Electric Power and Motor Drive System Analysis

12. Dynamic control of synchronous machines

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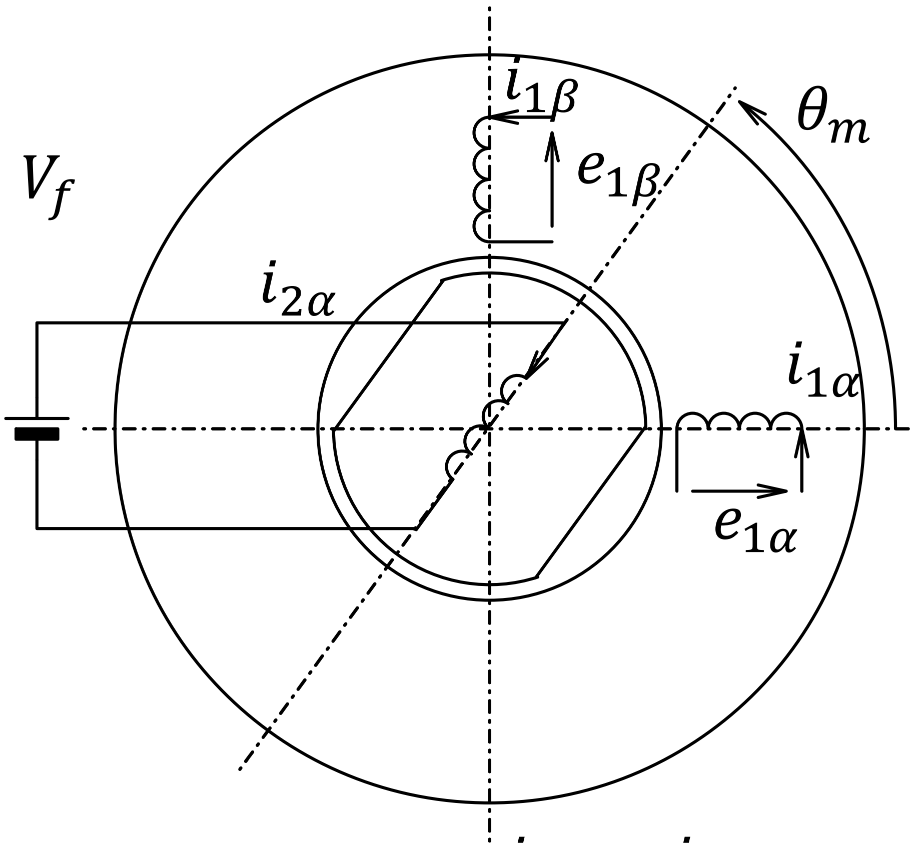
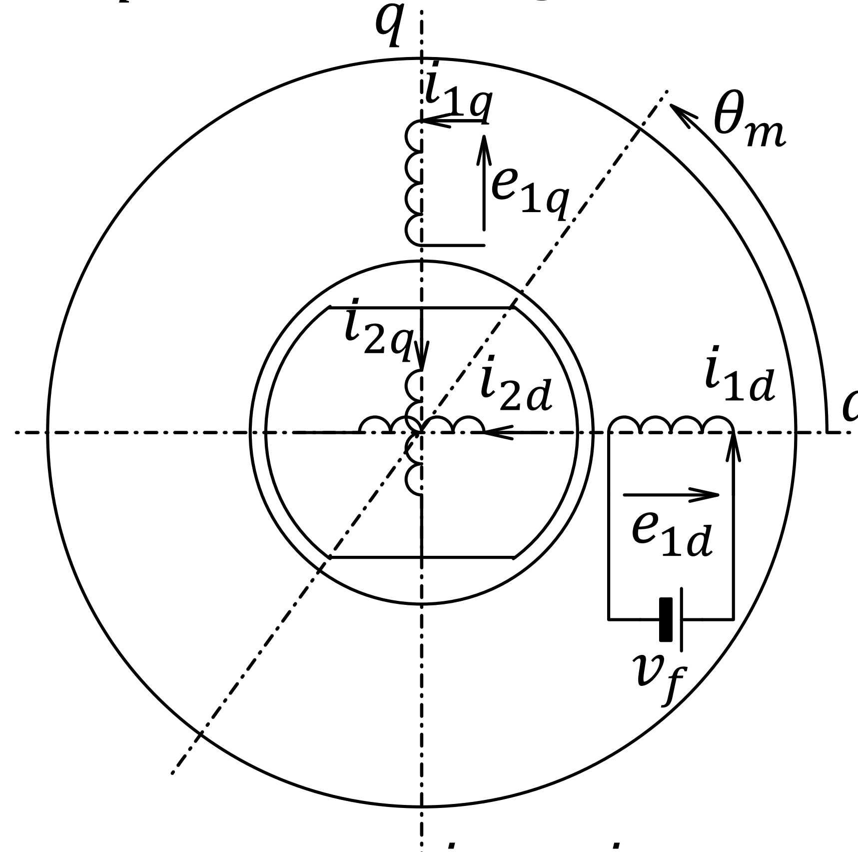
Synchronous machines

