Electric Power and Motor Drive System Analysis

11. Field oriented control of induction machines

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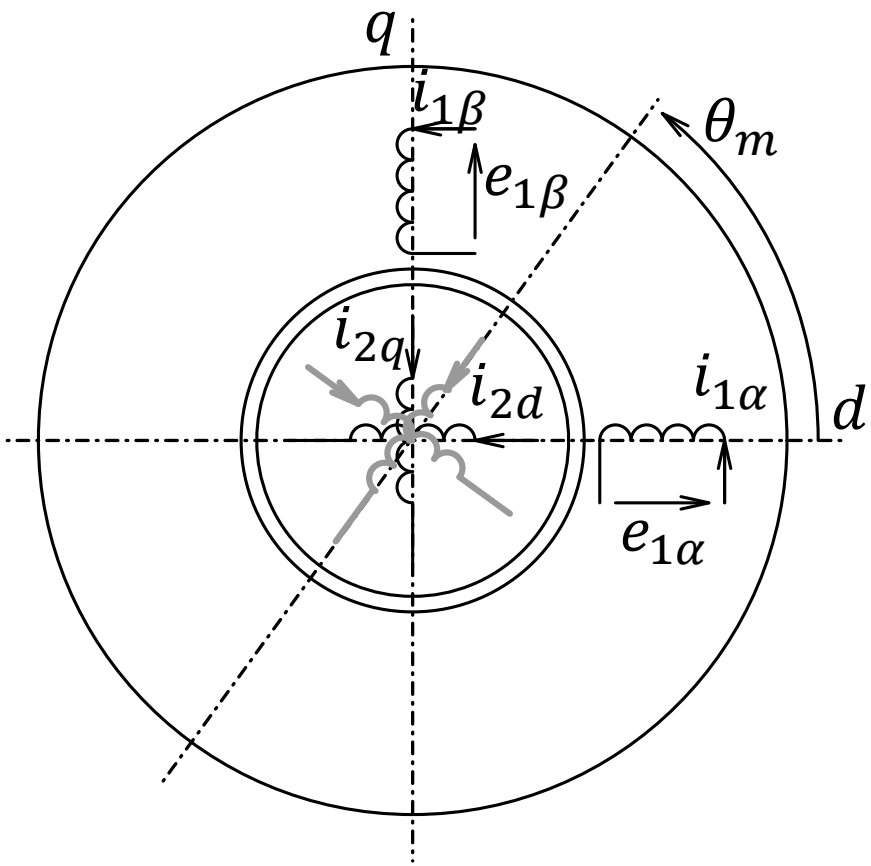
γ-δ transformation



Rotating transformation:

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = \begin{bmatrix} \cos\omega t & -\sin\omega t \\ \sin\omega t & \cos\omega t \end{bmatrix} \begin{bmatrix} x_d \\ x_q \end{bmatrix}$$

$$= [C(\theta\ )] \begin{bmatrix} x_d \\ x_q \end{bmatrix}$$

Stator rotating angle: $\theta = \omega t$

$$\begin{bmatrix} i_{1\alpha} \\ i_{1\beta} \end{bmatrix} = \begin{bmatrix} \cos\omega t & -\sin\omega t \\ \sin\omega t & \cos\omega t \end{bmatrix} \begin{bmatrix} i_{1\gamma} \\ i_{1\delta} \end{bmatrix}$$

$$= [C(\omega t\ )] \begin{bmatrix} i_{1\gamma} \\ i_{1\delta} \end{bmatrix}$$

Rotor rotating angle: $\theta = \omega t - \theta_m$

$$\begin{bmatrix} i_{2\alpha} \\ i_{2\beta} \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\omega t - \theta_m) & -\sin(\omega t - \theta_m) \\ \sin(\omega t - \theta_m) & \cos(\omega t - \theta_m) \end{bmatrix} \begin{bmatrix} i_{2\gamma} \\ i_{2\delta} \end{bmatrix}$$

$$= [C(\omega t - \theta_m)] \begin{bmatrix} i_{2\gamma} \\ i_{2\delta} \end{bmatrix}$$

$$\begin{bmatrix} e_{1\alpha} \\ e_{1\beta} \\ e_{2\alpha} \\ e_{2\beta} \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} \psi_{1\alpha} \\ \psi_{1\beta} \\ \psi_{2\alpha} \\ \psi_{2\beta} \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} L_{1\alpha}[I_2] & M[C(\theta_m)] \\ M[C(-\theta_m)] & L_{2\alpha}[I_2] \end{bmatrix}\begin{bmatrix} i_{1\alpha} \\ i_{1\beta} \\ i_{2\alpha} \\ i_{2\beta} \end{bmatrix}$$

$$\begin{bmatrix} e_{1\alpha} \\ e_{1\beta} \end{bmatrix} = \frac{d}{dt} L_{1\alpha} \begin{bmatrix} i_{1\alpha} \\ i_{1\beta} \end{bmatrix} + \frac{d}{dt} M[C(\theta_m)] \begin{bmatrix} i_{2\alpha} \\ i_{2\beta} \end{bmatrix}$$

$$\begin{bmatrix} e_{2\alpha} \\ e_{2\beta} \end{bmatrix} = \frac{d}{dt} M[C(-\theta_m)] \begin{bmatrix} i_{1\alpha} \\ i_{1\beta} \end{bmatrix} + \frac{d}{dt} L_{2\alpha} \begin{bmatrix} i_{2\alpha} \\ i_{2\beta} \end{bmatrix}$$

