2016 2Q

Wireless Communication Engineering

#6 Channel Fading and Diversity Combining

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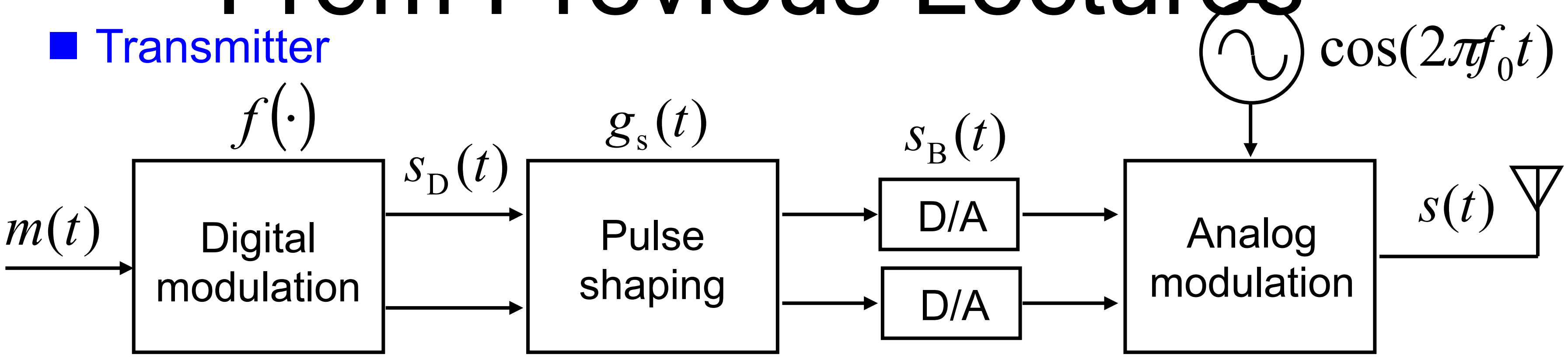
Course Schedule (1)

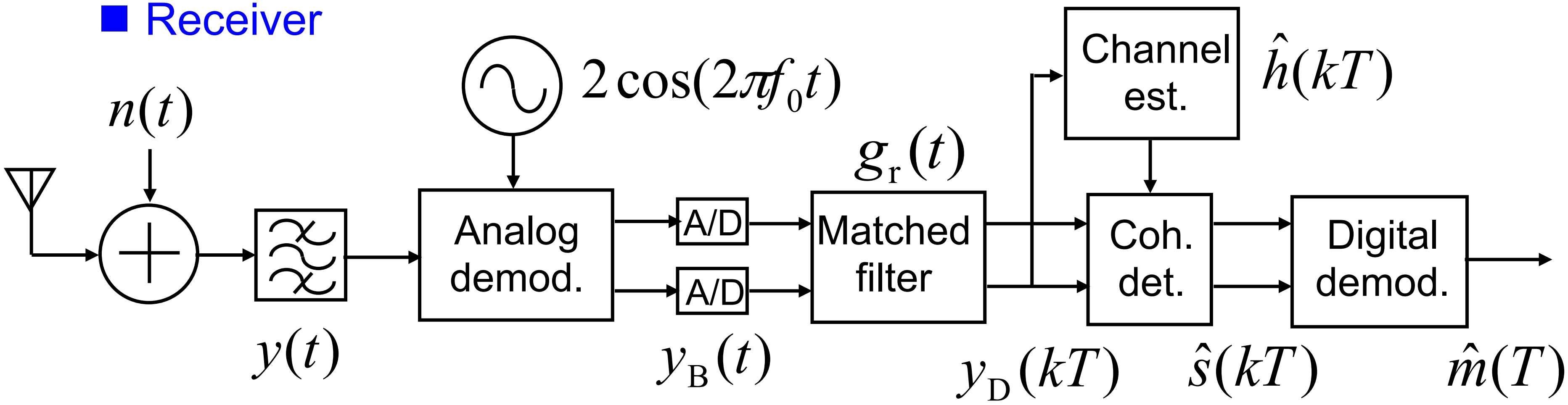
	Date	Text	Contents
#1	June 17	1, 7	Introduction to wireless communication systems
#2	June 17	2, 5, etc	Link budget design of wireless access
#3	June 24		Up/down conversion and equivalent baseband system
#4	June 24	3.3, 3.4	Digital modulation and pulse shaping
	July 1		No class
#5	July 8	3.5	Demodulation and detection error due to noise
#6	July 8	4.4	Channel fading and diversity combining

From Previous Lectures

- Transmitter



- Receiver



- Error rate of BPSK signal

$$P_{eb} = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{P|h|^2}{\sigma^2}}\right) = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\gamma}\right) \qquad \gamma = \frac{P|h|^2}{\sigma^2}$$

Contents

- Narrow band system
- Gain of propagation channel
- Rayleigh fading & probability distribution
- Error rate in fading channel
- Diversity technologies
- Maximum ratio combining diversity

Narrow Band System

Time invariant narrow band system

$$y_{\mathrm{B}}(t) = \int h_{\mathrm{B}}(\tau)\tilde{s}_{\mathrm{B}}(t-\tau)\mathrm{d}\tau = h_{\mathrm{B}} s_{\mathrm{B}}(t)$$

$$h_{\mathrm{B}} = h_{\mathrm{B}}(\tau_0) = h(\tau_0) e^{-j2\pi f_0 \tau_0}$$

Time variant narrow band system

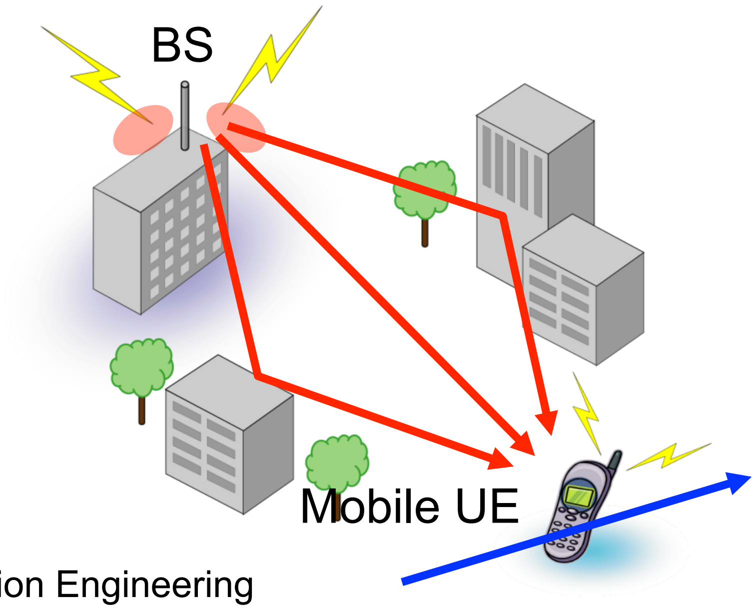
$$h_{\mathrm{B}} \rightarrow h_{\mathrm{B}}(d, \phi_{\mathrm{s}}, \phi_{\mathrm{r}}, t)$$