α-β model (rotating field winding)



$$v_{2\alpha} = v_f,\ i_{2\alpha} = i_f$$
$$i_{2\beta} = 0$$
$$\theta_m = \omega_m t - \frac{\pi}{2} + \delta$$

d-q model (rotating armature)



$$v_{1d} = v_f,\ i_{2d} = i_f$$
$$i_{1q} = 0$$
$$\theta_m = -\left(\omega_m t - \frac{\pi}{2} + \delta\right)$$

Voltage and current equation

α-β model (rotating field winding)

$$\begin{bmatrix} v_{1\alpha} \\ v_{1\beta} \\ v_f \end{bmatrix} = \begin{bmatrix} R_1 + L_{1\alpha}\dfrac{d}{dt} & 0 & \begin{matrix} M\cos\theta_m \dfrac{d}{dt} \\ -M\omega_m \sin\theta_m \end{matrix} \\ 0 & R_1 + L_{1\alpha}\dfrac{d}{dt} & \begin{matrix} M\sin\theta_m \dfrac{d}{dt} \\ +M\omega_m \cos\theta_m \end{matrix} \\ \begin{matrix} M\cos\theta_m \dfrac{d}{dt} \\ -M\omega_m \sin\theta_m \end{matrix} & \begin{matrix} M\sin\theta_m \dfrac{d}{dt} \\ +M\omega_m \cos\theta_m \end{matrix} & R_2 + L_{2\alpha}\dfrac{d}{dt} \end{bmatrix} \begin{bmatrix} i_{1\alpha} \\ i_{1\beta} \\ i_f \end{bmatrix}$$

Assuming a cylindrical rotor without saliency

Voltage and current equation

$$\begin{bmatrix} v_{1d} \\ v_{1q} \\ v_{2d} \\ v_{2q} \end{bmatrix} = \begin{bmatrix} R_1 + L_{1d}\frac{d}{dt} & 0 & M_d\frac{d}{dt} & 0 \\ 0 & R_1 + L_{1q}\frac{d}{dt} & 0 & M_q\frac{d}{dt} \\ M_d\frac{d}{dt} & M_q\frac{d\theta_m}{dt} & R_2 + L_{2d}\frac{d}{dt} & L_{2q}\frac{d\theta_m}{dt} \\ -M_d\frac{d\theta_m}{dt} & M_q\frac{d}{dt} & -L_{2d}\frac{d\theta_m}{dt} & R_2 + L_{2q}\frac{d}{dt} \end{bmatrix} \begin{bmatrix} i_{1d} \\ i_{1q} \\ i_{2d} \\ i_{2q} \end{bmatrix}$$

$$\begin{bmatrix} v_f \\ v_{2d} \\ v_{2q} \end{bmatrix} = \begin{bmatrix} R_1 + L_{1d}\frac{d}{dt} & M_d\frac{d}{dt} & 0 \\ M_d\frac{d}{dt} & R_2 + L_{2d}\frac{d}{dt} & L_{2q}\frac{d\theta_m}{dt} \\ -M_d\frac{d\theta_m}{dt} & -L_{2q}\frac{d\theta_m}{dt} & R_2 + L_{2q}\frac{d}{dt} \end{bmatrix} \begin{bmatrix} i_f \\ i_{2d} \\ i_{2q} \end{bmatrix}$$

