neighbor search



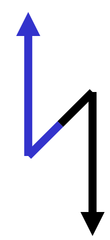
Navier-Stokes in Lagrangian frame

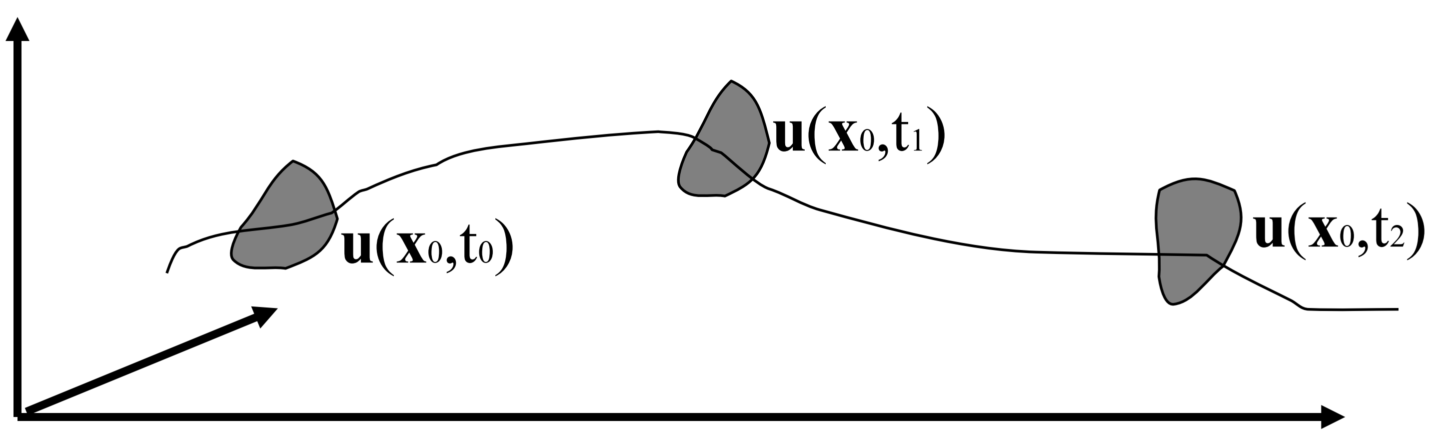
$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho_i}\nabla p_i + \nu \nabla^2 u_i$$



Eulerian

Lagrangian





$$\frac{Du_i}{Dt} = -\frac{1}{\rho_i}\nabla p_i + \nu \nabla^2 u_i$$

Pressure

$$\frac{Du_i}{Dt} = \boxed{-\frac{1}{\rho_i}\nabla p_i} + \nu \nabla^2 u_i$$

$$u_i = \sum_j u_j \frac{m_j}{\rho_j} W(\mathbf{x}_i - \mathbf{x}_j)$$

$$\downarrow$$

$$-\frac{1}{\rho_i}\nabla p_i = -\sum_j p_j \frac{m_j}{\rho_i \rho_j} \nabla W(\mathbf{x}_i - \mathbf{x}_j)$$

$$pV = nRT = k \qquad V = \frac{1}{\rho}$$

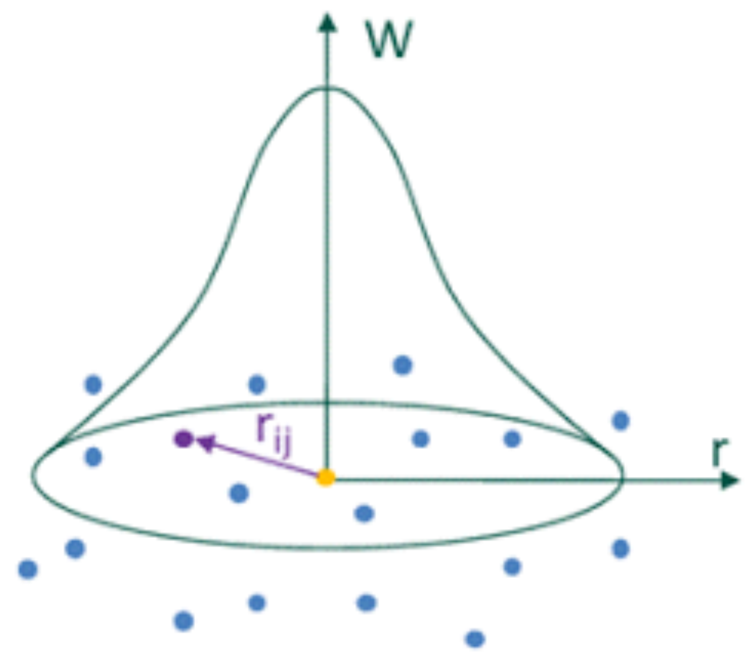
$$p_i = k\rho_i$$

Density

$$u_i = \sum_j u_j \frac{m_j}{\rho_j} W(\mathbf{x}_i - \mathbf{x}_j)$$

$$\rho_i = \sum_j \rho_j \frac{m_j}{\rho_j} W(\mathbf{x}_i - \mathbf{x}_j)$$

$$= \sum_j m_j W(\mathbf{x}_i - \mathbf{x}_j)$$



(density) = weighted ambient point mass

Viscosity

$$\frac{Du_i}{Dt} = -\frac{1}{\rho_i}\nabla p_i + \boxed{\nu\nabla^2 u_i}$$

$$\nabla^2 u_i = \sum_j u_j \frac{m_j}{\rho_j}\nabla^2 W(\mathbf{x}_i - \mathbf{x}_j)$$

$$\nu\nabla^2 u_i = \mu \sum_j u_j \frac{m_j}{\rho_i \rho_j}\nabla^2 W(\mathbf{x}_i - \mathbf{x}_j)$$