d : Distance between Tx & Rx

ϕ : Tx & Rx antenna angle

t : Mobility of UE

Mobile communication



Gain of Propagation Channel

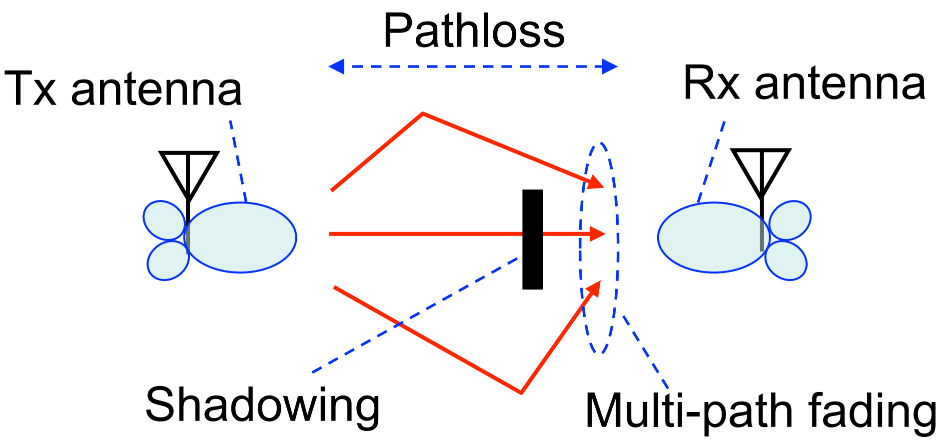
Gain of narrow band propagation channel

$$G_{\mathrm{prop}} = \left|h_{\mathrm{B}}\left(d, \phi_{\mathrm{s}}, \phi_{\mathrm{r}}, t\right)\right|^2$$

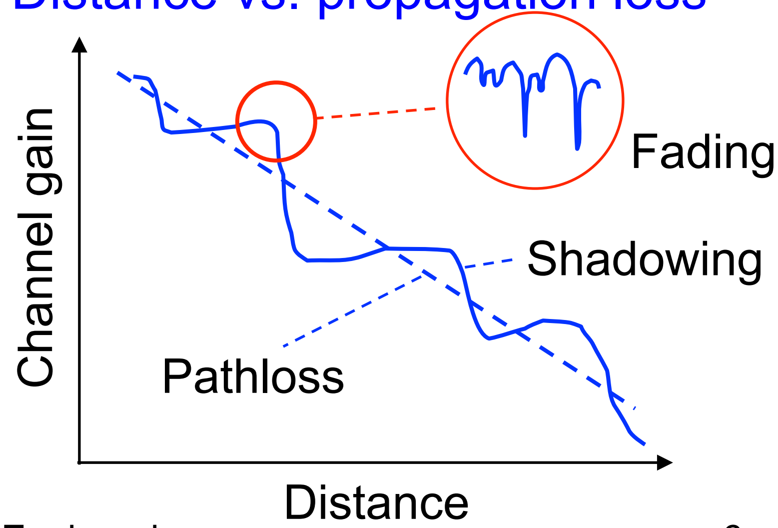
$$= G_{\mathrm{rant}}\left(\phi_{\mathrm{r}}\right) G_{\mathrm{fading}}(t) G_{\mathrm{shadow}}(t) G_{\mathrm{pl}}(d) G_{\mathrm{sant}}\left(\phi_{\mathrm{s}}\right)$$

Rx antenna gain, Multi-path fading, Shadowing, Pathloss, Tx antenna gain

Narrow band propagation channel



Distance vs. propagation loss



Standing Wave & Fading

Multi-path propagation channel

$$h_{\mathrm{B}}(t) = \sum_i h_{\mathrm{B}i}(t)$$

$$h_{\mathrm{B}i}(t) = h_{\mathrm{pl}}(d)e^{j2\pi f_0 \tau_i(t)}$$

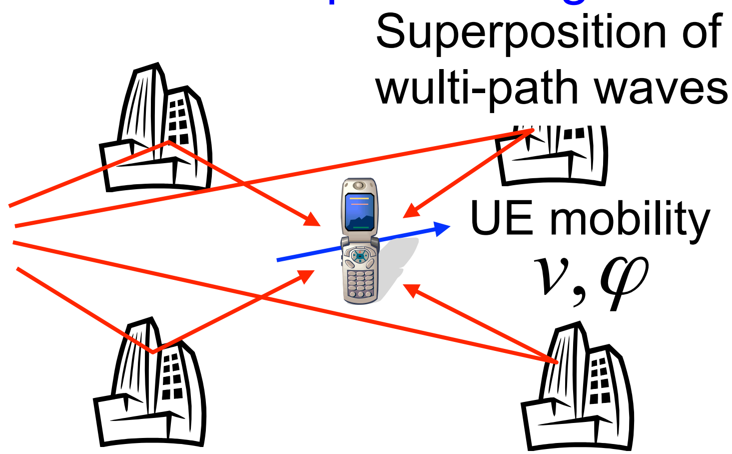
Doppler shift (phase shift due to mobility)