Upper matrix

$$\begin{bmatrix} e_{1\alpha} \\ e_{1\beta} \end{bmatrix} = \frac{d}{dt} L_{1\alpha} \begin{bmatrix} i_{1\alpha} \\ i_{1\beta} \end{bmatrix} + \frac{d}{dt} M[C(\theta_m)] \begin{bmatrix} i_{2\alpha} \\ i_{2\beta} \end{bmatrix}$$

$$\begin{bmatrix} e_{1\gamma} \\ e_{1\delta} \end{bmatrix} = [C(-\omega t\ )] \begin{bmatrix} e_{1\alpha} \\ e_{1\beta} \end{bmatrix}$$

$$= [C(-\omega t\ )] \frac{d}{dt} L_{1\alpha} [C(\omega t)] \begin{bmatrix} i_{1\gamma} \\ i_{1\delta} \end{bmatrix} + [C(-\omega t\ )] \frac{d}{dt} M[C(\theta_m)][C(\omega t - \theta_m)] \begin{bmatrix} i_{2\gamma} \\ i_{2\delta} \end{bmatrix}$$

$$\frac{d}{dt}[C(\omega t)] = \frac{d}{dt} \begin{bmatrix} \cos \omega t & -\sin \omega t \\ \sin \omega t & \cos \omega t \end{bmatrix} = \omega \begin{bmatrix} -\sin \omega t & -\cos \omega t \\ \cos \omega t & -\sin \omega t \end{bmatrix} = \omega \left[C\left(\omega t + \frac{\pi}{2}\right) \right]$$

$$\begin{bmatrix} e_{1\gamma} \\ e_{1\delta} \end{bmatrix} = \frac{d}{dt} L_{1\alpha} \begin{bmatrix} i_{1\gamma} \\ i_{1\delta} \end{bmatrix} + \omega L_{1\alpha} \left[C\left(\frac{\pi}{2}\right) \right] \begin{bmatrix} i_{1\gamma} \\ i_{1\delta} \end{bmatrix} + \frac{d}{dt} M \begin{bmatrix} i_{2\gamma} \\ i_{2\delta} \end{bmatrix} + \omega M \left[C\left(\frac{\pi}{2}\right) \right] \begin{bmatrix} i_{2\gamma} \\ i_{2\delta} \end{bmatrix}$$

$$\begin{bmatrix} e_{1\gamma} \\ e_{1\delta} \end{bmatrix} = \begin{bmatrix} L_{1\alpha} \frac{d}{dt} & -\omega L_{1\alpha} & M \frac{d}{dt} & -\omega M \\ \omega L_{1\alpha} & L_{1\alpha} \frac{d}{dt} & \omega M & M \frac{d}{dt} \end{bmatrix} \begin{bmatrix} i_{1\gamma} \\ i_{1\delta} \\ i_{2\gamma} \\ i_{2\delta} \end{bmatrix}$$

Lower matrix

$$\begin{bmatrix} e_{2\alpha} \\ e_{2\beta} \end{bmatrix} = \frac{d}{dt} M[C(-\theta_m)] \begin{bmatrix} i_{1\alpha} \\ i_{1\beta} \end{bmatrix} + \frac{d}{dt} L_{2\alpha} \begin{bmatrix} i_{2\alpha} \\ i_{2\beta} \end{bmatrix}$$

$$\begin{bmatrix} e_{2\gamma} \\ e_{2\delta} \end{bmatrix} = [C(\theta_m - \omega t)] \begin{bmatrix} e_{2\alpha} \\ e_{2\beta} \end{bmatrix}$$

$$= [C(\theta_m - \omega t)] \frac{d}{dt} M[C(-\theta_m)][C(\omega t\ )] \begin{bmatrix} i_{1\gamma} \\ i_{1\delta} \end{bmatrix}$$

$$+ [C(\theta_m - \omega t)] \frac{d}{dt} L_{2\alpha}[C(\omega t - \theta_m)] \begin{bmatrix} i_{2\gamma} \\ i_{2\delta} \end{bmatrix}$$

$$= M \frac{d}{dt} \begin{bmatrix} i_{1\gamma} \\ i_{1\delta} \end{bmatrix} + (\omega - \omega_m) M \left[C\left(\frac{\pi}{2}\ \right) \right] \begin{bmatrix} i_{1\gamma} \\ i_{1\delta} \end{bmatrix} + L_{2\alpha} \frac{d}{dt} \begin{bmatrix} i_{2\gamma} \\ i_{2\delta} \end{bmatrix}$$

$$+ (\omega - \omega_m) L_{2\alpha} \left[C\left(\frac{\pi}{2}\ \right) \right] \begin{bmatrix} i_{2\gamma} \\ i_{2\delta} \end{bmatrix}$$

$$\begin{bmatrix} e_{2\gamma} \\ e_{2\delta} \end{bmatrix} = \begin{bmatrix} M \frac{d}{dt} & -(\omega - \omega_m) M & L_{2\alpha} \frac{d}{dt} & -(\omega - \omega_m) L_{2\alpha} \\ (\omega - \omega_m) M & M \frac{d}{dt} & (\omega - \omega_m) L_{2\alpha} & L_{2\alpha} \frac{d}{dt} \end{bmatrix} \begin{bmatrix} i_{2\gamma} \\ i_{2\delta} \\ i_{2\gamma} \\ i_{2\delta} \end{bmatrix}$$