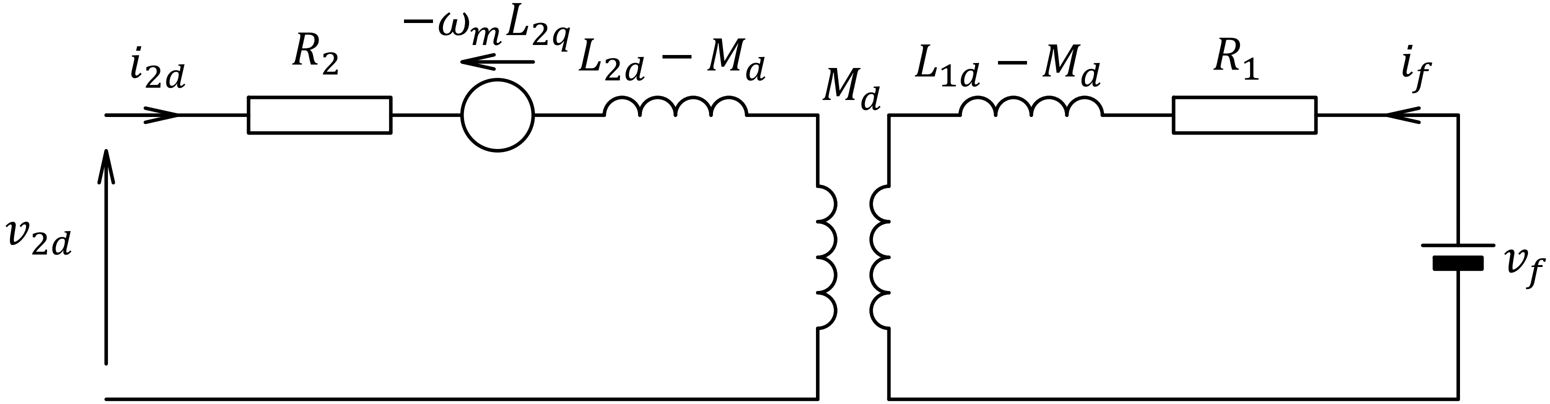
Voltage and current equation

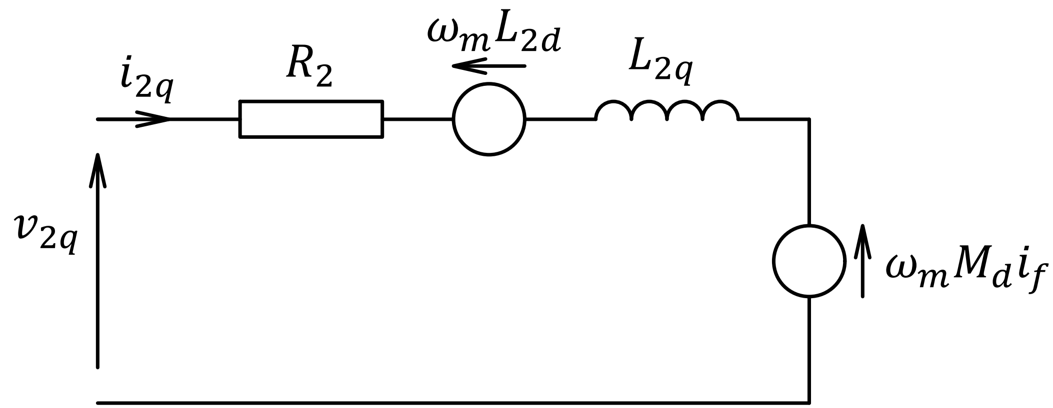
$$\begin{bmatrix} v_f \\ v_{2d} \\ v_{2q} \end{bmatrix} = \begin{bmatrix} R_1 + L_{1d}\frac{d}{dt} & M_d\frac{d}{dt} & 0 \\ M_d\frac{d}{dt} & R_2 + L_{2d}\frac{d}{dt} & L_{2q}\frac{d\theta_m}{dt} \\ -M_d\frac{d\theta_m}{dt} & -L_{2q}\frac{d\theta_m}{dt} & R_2 + L_{2q}\frac{d}{dt} \end{bmatrix} \begin{bmatrix} i_f \\ i_{2d} \\ i_{2q} \end{bmatrix}$$

$$\frac{d\theta_m}{dt} = -\omega_m$$

$$\begin{bmatrix} v_f \\ v_{2d} \\ v_{2q} \end{bmatrix} = \begin{bmatrix} R_1 + L_{1d}\frac{d}{dt} & M_d\frac{d}{dt} & 0 \\ M_d\frac{d}{dt} & R_2 + L_{2d}\frac{d}{dt} & -\omega_m L_{2q} \\ \omega_m M_d & \omega_m L_{2d} & R_2 + L_{2q}\frac{d}{dt} \end{bmatrix} \begin{bmatrix} i_f \\ i_{2d} \\ i_{2q} \end{bmatrix}$$

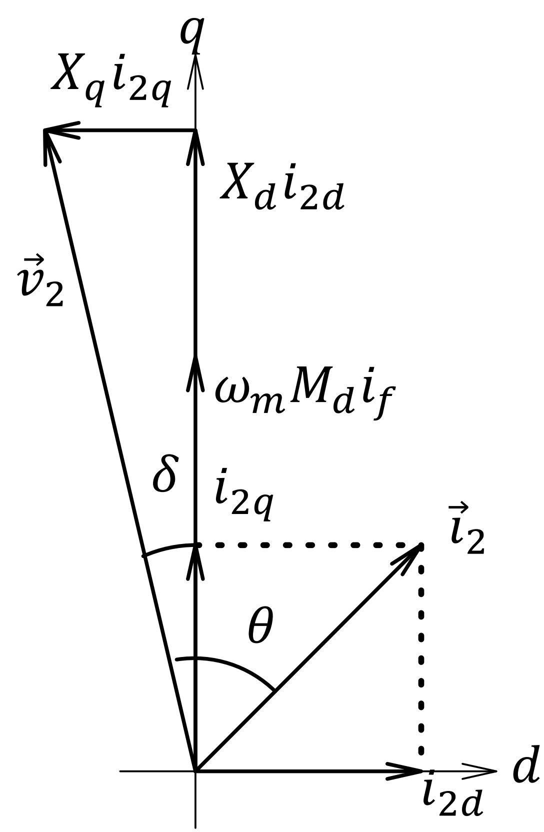
$$
\begin{bmatrix} v_f \\ v_{2d} \\ v_{2q} \end{bmatrix} = \begin{bmatrix} R_1 + L_{1d}\dfrac{d}{dt} & M_d\dfrac{d}{dt} & 0 \\ M_d\dfrac{d}{dt} & R_2 + L_{2d}\dfrac{d}{dt} & -\omega_m L_{2q} \\ \omega_m M_d & \omega_m L_{2d} & R_2 + L_{2q}\dfrac{d}{dt} \end{bmatrix} \begin{bmatrix} i_f \\ i_{2d} \\ i_{2q} \end{bmatrix}
$$





Vector diagram representation

$$\begin{bmatrix} v_f \\ v_{2d} \\ v_{2q} \end{bmatrix} = \begin{bmatrix} R_1 + L_{1d}\frac{d}{dt} & M_d\frac{d}{dt} & 0 \\ M_d\frac{d}{dt} & R_2 + L_{2d}\frac{d}{dt} & -\omega_m L_{2q} \\ \omega_m M_d & \omega_m L_{2d} & R_2 + L_{2q}\frac{d}{dt} \end{bmatrix} \begin{bmatrix} i_f \\ i_{2d} \\ i_{2q} \end{bmatrix}$$