$$\tau_i(t) = \tau_0 + \frac{vt\cos\varphi_i}{c}$$

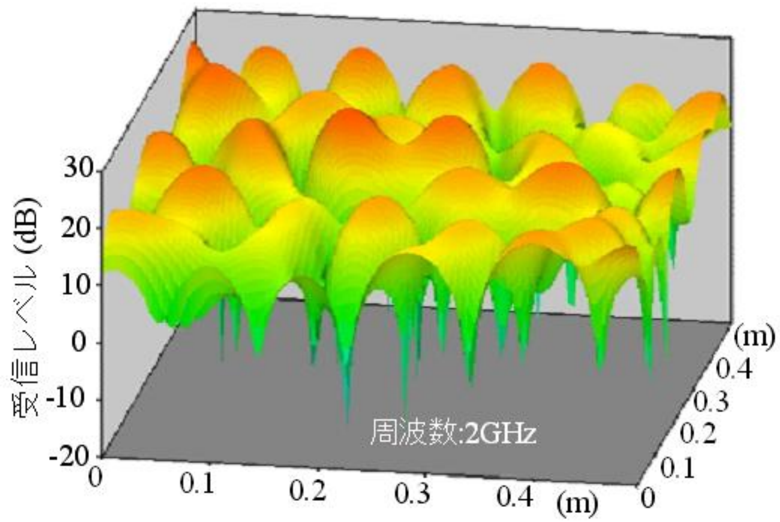
$$h_{\mathrm{B}i}(t) = h_{\mathrm{pl}}(d)e^{j\theta_0 + 2\pi f_{\mathrm{d}i} t}$$

$$f_{\mathrm{d}i} = \frac{v\cos\theta_i}{\lambda_0}$$

Multi-path fading



Standing wave



Rayleigh Fading

Multi-path channel

$$h_B(t) = \sum_i h_{pl}(d) e^{j\theta_i(t)} = x + jy$$

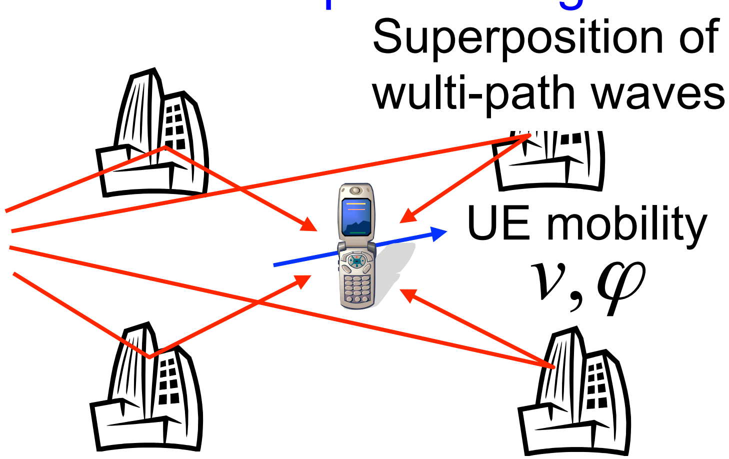
Central limit theorem

Sum of independent random variables

↓

Gaussian distribution

Multi-path fading

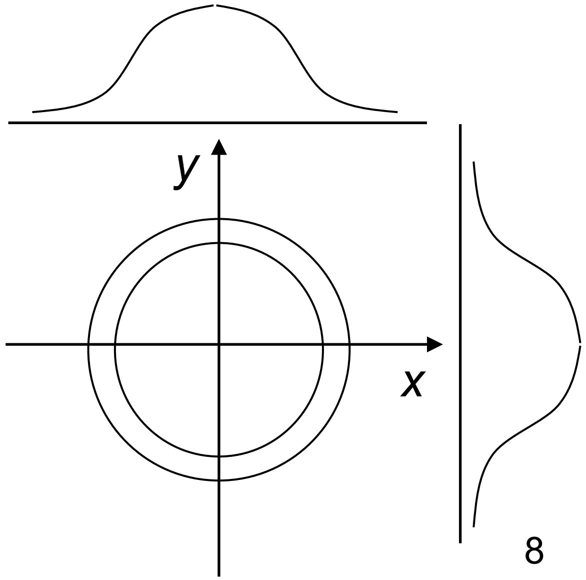


Complex Gaussian distribution

$$f(x) = \frac{1}{\sqrt{\pi\sigma_h^2}} \exp\left(-\frac{x^2}{\sigma_h^2}\right)$$

$$f(x, y) = \frac{1}{\pi\sigma_h^2} \exp\left(-\frac{x^2 + y^2}{\sigma_h^2}\right)$$

Complex Gaussian dist.



Probability of Fading

Cartesian to polar conversion

$$f(x,y) = \frac{1}{\pi\sigma_{\mathrm{h}}^2}\exp\left(-\frac{x^2+y^2}{\sigma_{\mathrm{h}}^2}\right)$$

$$x = r\cos\phi \quad y = r\sin\phi$$

$$f(r,\phi) = f(x,y)\left|\det\mathbf{J}\right| = \frac{r}{\pi\sigma_{\mathrm{h}}^2}\exp\left(-\frac{r^2}{\sigma_{\mathrm{h}}^2}\right)$$

Jacobian

$$\mathrm{d}x\,\mathrm{d}y = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \phi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \phi} \end{vmatrix} \mathrm{d}r\,\mathrm{d}\phi$$

$$= \left|\det\mathbf{J}\right|\mathrm{d}r\,\mathrm{d}\phi$$

Uniform phase distribution

Rayleigh distribution (amplitude & power)

$$f(r) = \int_0^{2\pi} f(r,\phi)\,\mathrm{d}\phi = \frac{2r}{\sigma_{\mathrm{h}}^2}\exp\left(-\frac{r^2}{\sigma_{\mathrm{h}}^2}\right)$$

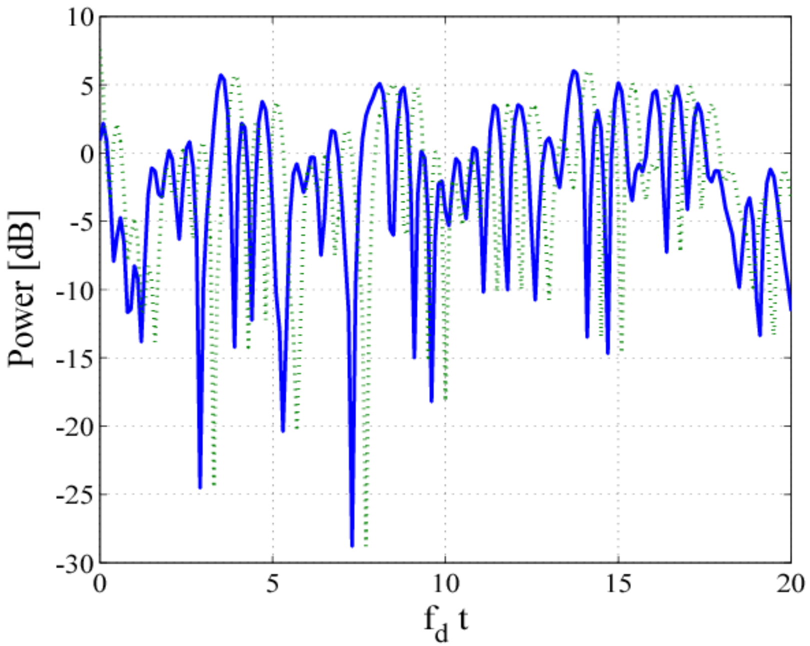
$$f(g) = \frac{1}{2\sqrt{g}} f(r) = \frac{1}{\sigma_{\mathrm{h}}^2}\exp\left(-\frac{g}{\sigma_{\mathrm{h}}^2}\right)$$