EMF and torque in γ-δ coordinate

$$\begin{bmatrix} e_{1\gamma} \\ e_{1\delta} \\ e_{2\gamma} \\ e_{2\delta} \end{bmatrix} = \begin{bmatrix} L_{1\alpha}\dfrac{d}{dt} & -\omega L_{1\alpha} & M\dfrac{d}{dt} & -\omega M \\ \omega L_{1\alpha} & L_{1\alpha}\dfrac{d}{dt} & \omega M & M\dfrac{d}{dt} \\ M\dfrac{d}{dt} & -(\omega-\omega_m)M & L_{2\alpha}\dfrac{d}{dt} & -(\omega-\omega_m)L_{2\alpha} \\ (\omega-\omega_m)M & M\dfrac{d}{dt} & (\omega-\omega_m)L_{2\alpha} & L_{2\alpha}\dfrac{d}{dt} \end{bmatrix} \begin{bmatrix} i_{2\gamma} \\ i_{2\delta} \\ i_{2\gamma} \\ i_{2\delta} \end{bmatrix}$$

$$T = M\left(i_{1q}i_{2d} - i_{1d}i_{2q}\right)$$

$$\begin{bmatrix} i_{1d} \\ i_{1q} \end{bmatrix} = [C(\omega t)] \begin{bmatrix} i_{1\gamma} \\ i_{1\delta} \end{bmatrix}, \begin{bmatrix} i_{2d} \\ i_{2q} \end{bmatrix} = [C(\omega t)] \begin{bmatrix} i_{2\gamma} \\ i_{2\delta} \end{bmatrix}$$

$$T = M\left(i_{1\delta}i_{2\gamma} - i_{1\gamma}i_{2\delta}\right)$$

Voltage and current equation

$$\begin{bmatrix} v_{1\gamma} \\ v_{1\delta} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_1 + L_{1\alpha}\dfrac{d}{dt} & -\omega L_{1\alpha} & M\dfrac{d}{dt} & -\omega M \\ \omega L_{1\alpha} & R_1 + L_{1\alpha}\dfrac{d}{dt} & \omega M & M\dfrac{d}{dt} \\ M\dfrac{d}{dt} & -(\omega - \omega_m)M & R_2 + L_{2\alpha}\dfrac{d}{dt} & -(\omega - \omega_m)L_{2\alpha} \\ (\omega - \omega_m)M & M\dfrac{d}{dt} & (\omega - \omega_m)L_{2\alpha} & R_2 + L_{2\alpha}\dfrac{d}{dt} \end{bmatrix} \begin{bmatrix} i_{1\gamma} \\ i_{1\delta} \\ i_{2\gamma} \\ i_{2\delta} \end{bmatrix}$$

$$T = M\left(i_{1\delta} i_{2\gamma} - i_{1\gamma} i_{2\delta}\right)$$

Magnetization current

$$\begin{bmatrix}0\\0\end{bmatrix} = \begin{bmatrix} M\dfrac{d}{dt} & -(\omega-\omega_m)M & R_2 + L_{2\alpha}\dfrac{d}{dt} & -(\omega-\omega_m)L_{2\alpha} \\ (\omega-\omega_m)M & M\dfrac{d}{dt} & (\omega-\omega_m)L_{2\alpha} & R_2 + L_{2\alpha}\dfrac{d}{dt} \end{bmatrix}\begin{bmatrix} i_{1\gamma} \\ i_{1\delta} \\ i_{2\gamma} \\ i_{2\delta}\end{bmatrix}$$

$$\begin{bmatrix}\psi_{2\gamma}\\ \psi_{2\delta}\end{bmatrix} = \begin{bmatrix} M & 0 & L_{2\alpha} & 0 \\ 0 & M & 0 & L_{2\alpha}\end{bmatrix}\begin{bmatrix} i_{1\gamma} \\ i_{1\delta} \\ i_{2\gamma} \\ i_{2\delta}\end{bmatrix} = M\begin{bmatrix} i_{1\gamma} \\ i_{1\delta}\end{bmatrix} + L_{2\alpha}\begin{bmatrix} i_{2\gamma} \\ i_{2\delta}\end{bmatrix}$$

define $\begin{bmatrix}\psi_{2\gamma}\\ \psi_{2\delta}\end{bmatrix} = M\begin{bmatrix} i_{0\gamma} \\ i_{0\delta}\end{bmatrix}$