Power and Torque

$$p = \begin{bmatrix} i_f \\ i_{2d} \\ i_{2q} \end{bmatrix}^T \begin{bmatrix} v_f \\ v_{2d} \\ v_{2q} \end{bmatrix} = \begin{bmatrix} i_f \\ i_{2d} \\ i_{2q} \end{bmatrix}^T \begin{bmatrix} R_1 + L_{1d}\frac{d}{dt} & M_d \frac{d}{dt} & 0 \\ M_d \frac{d}{dt} & R_2 + L_{2d}\frac{d}{dt} & -\omega_m L_{2q} \\ \omega_m M_d & \omega_m L_{2d} & R_2 + L_{2q}\frac{d}{dt} \end{bmatrix} \begin{bmatrix} i_f \\ i_{2d} \\ i_{2q} \end{bmatrix}$$

$= R_1 {i_f}^2 + R_2 {i_{2d}}^2 + R_2 {i_{2q}}^2$ **resistance loss**

$+i_f L_{1d} \frac{di_f}{dt} + i_{2d} L_{2d} \frac{di_{2d}}{dt} + i_{2q} L_{2q} \frac{di_{2q}}{dt} + i_f M_d \frac{di_{2d}}{dt} + i_{2d} M_d \frac{di_f}{dt}$ **inductance**

$+i_{2q}\omega_m M_d i_f + i_{2q}\omega_m L_{2d} i_{2d} - i_{2d}\omega_m L_{2q} i_{2q}$ **mechanical output**

Instantaneous Power:

$$p_o = \omega_m M_d i_f i_{2q} + \omega_m (L_{2d} - L_{2q}) i_{2d} i_{2q}$$

Instantaneous Torque:

$$T = \frac{p_o}{\omega_m} = M_d i_f i_{2q} + (L_{2d} - L_{2q}) i_{2d} i_{2q}$$

Steady State Analysis for a constant voltage constant frequency supply:

$$v_f = V_f,\ v_{2d} = -V_2 \sin\delta,\ v_{2q} = V_2 \cos\delta,\ \omega_m = \omega$$

$$V_f = R_1 i_f$$

$$v_{2d} = R_2 i_{2d} - \omega L_{2q} i_{2q} = R_2 i_{2d} - X_q i_{2q}$$

$$v_{2q} = \omega M_d i_f + \omega L_{2d} i_{2d} + R_2 i_{2q} = E_0 + X_d i_{2d} + R_2 i_{2q}$$

$$i_{2d} = \frac{X_q\left(v_{2q} - E_0\right) + R_2 v_{2d}}{{R_2}^2 + X_d X_q} \approx -\frac{E_0 - V_2 \cos\delta}{X_d}$$

$$i_{2q} = -\frac{R_2\left(E_0 - v_{2q}\right) + X_d v_{2d}}{{R_2}^2 + X_d X_q} \approx \frac{V_2 \sin\delta}{X_q}$$

$$p_o = E_0 i_{2q} + \left(L_{2d} - L_{2q}\right) i_{2d} i_{2q}$$

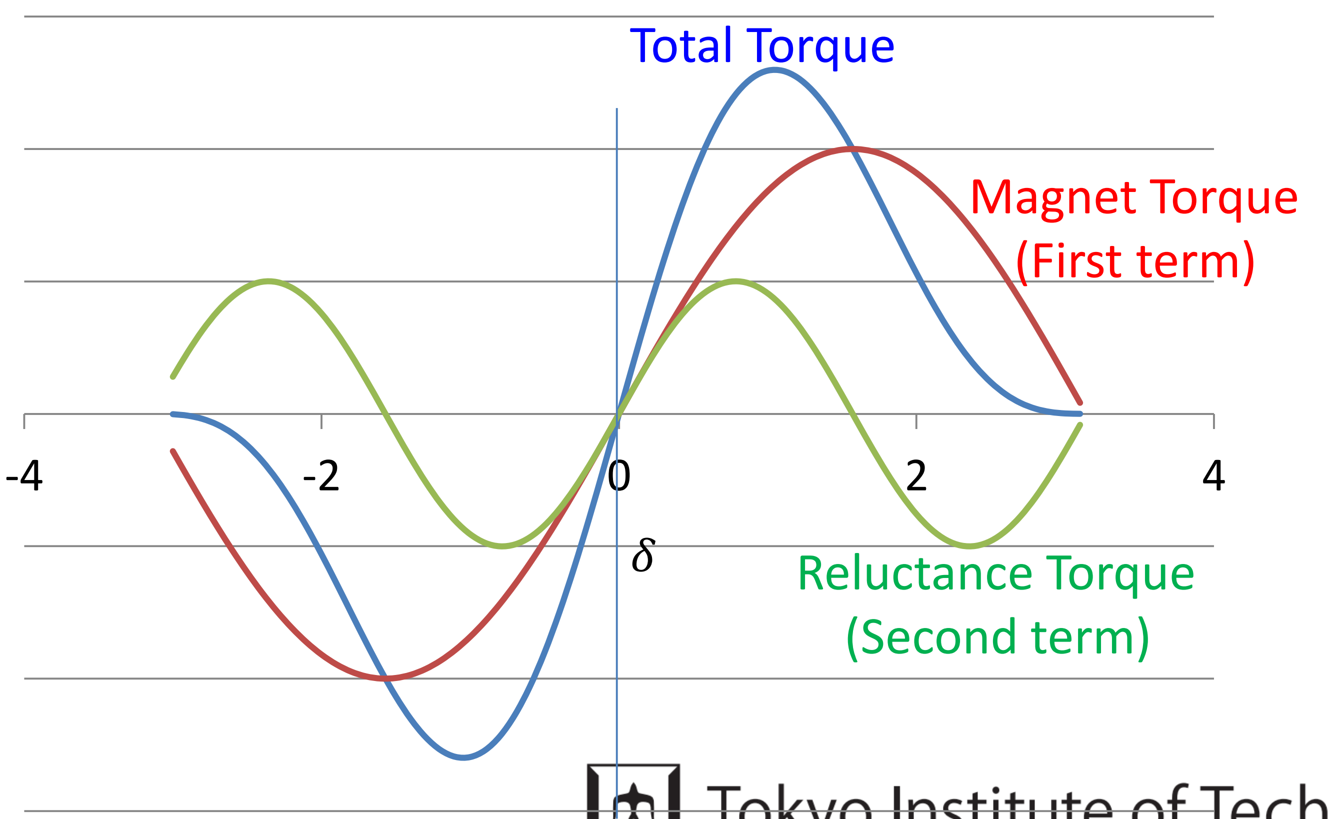
$$= \frac{E_0 V_2 \sin\delta}{X_q} + \left(X_d - X_q\right) \frac{-E_0 + V_2 \cos\delta}{X_d} \frac{V_2 \sin\delta}{X_q}$$