$$r = \sqrt{g}$$

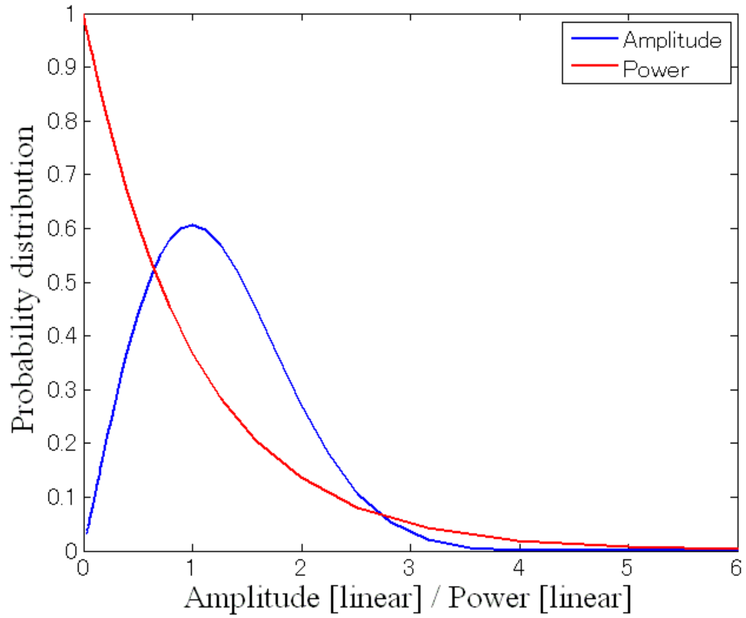
Rayleigh distribution

Rayleigh Distribution

Fading variation



Probability distribution



Cumulative Distribution

Rayleigh distribution

$$f(g) = \frac{1}{\sigma_h^2} \exp\left(-\frac{g}{\sigma_h^2} \right)$$

Cumulative probability distribution (CDF)

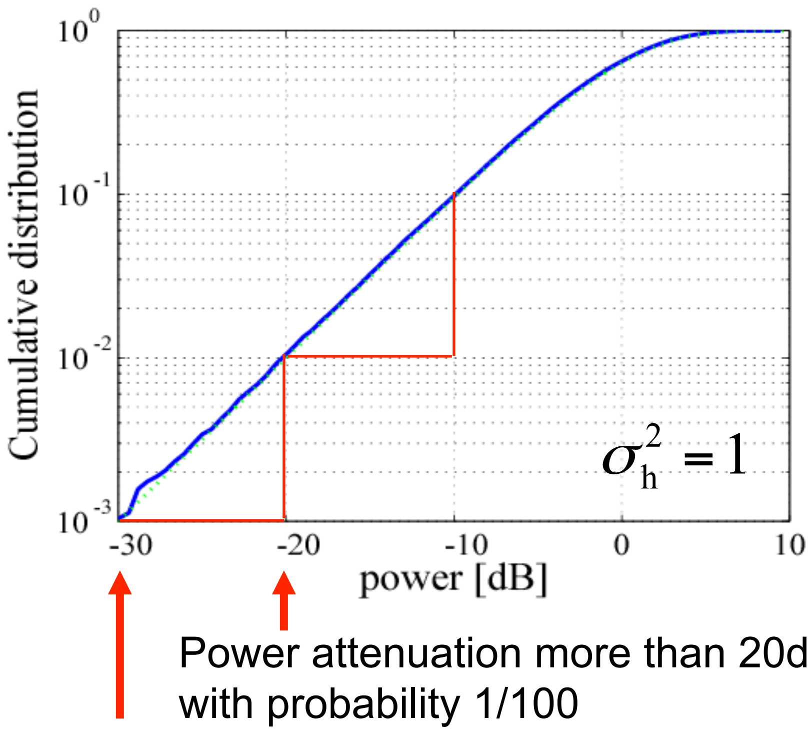
$$\tilde{f}(g) = \int_0^{\tilde{g}} f(g)\mathrm{d}g = 1 - \exp\left(-\frac{\tilde{g}}{\sigma_h^2} \right)$$

Taylor expansion

$$\tilde{f}(p) = \tilde{f}(0) + g\tilde{f}'(0) + \frac{g^2}{2}\tilde{f}''(0) + \cdots$$

$$= \frac{g}{\sigma_h^2} + \frac{g^2}{2\left(\sigma_h^2\right)^2} + \cdots$$

CDF



Power attenuation more than 20dB with probability 1/100

Power attenuation more than 30dB with probability 1/1000

CDF of SNR

Signal-to-Noise Ratio (SNR)