$$\psi_{2\delta} = 0 \;\Rightarrow\; i_{0\delta} = 0$$

$$\omega_s = \omega - \omega_m$$

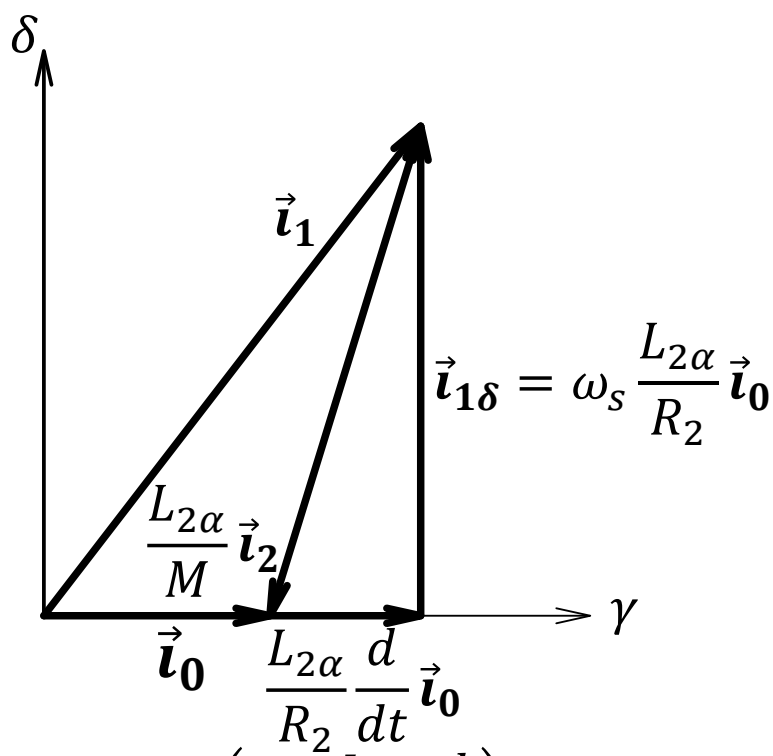
$$\begin{bmatrix} i_{0\gamma} \\ i_{0\delta}\end{bmatrix} = \begin{bmatrix} i_{1\gamma} \\ i_{1\delta}\end{bmatrix} + \frac{L_{2\alpha}}{M}\begin{bmatrix} i_{2\gamma} \\ i_{2\delta}\end{bmatrix}$$

$$\begin{bmatrix}0\\0\end{bmatrix} = M\frac{d}{dt}\begin{bmatrix} i_{0\gamma} \\ i_{0\delta}\end{bmatrix} + \omega_s M\begin{bmatrix} 0 & -1 \\ 1 & 0\end{bmatrix}\begin{bmatrix} i_{0\gamma} \\ i_{0\delta}\end{bmatrix} + R_2\begin{bmatrix} i_{2\gamma} \\ i_{2\delta}\end{bmatrix}$$

Secondary current relationship

$$\begin{bmatrix}0\\0\end{bmatrix} = M\frac{d}{dt}\begin{bmatrix}i_{0\gamma}\\i_{0\delta}\end{bmatrix} + \omega_s M\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}\begin{bmatrix}i_{0\gamma}\\i_{0\delta}\end{bmatrix} + R_2\begin{bmatrix}i_{2\gamma}\\i_{2\delta}\end{bmatrix}$$

$$\begin{bmatrix}0\\0\end{bmatrix} = \frac{L_{2\alpha}}{R_2}\frac{d}{dt}\begin{bmatrix}i_{0\gamma}\\i_{0\delta}\end{bmatrix} + \omega_s\frac{L_{2\alpha}}{R_2}\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}\begin{bmatrix}i_{0\gamma}\\i_{0\delta}\end{bmatrix} + \frac{L_{2\alpha}}{M}\begin{bmatrix}i_{2\gamma}\\i_{2\delta}\end{bmatrix}$$



$$\vec{\boldsymbol{i}}_{\mathbf{1\gamma}} = \left(1 + \frac{L_{2\alpha}}{R_2}\frac{d}{dt}\right)\vec{\boldsymbol{i}}_{\mathbf{0}}$$

$$i_{0\delta} = 0$$

$$i_{0\gamma} = i_0$$

$$\frac{L_{2\alpha}}{M}\begin{bmatrix}i_{2\gamma}\\i_{2\delta}\end{bmatrix} + \frac{L_{2\alpha}}{R_2}\begin{bmatrix}\frac{d}{dt}\\\omega_s\end{bmatrix}i_0 = \begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix}i_{0\gamma}\\i_{0\delta}\end{bmatrix} = \begin{bmatrix}i_{1\gamma}\\i_{1\delta}\end{bmatrix} + \frac{L_{2\alpha}}{M}\begin{bmatrix}i_{2\gamma}\\i_{2\delta}\end{bmatrix}$$