$$= \frac{E_0 V_2 \sin\delta}{X_d} + \left(\frac{1}{X_q} - \frac{1}{X_d}\right) \frac{{V_2}^2 \sin 2\delta}{X_q}$$

Torque in case of CVCF

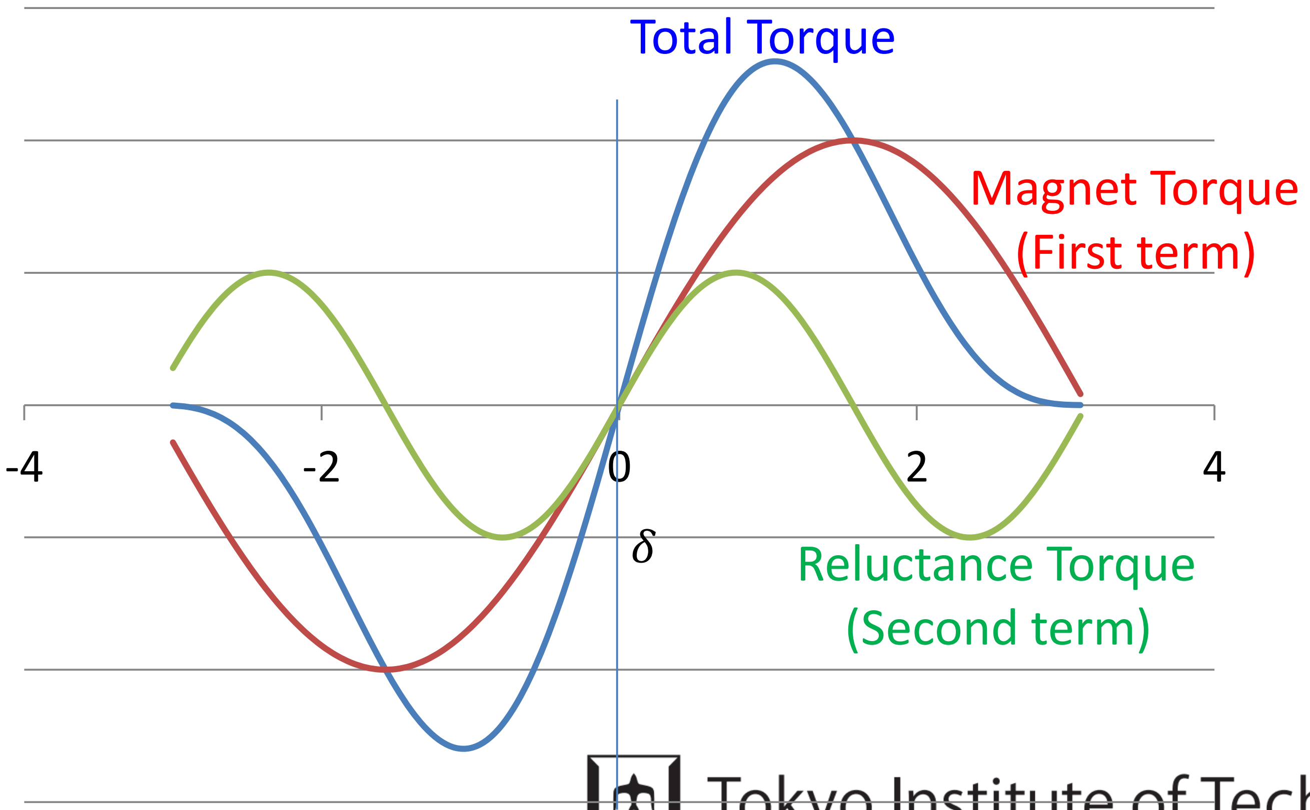
$$p_o = \frac{E_0 V_2 \sin\delta}{X_d} + \left(\frac{1}{X_q} - \frac{1}{X_d}\right)\frac{{V_2}^2 \sin 2\delta}{X_q}$$

$$T = \frac{1}{\omega}\left[\frac{E_0 V_2 \sin\delta}{X_d} + \left(\frac{1}{X_q} - \frac{1}{X_d}\right)\frac{{V_2}^2 \sin 2\delta}{X_q}\right]$$