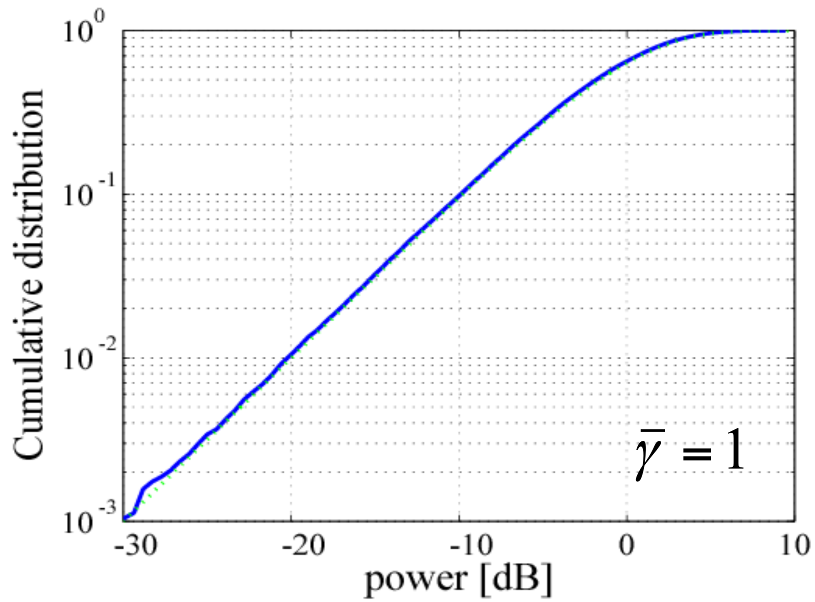
$$\gamma = \frac{P\left|h_{\mathrm{B}}(t)\right|^2}{\sigma^2} = \frac{Pg(t)}{\sigma^2}$$

Rayleigh fading

$$f(g) = \frac{1}{\sigma_{\mathrm{h}}^2}\exp\left(-\frac{g}{\sigma_{\mathrm{h}}^2}\right)$$

$$\sigma_{\mathrm{h}}^2 = \mathrm{E}\left[\left|h_{\mathrm{B}}(t)\right|^2\right] = \bar{g}$$

CDF



PDF of SNR

$$f(\gamma) = \frac{1}{\bar{\gamma}}\exp\left(-\frac{\gamma}{\bar{\gamma}}\right) \qquad \bar{\gamma} = \frac{P\bar{g}}{\sigma^2}$$

Average SNR

Error Rate in Fading Channel

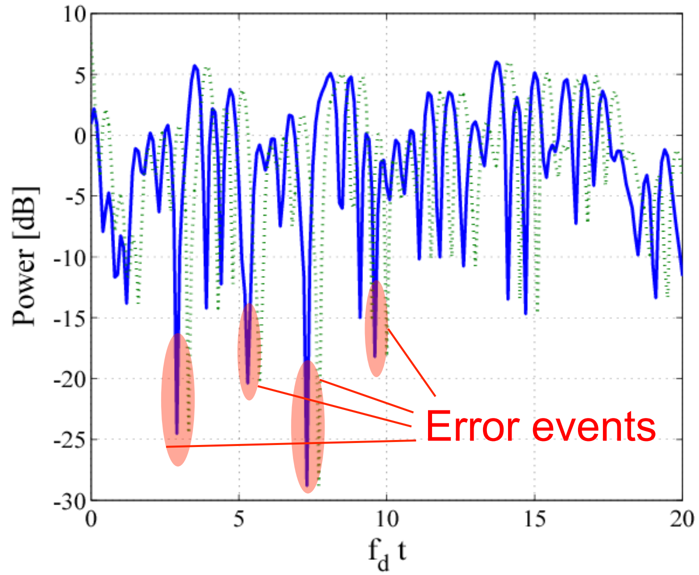
BER of BPSK

$$P_{eb}(\gamma) = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\gamma}\right) \qquad \gamma = \frac{P|h|^2}{\sigma^2}$$

Rayleigh fading channel

$$f(\gamma) = \frac{1}{\bar{\gamma}}\exp\left(-\frac{\gamma}{\bar{\gamma}}\right) \qquad \bar{\gamma} = \mathrm{E}\left[\frac{P|h(t)|^2}{\sigma^2}\right]$$

Fading channel



Average BER

$$\bar{P}_{eb}(\bar{\gamma}) = \int P_{eb}(\gamma) f(\gamma)\,\mathrm{d}\gamma$$

$$= \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right)$$

Mathematical formulas

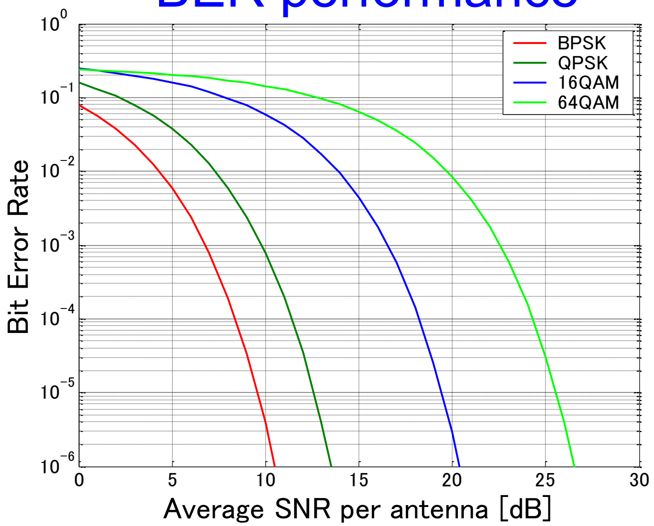
$$\int f(x)g'(x)\,\mathrm{d}x = f(x)g(x) - \int f'(x)g(x)\,\mathrm{d}x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\operatorname{erfc}(x) = -\frac{2}{\sqrt{\pi}}\exp\left(-x^2\right)$$

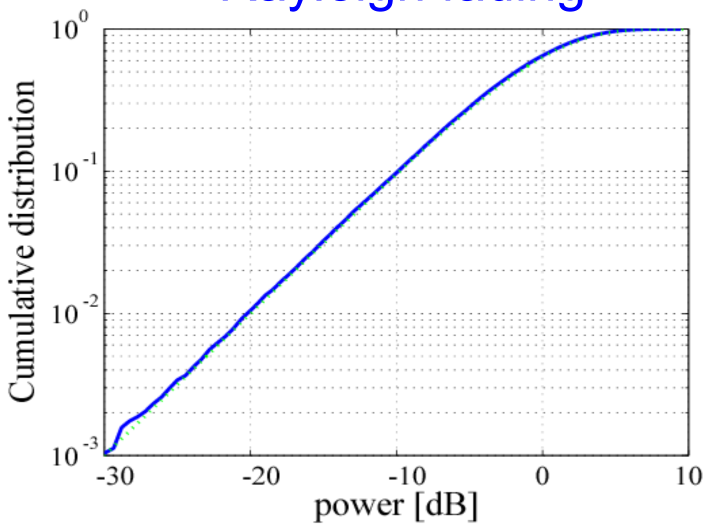
$$\int_0^\infty \exp\left(-ax^2\right)\mathrm{d}x = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