$$i_{1\gamma} = \left(1 + \frac{L_{2\alpha}}{R_2}\frac{d}{dt}\right)i_0$$

$$i_{1\delta} = \omega_s\frac{L_{2\alpha}}{R_2}i_0$$

Torque representation

$$T = M\left(i_{1\delta} i_{2\gamma} - i_{1\gamma} i_{2\delta}\right)$$

$$i_{1\gamma} = \left(1 + \frac{L_{2\alpha}}{R_2}\frac{d}{dt}\right) i_0$$

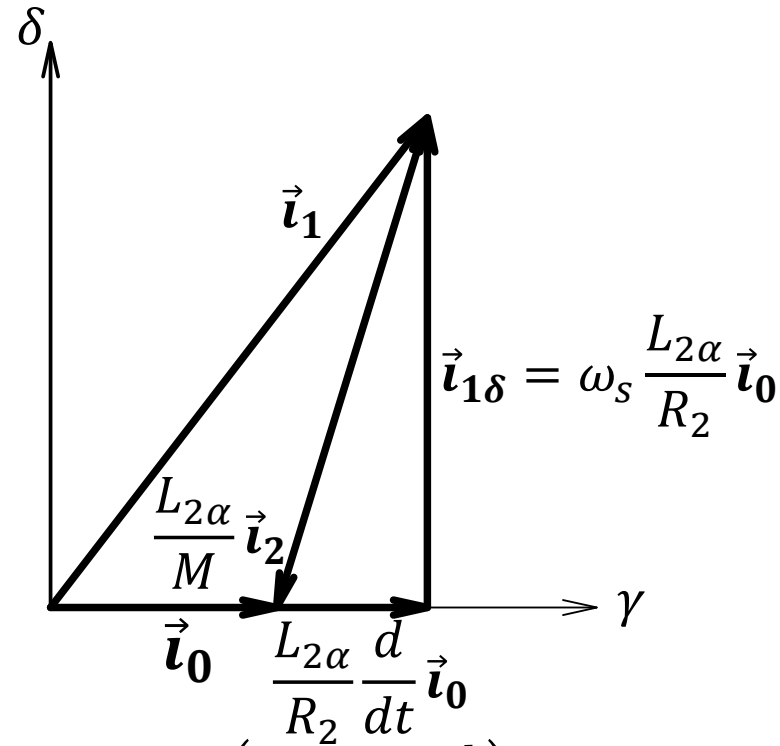
$$i_{1\delta} = \omega_s \frac{L_{2\alpha}}{R_2} i_0$$

$$\begin{bmatrix} i_0 \\ 0 \end{bmatrix} = \begin{bmatrix} i_{1\gamma} \\ i_{1\delta} \end{bmatrix} + \frac{L_{2\alpha}}{M}\begin{bmatrix} i_{2\gamma} \\ i_{2\delta} \end{bmatrix}$$

$$i_{2\gamma} = -\frac{M}{R_2}\frac{d}{dt} i_0$$

$$i_{2\delta} = -\omega_s \frac{L_{2\alpha}}{R_2} i_0 = -i_{1\delta}$$

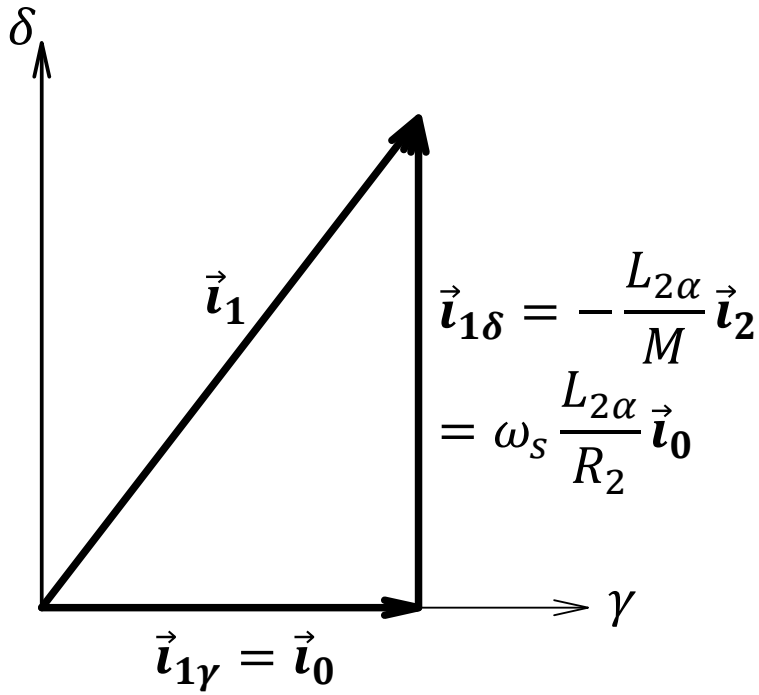
$$T = M\left(i_{1\delta} i_{2\gamma} - i_{1\gamma} i_{2\delta}\right)$$
$$= M\left(i_{1\delta} i_{2\gamma} + i_{1\gamma} i_{1\delta}\right)$$
$$= M i_{1\delta}\left(i_{2\gamma} + i_{1\gamma}\right)$$
$$= M i_{1\delta}\left(1 + \frac{L_{2\alpha} - M}{R_2}\frac{d}{dt}\right) i_0$$



$$\vec{\boldsymbol{i}}_{\mathbf{1\gamma}} = \left(1 + \frac{L_{2\alpha}}{R_2}\frac{d}{dt}\right)\vec{\boldsymbol{i}}_{\mathbf{0}}$$