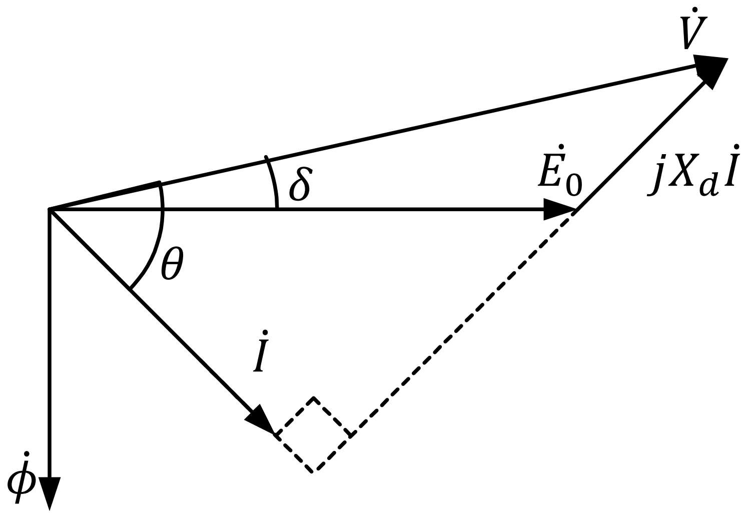
Torque by a current-control inverter

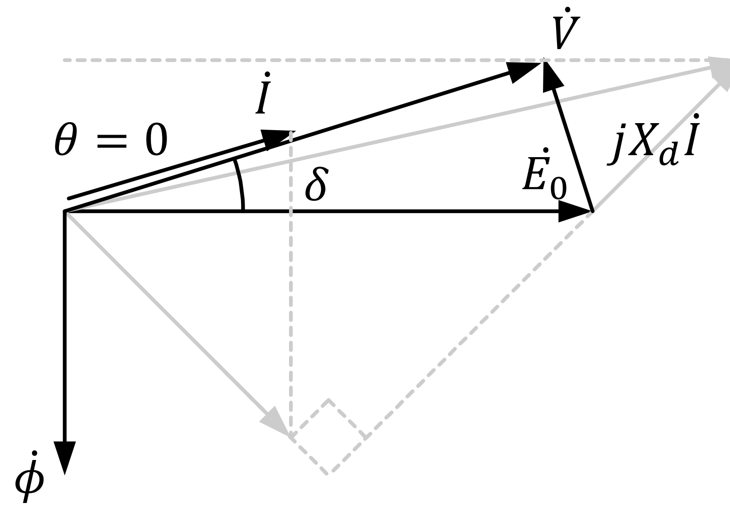
$$i_{2d} = \sqrt{3} I_2 \cos\beta$$
$$i_{2q} = \sqrt{3} I_2 \sin\beta$$
$$T = M_d i_f i_{2q} + \left(L_{2d} - L_{2q}\right) i_{2d} i_{2q}$$
$$T = \sqrt{3} M_d i_f I_2 \sin\beta + \frac{3}{2}\left(L_{2d} - L_{2q}\right) {I_2}^2 \sin 2\beta$$



Unity power factor control

$$X_d = X_q$$





- Unity power factor： $\cos\theta = 1$
- Direction of voltage and current vector
- Torque： $T = \dfrac{VE_0 \sin\delta}{\omega X_d}$