BER Performance in Fading Channel

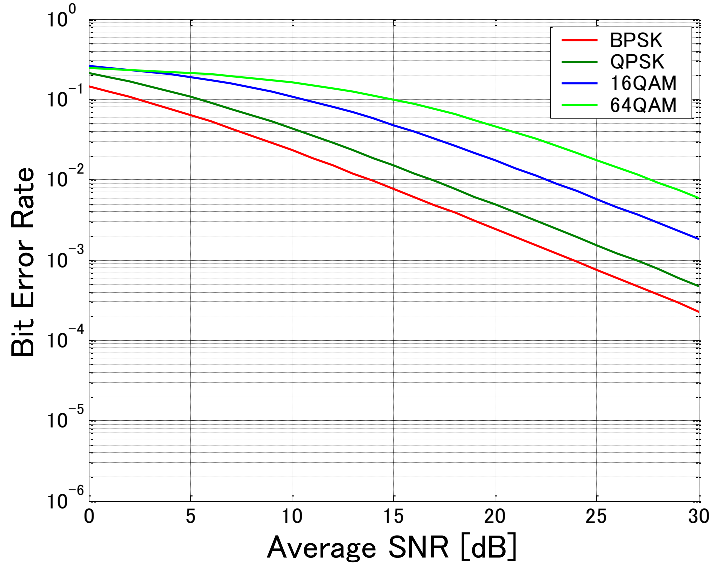
BER performance



Rayleigh fading



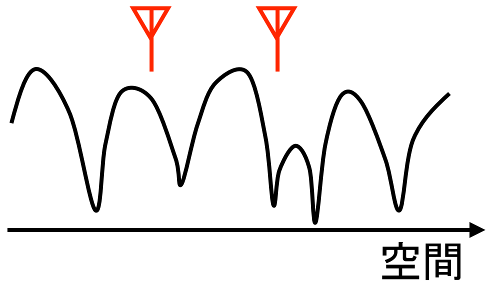
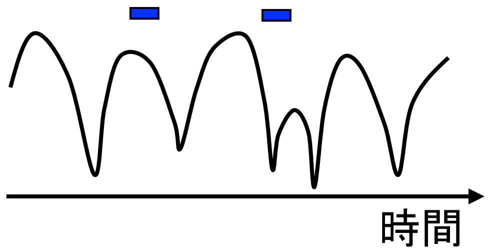
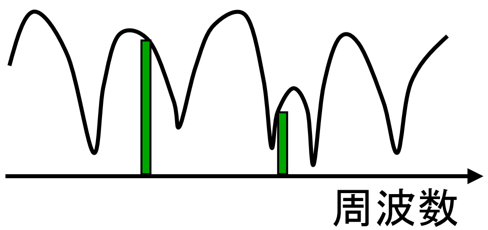
Average BER performance



One of biggest problem
in wireless communications

Diversity Technologies

２つ以上の{アンテナ，シンボル，サブキャリア}を用いたフェージング対策技術

Antenna	アレーアンテナ マルチパス環境で有効	
Time	再送、インターリーバ ドップラ変動環境で有効	
Frequency	OFDM 遅延波がある環境で有効	

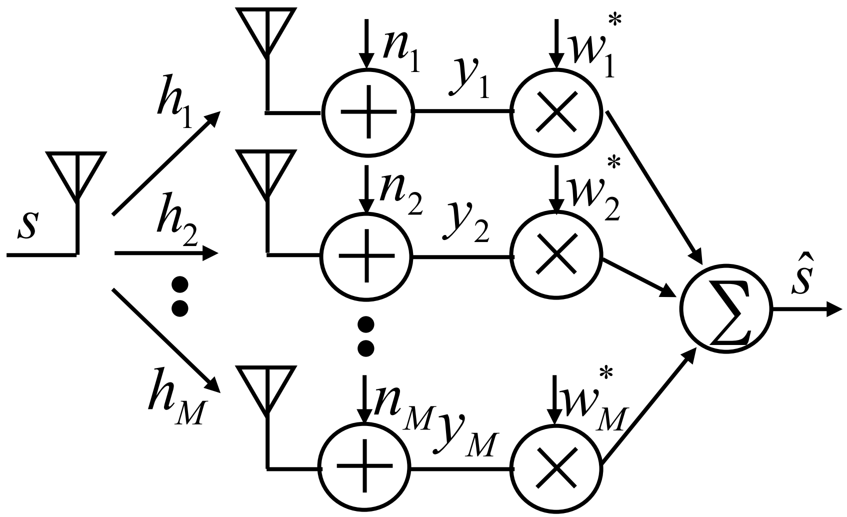
Antenna Signal Processing

Receive signal model

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_M \end{bmatrix} s + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_M \end{bmatrix}$$

$$\mathbf{y} = \mathbf{h}s + \mathbf{n}$$

Array antenna receiver



Weighted combining

$$\hat{s} = \mathbf{w}^H \mathbf{y} = \mathbf{w}^H \mathbf{h} s + \mathbf{w}^H \mathbf{n}$$

$$\mathbf{w} = \begin{bmatrix} w_1 & w_2 & \cdots & w_M \end{bmatrix}^T$$

Maximum Ratio Combining

SNR after weighted combining

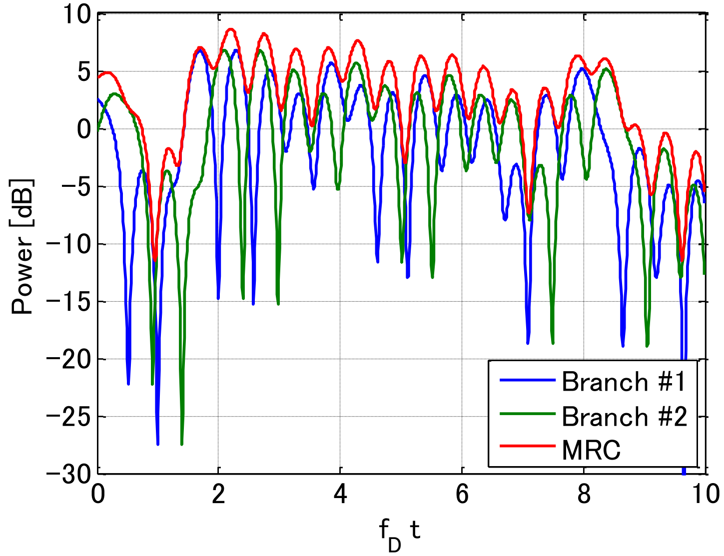
$$\gamma = \frac{\mathrm{E}[|\mathbf{w}^H \mathbf{h} s|^2]}{\mathrm{E}[|\mathbf{w}^H \mathbf{n}|^2]} = \frac{|\mathbf{w}^H \mathbf{h}|^2 \, \mathrm{E}[|s|^2]}{\mathbf{w}^H \, \mathrm{E}[\mathbf{n}\mathbf{n}^H]\mathbf{w}} = \frac{|\mathbf{w}^H \mathbf{h}|^2 P}{|\mathbf{w}|^2 \sigma^2}$$