Torque equation at a constant i_0

constant $\vec{\boldsymbol{i}}_{\mathbf{0}} \Rightarrow \dfrac{d}{dt}\vec{\boldsymbol{i}}_{\mathbf{0}}$

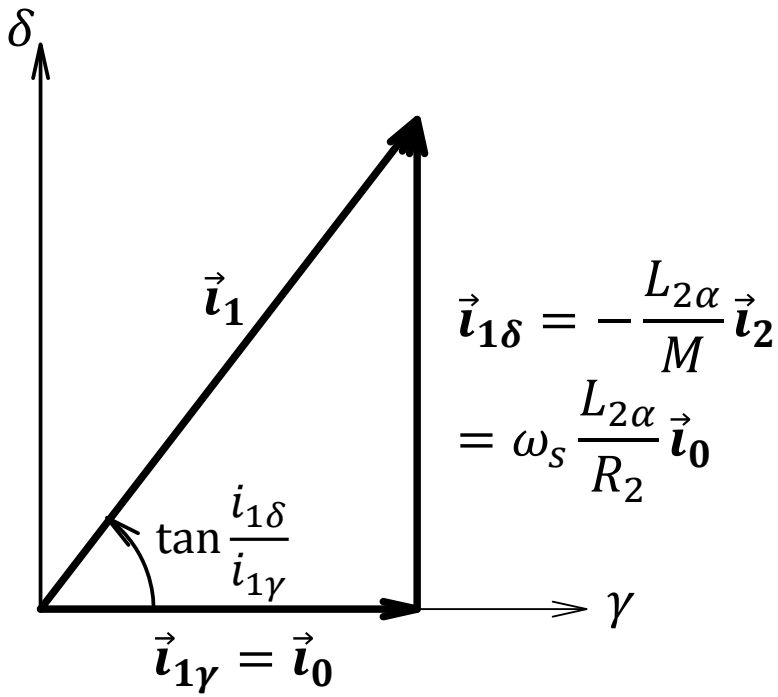


$$T = M\left(i_{1\delta} i_{2\gamma} - i_{1\gamma} i_{2\delta}\right)$$
$$= M i_{1\delta}\left(1 + \frac{L_{2\alpha} - M}{R_2}\frac{d}{dt}\right) i_0$$
$$= M i_0 i_{1\delta}$$
$$= \omega_s M \frac{L_{2\alpha}}{R_2} {i_0}^2$$