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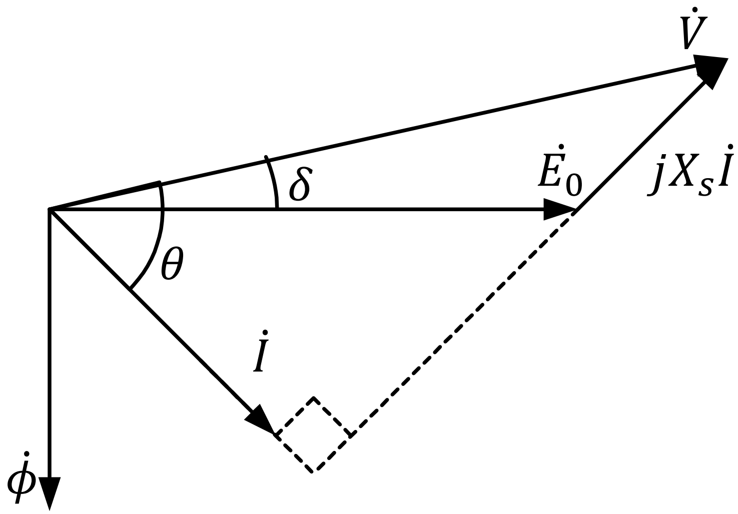
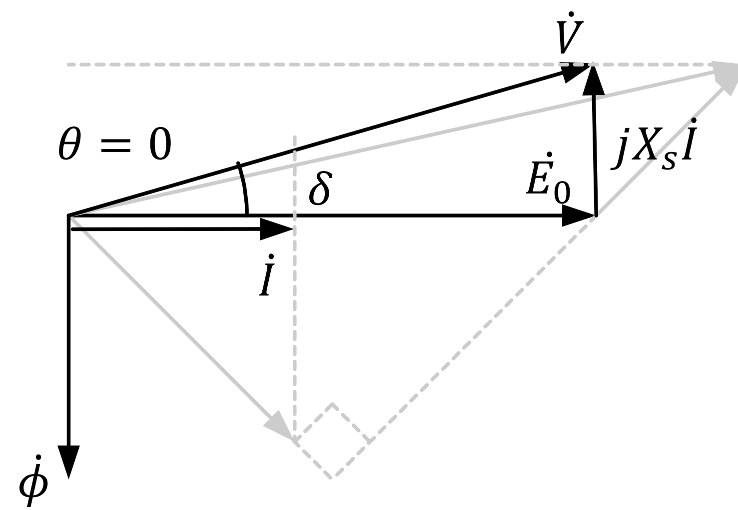
Maximum torque control

$$X_d = X_q$$





- Minimizing the current at a specific power
- Make the current phase angle $\beta = \frac{\pi}{2}$
- Power factor： $\cos\theta \neq 1$

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Maximum torque control

$$T = \sqrt{3} M_d i_f I_2 \sin\beta + \frac{3}{2}\left(L_{2d} - L_{2q}\right) {I_2}^2 \sin 2\beta$$

$$\frac{\partial T}{\partial \beta} = \sqrt{3} M_d i_f I_2 \cos\beta + 3\left(L_{2d} - L_{2q}\right) {I_2}^2 \cos 2\beta$$

Requirement:

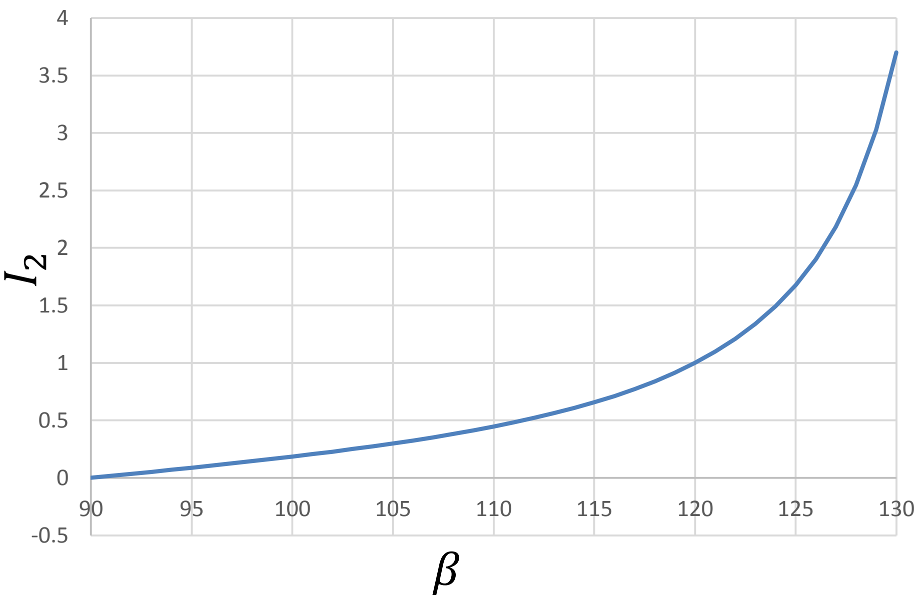
$$\sqrt{3} M_d i_f I_2 \cos\beta + 3\left(L_{2d} - L_{2q}\right) {I_2}^2 \cos 2\beta = 0$$

$$\sqrt{3} M_d i_f \cos\beta + 3\left(L_{2d} - L_{2q}\right) I_2 \cos 2\beta = 0$$

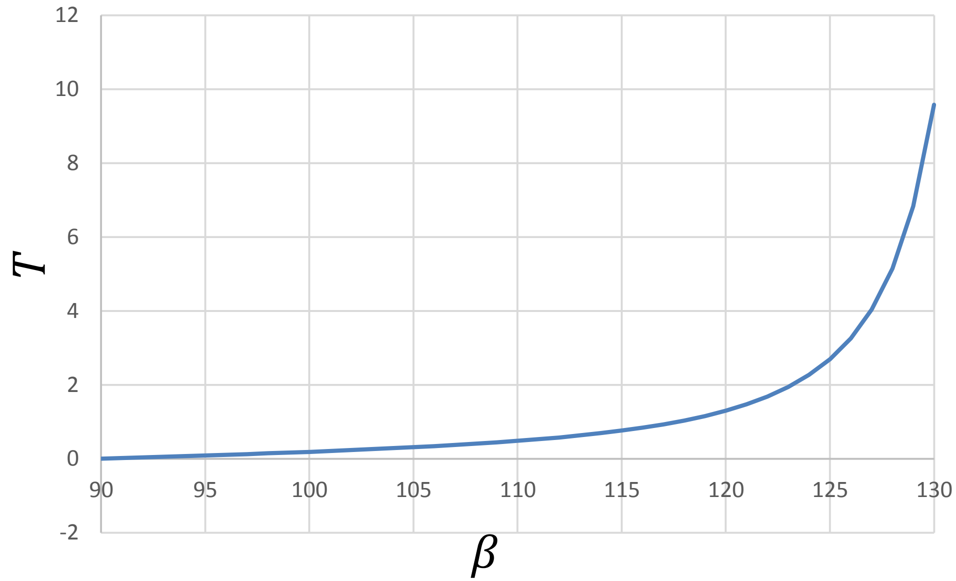
$$I_2 = -\frac{M_d i_f \cos\beta}{\sqrt{3}\left(L_{2d} - L_{2q}\right) \cos 2\beta}$$