Vector inner product

Vector outer product

$$\mathrm{E}[\mathbf{n}\mathbf{n}^H] = \sigma^2 \mathbf{I}_M$$ ← Uncorrelated noise

Performance of MRC



Maximum Ratio Combining (MRC)

$$\mathbf{w}_{\mathrm{opt}} = \arg\max \gamma = \alpha \mathbf{h}$$

$$\gamma_{\mathrm{opt}} = \frac{|\mathbf{h}|^2 P}{\sigma^2} = \frac{\sum_{i=1}^{M} |h_i|^2 P}{\sigma^2} = \sum_{i=1}^{M} \gamma_i$$ ← Sum of branch (antenna) SNR

PDF & Characteristic Function

PDF of sum of random variables

Independent random variables x y

$$f(x) \quad f(y) \quad f(x,y) = f(x)f(y)$$

$z = x + y$ ← Sum of random variables

$f(z) = \int f(x) f(z-x) \mathrm{d}x$ ← Convolution

Characteristic function of PDF

$$f(\gamma) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \varphi(t) \exp(-j\gamma t) \mathrm{d}\gamma \longleftrightarrow \varphi(t) = \int_{0}^{\infty} f(\gamma) \exp(j\gamma t) \mathrm{d}\gamma$$

Theorem on Fourier transformation

$$\gamma = \sum_i \gamma_i \longleftrightarrow \varphi(t) = \prod_i \varphi_i(t)$$

CDF of SNR after MRC

SNR after MRC

$$\gamma = \sum_{i=1}^{M} \gamma_i$$

Characteristic function of each antenna

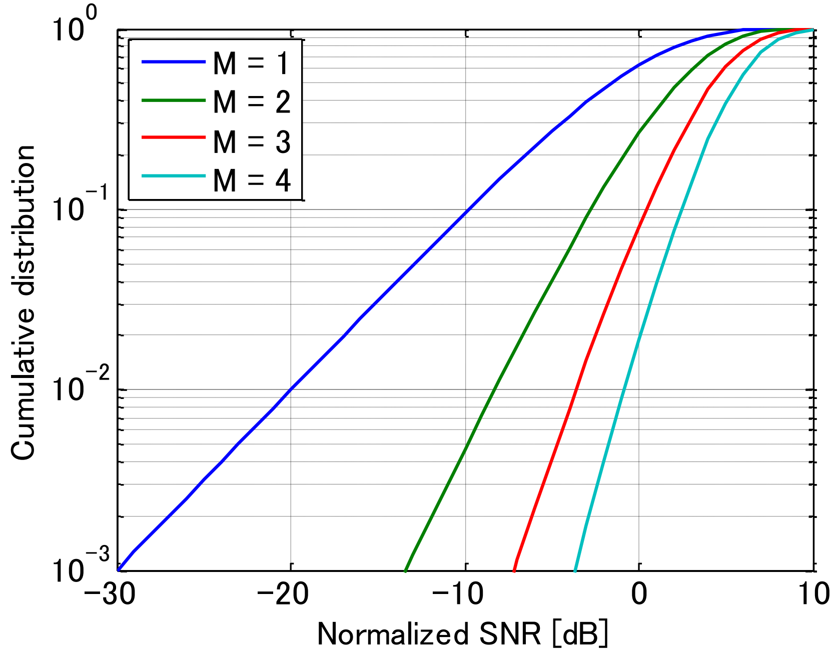
$$f(\gamma_i) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma_i}{\bar{\gamma}}\right) \leftrightarrow \varphi_i(t) = \frac{1}{1 - j\bar{\gamma}t}$$

PDF of SNR after MRC

$$\varphi(t) = \prod_{i=1}^{M} \varphi_i(t) = \left(\frac{1}{1 - j\bar{\gamma}t}\right)^M$$

$$f(\gamma) = \frac{1}{(M-1)!\bar{\gamma}^M} \gamma^{M-1} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right)$$ ← Gamma dist. (χ square dist.)

CDF



BER of MRC Diversity

Average BER

$$\overline{P}_{\mathrm{e}}(\bar{\gamma}) = \int P_{\mathrm{e}}(\gamma) f(\gamma) \mathrm{d}\gamma$$

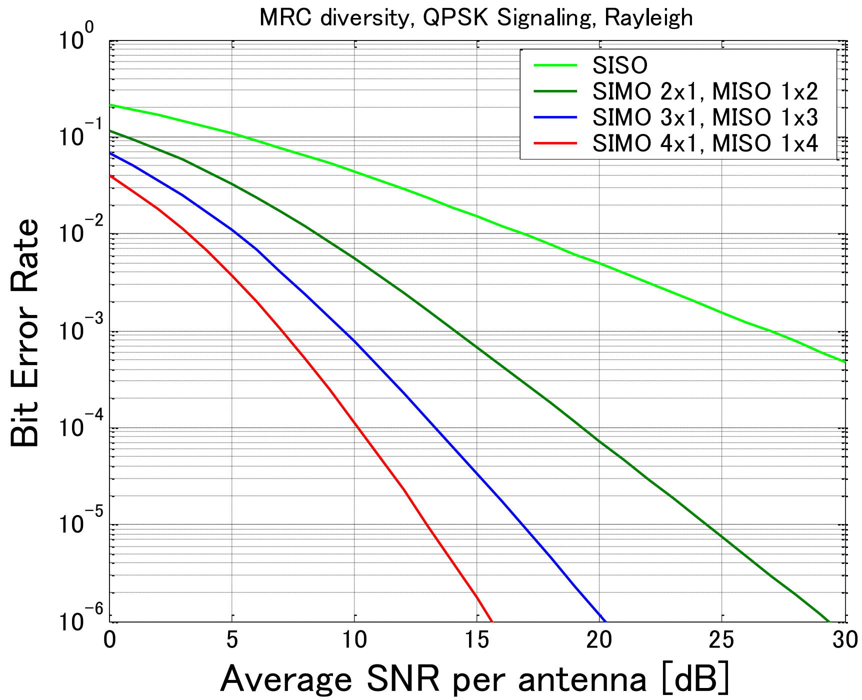
BER in AWGN (Gaussian Noise)

$$P_{\mathrm{e}} = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\gamma}\right)$$