Control

Torque reference: T^*

Magnetization current reference: ${i_0}^*$



$$T = M i_0 i_{1\delta}$$

$${i_{1\delta}}^* = \frac{T^*}{M {i_0}^*}$$

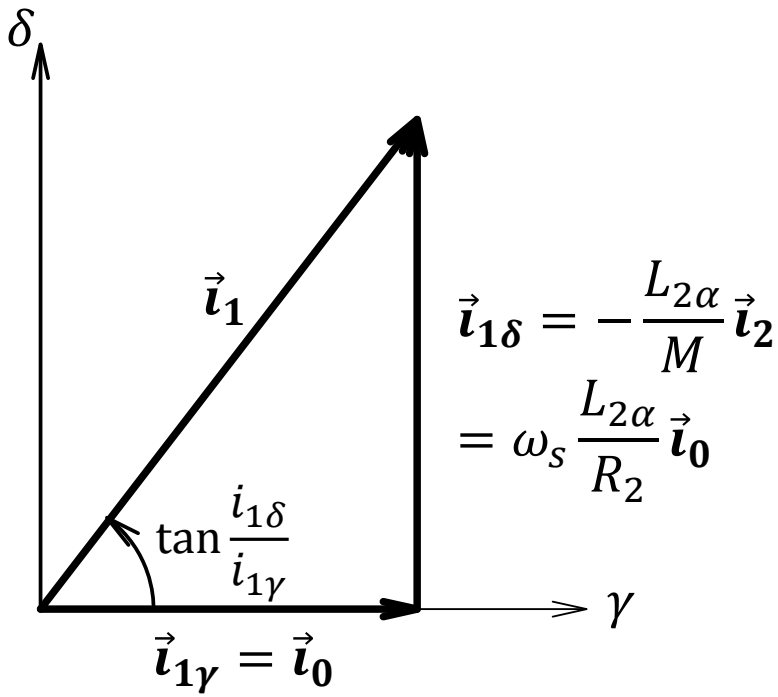
$${i_{1\gamma}}^* = {i_0}^*$$

$${I_1}^* = \sqrt{{i_{1\gamma}}^{*2} + {i_{1\delta}}^{*2}}$$

Control

Torque reference: T^*

Magnetization current reference: ${i_0}^*$



$$i_{1\delta} = \omega_s \frac{L_{2\alpha}}{R_2} i_0$$

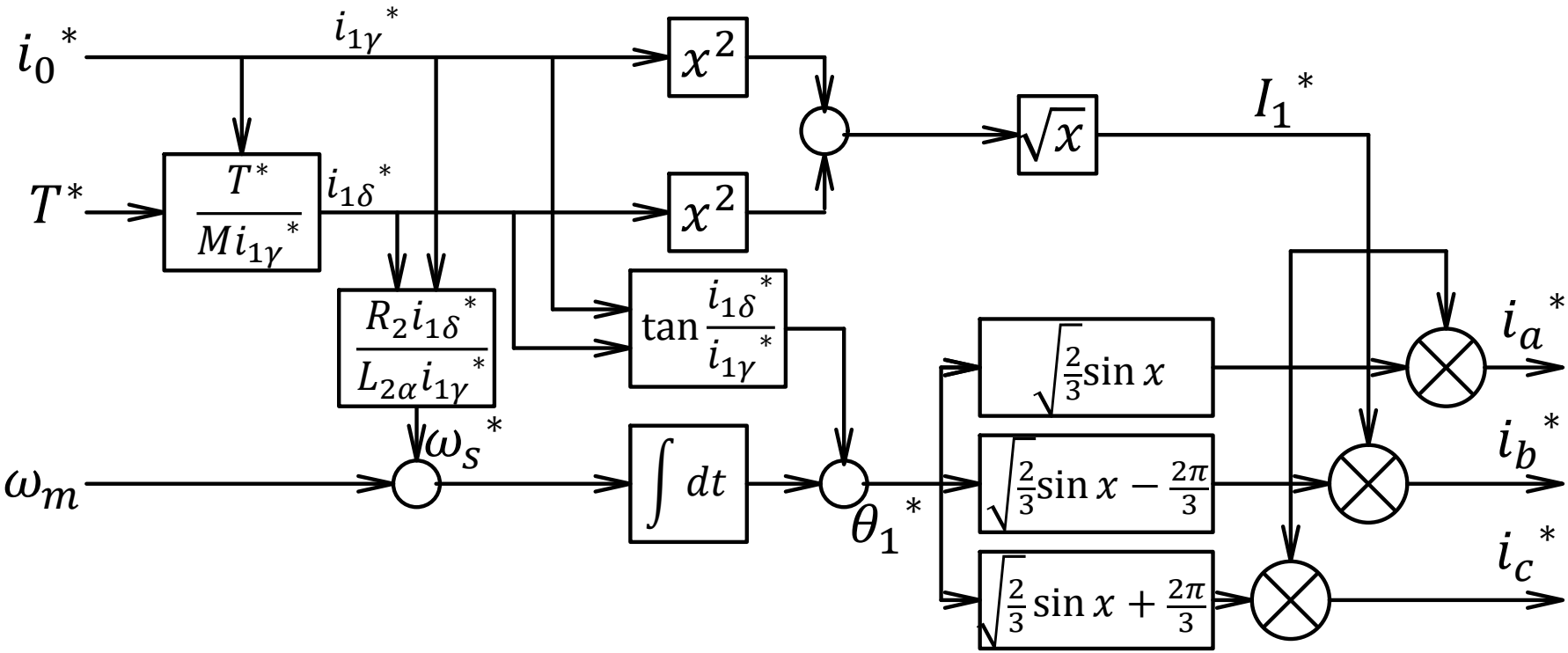
$${\omega_s}^* = \frac{R_2}{L_{2\alpha}{i_0}^*}{i_{1\delta}}^*$$

$$\omega_s = \omega - \omega_m$$

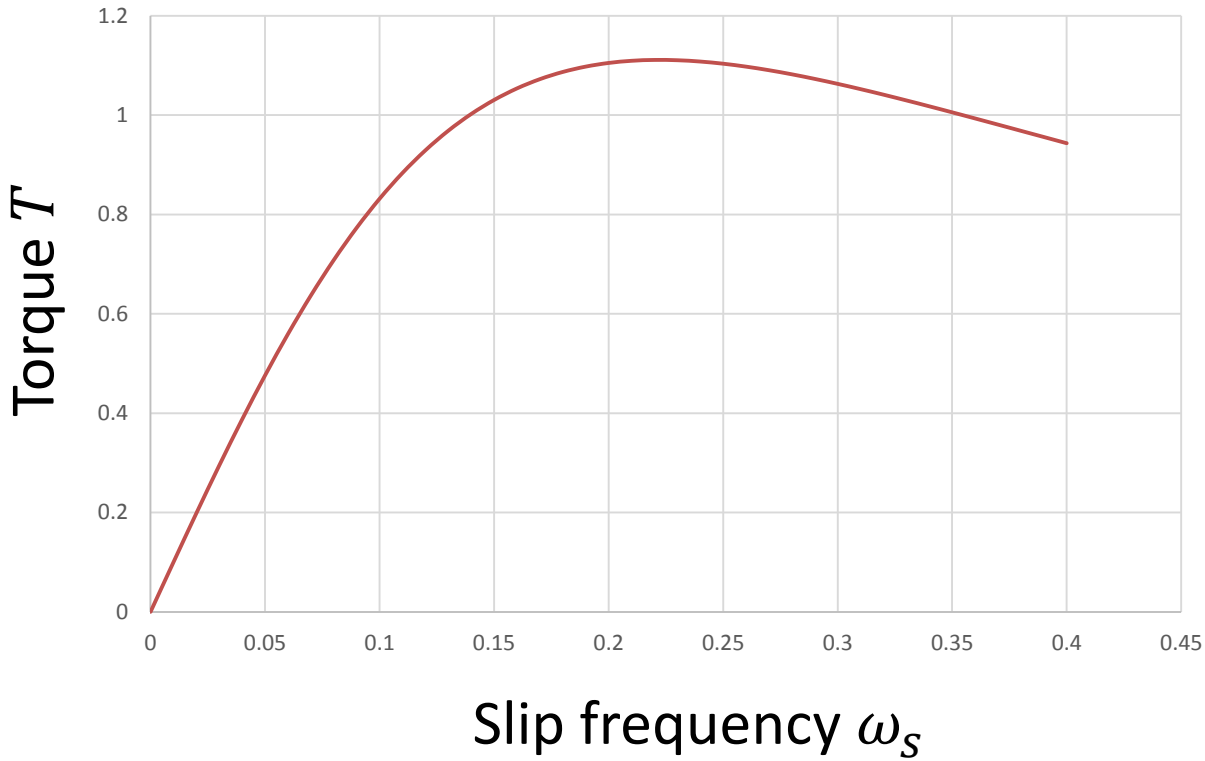
$$\theta^* = \int {\omega_s}^* + \omega_m \; dt$$