Example

Current amplitude and phase angle



Current phase angle β and torque T



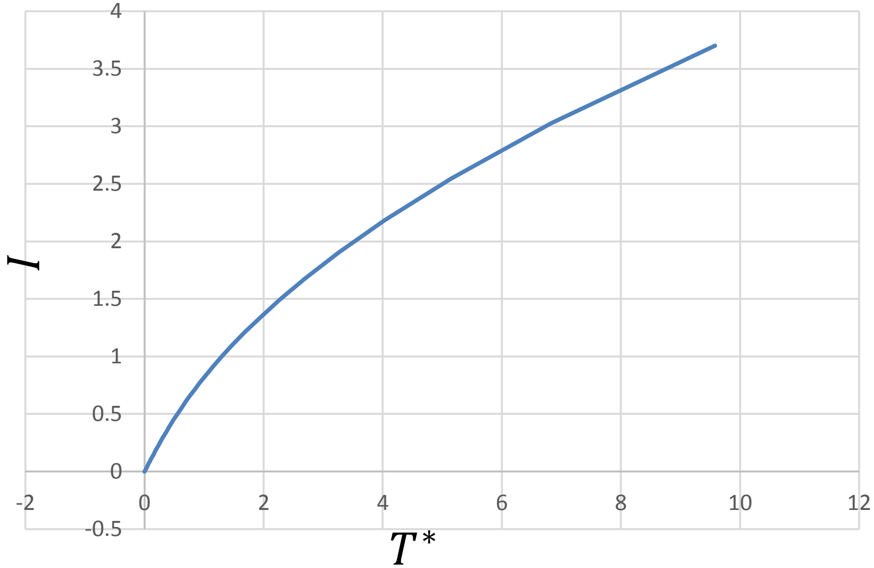
$$I = -\frac{\Psi \cos\beta}{\sqrt{3}\left(L_d - L_q\right)\cos 2\beta}$$

$$T = \sqrt{3} M_d i_f I_2 \sin\beta + \frac{3}{2}\left(L_{2d} - L_{2q}\right) {I_2}^2 \sin 2\beta$$

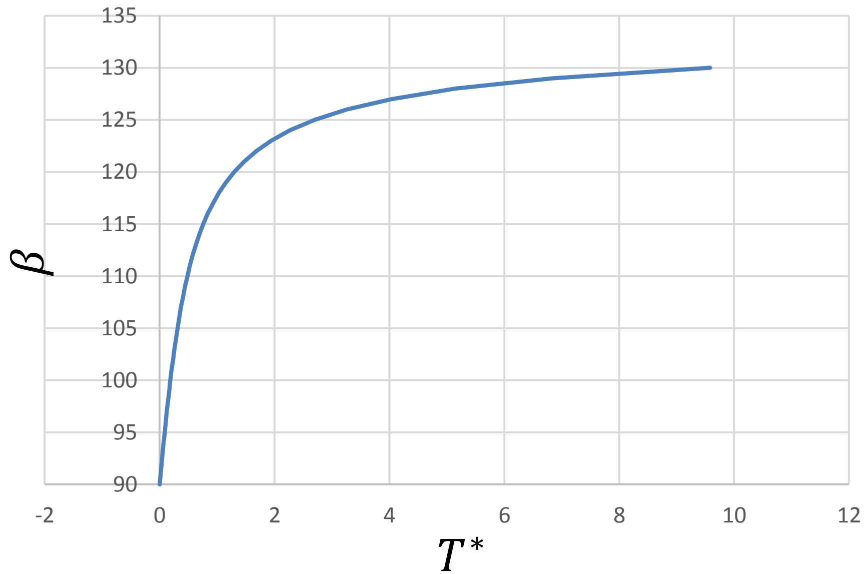
- Maximum torque appears at a phase angle range $\beta = \frac{\pi}{2} \sim \frac{3\pi}{4}$
- These characteristics are nonlinear and difficult to be solved.

Maximum torque control

Torque reference and required rms current



Torque reference and phase angle β



Inverse function of $I = -\frac{\Psi \cos\beta}{\sqrt{3}(L_d - L_q)\cos 2\beta}$

and $T = \sqrt{3} M_d i_f I_2 \sin\beta + \frac{3}{2}(L_{2d} - L_{2q}) {I_2}^2 \sin 2\beta$

Look-up table is applicable to obtain the above results.

Today's Topic for Discussion

- When should we use the unity-power factor and maximum torque control method?
- What is the difference between them?