Verlet integration

$$\frac{Du_i}{Dt} = -\frac{1}{\rho_i}\nabla p_i + \nu\nabla^2 u_i$$

$$x : \text{position}$$

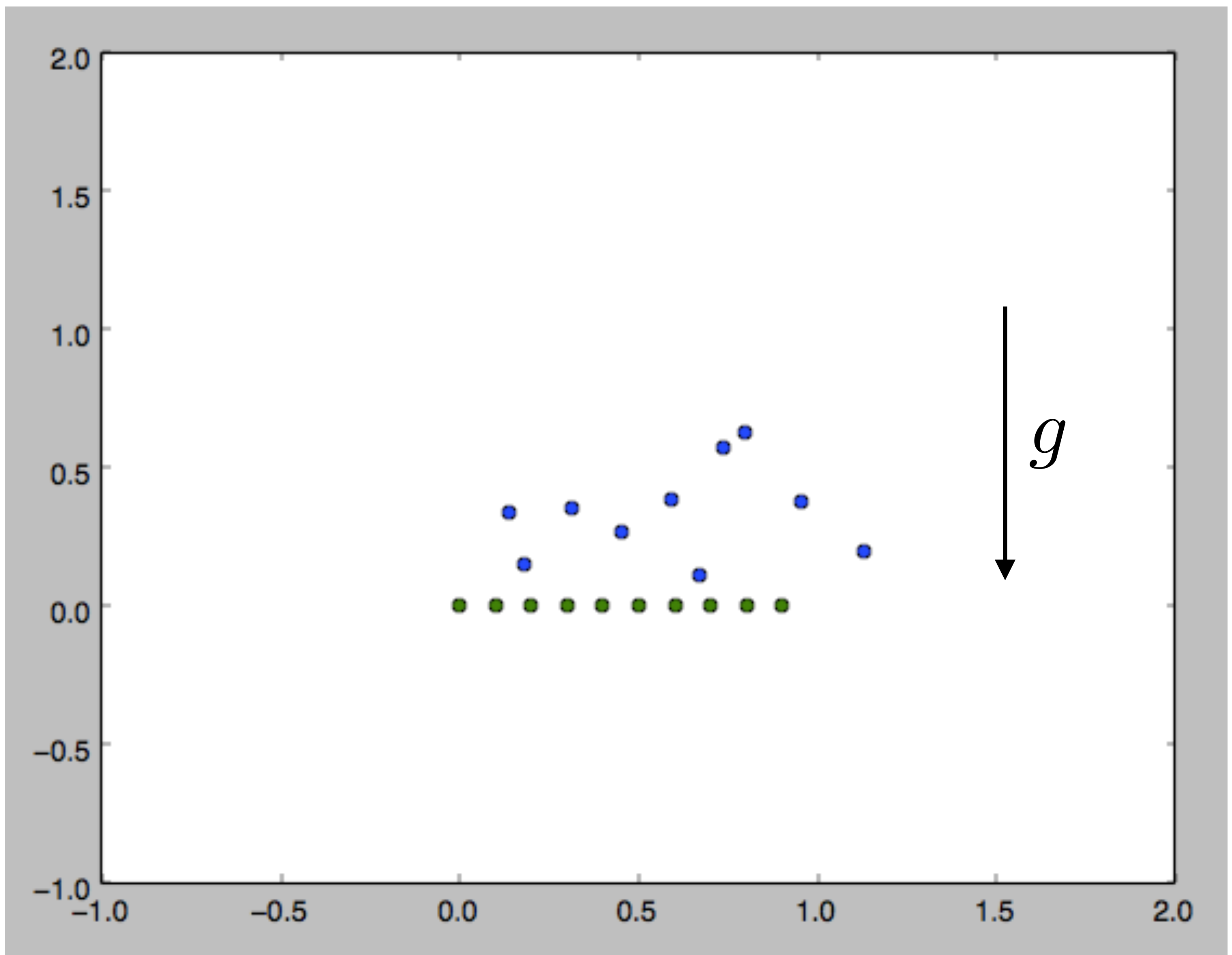
$$u = \frac{Dx}{Dt} : \text{velocity}$$

$$\frac{Du}{Dt} = \frac{D^2 x}{Dt^2} : \text{acceleration}$$

$$x(t+\Delta t) = 2x(t) - x(t-\Delta t) + \frac{d^2 x}{dt^2}(t)\Delta t^2 + \mathcal{O}(\Delta t^4)$$

Gravity

$$\frac{Du_i}{Dt} = -\frac{1}{\rho_i}\nabla p_i + g_i$$



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