$${\theta_1}^* = \theta^* + \tan\frac{{i_{1\delta}}^*}{{i_{1\gamma}}^*}$$

Control block



Slip frequency control
Slip angular frequency ω_s vs Torque



- Torque proportional to the slip frequency
- Manipulate the slip frequency to control torque

Approximation

- Torque equation:

$$T = \frac{P_O}{\omega_m} = \frac{P_O}{\omega - \omega_s} = 3\frac{r_2{}'}{\omega_s}\frac{(E_1/\omega)^2}{\left(\frac{r_2{}'}{\omega_s}\right)^2 + \ell_2{}'^2}$$

- Linearization of torque:

$$\omega_s \ll \omega_{s\max} = \frac{r_2{}'}{\ell_2{}'}$$

$$T \approx 3\frac{(E_1/\omega)^2}{r_2{}'}\omega_s$$

- Secondary current linearlization:

$$I_2' = \frac{E_1/\omega}{\sqrt{\left(\frac{r_2{}'}{\omega_s}\right)^2 + \ell_2{}'^2}} \approx \frac{E_1/\omega}{r_2{}'}\omega_s$$

Slip frequency control

- Slip frequency reference:

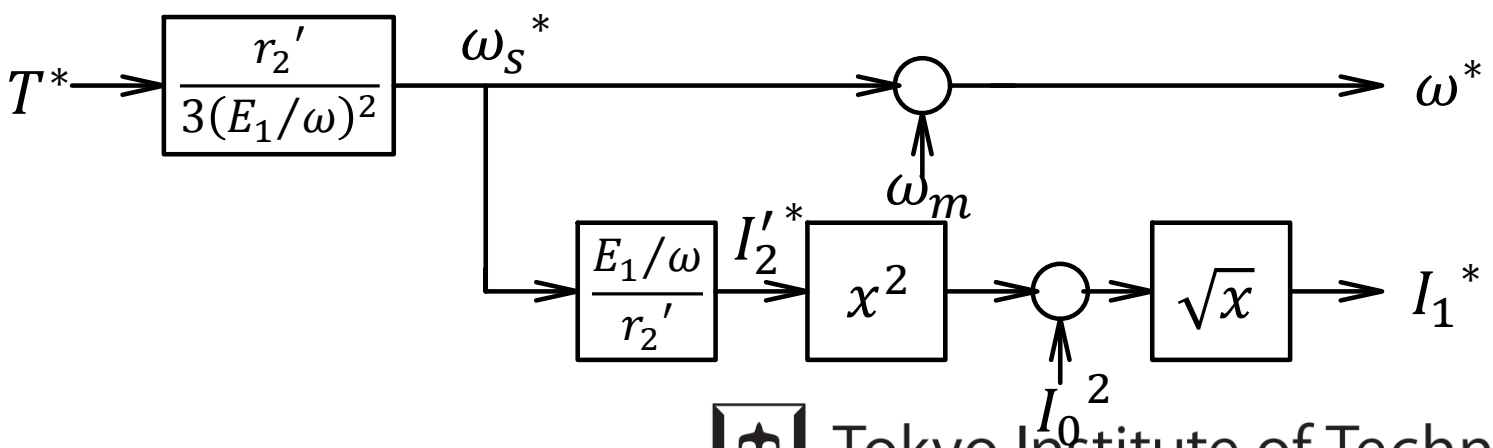
$$\omega_s{}^* = \frac{r_2{}'}{3(E_1/\omega)^2} T^*$$

- Secondary current reference:

$$I_2'{}^* = \frac{E_1/\omega}{r_2{}'} \omega_s$$

- Primary current reference:

$$I_1{}^* = \sqrt{I_0{}^2 + I_2'{}^{*2}}$$



Today’s Topic for Discussion

- What the difference between the traditional slip frequency control and field oriented torque control?