PDF of SNR after MRC

$$f(\gamma) = \frac{1}{(M-1)!\bar{\gamma}^{M}} \gamma^{M-1} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right)$$

BER performance



$$\overline{P}_{\mathrm{e}}(\bar{\gamma}) = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right)^{M} \sum_{i=0}^{M-1} \binom{M-1+i}{i} \left(\frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right)\right)^{i}$$

Summary

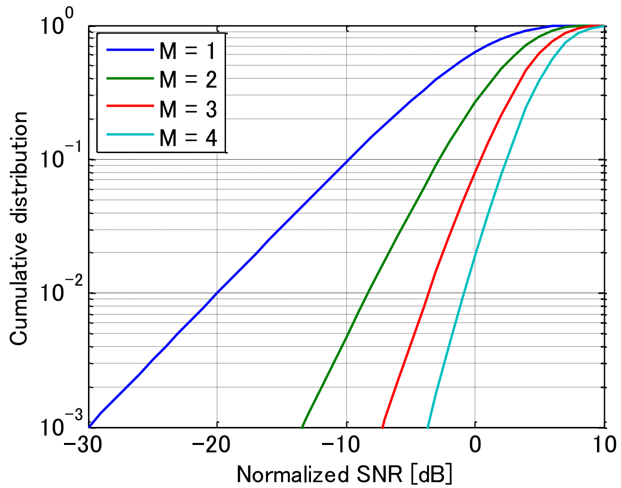
- Error rate in fading channel

$$P_{\mathrm{eb}}(\gamma) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\gamma}\right) \quad \Rightarrow \quad \bar{P}_{\mathrm{eb}}(\bar{\gamma}) = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right)$$

- Antenna signal processing

$$\mathbf{y} = \mathbf{h}s + \mathbf{n}$$

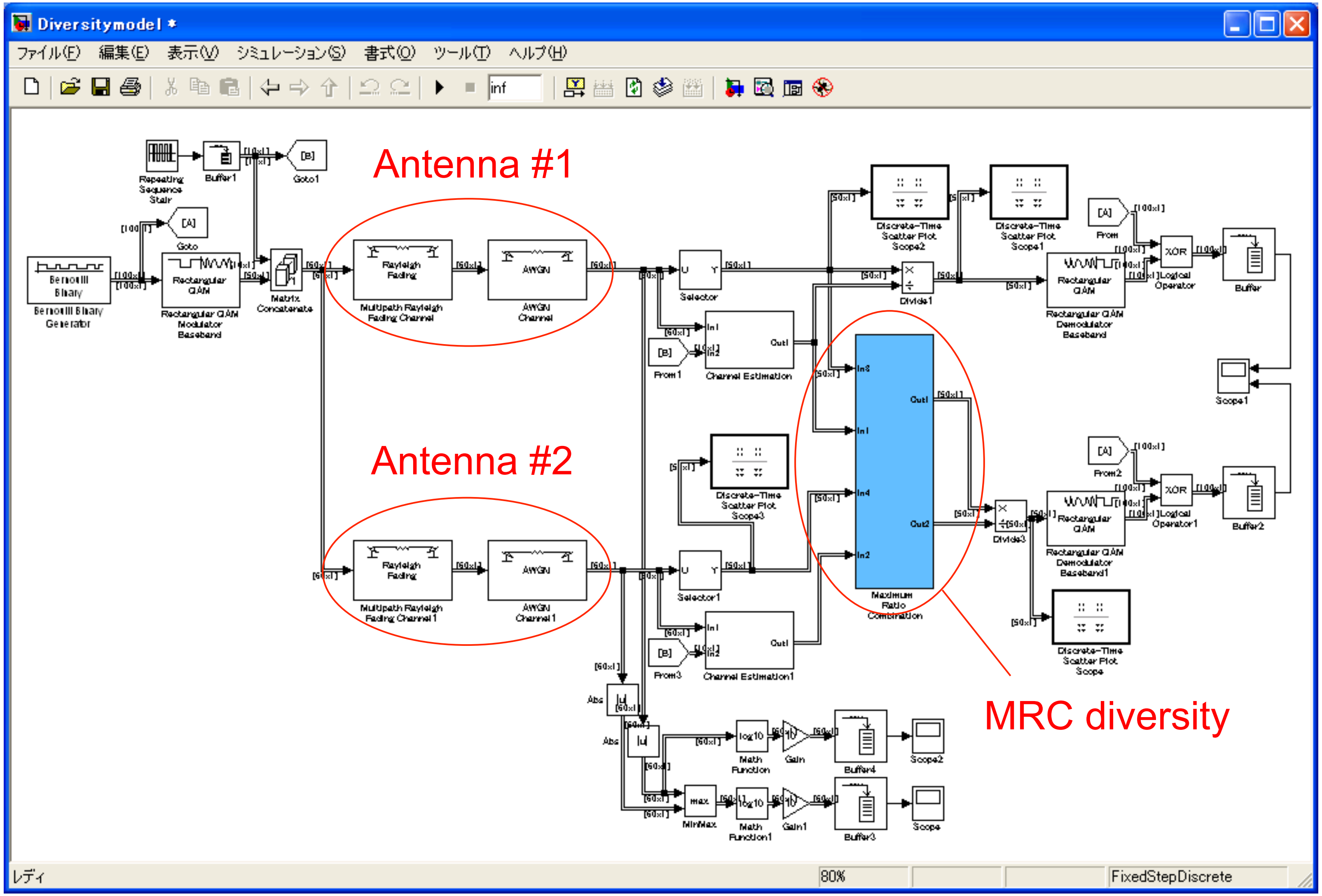
$$\hat{s} = \mathbf{w}^H\mathbf{y} = \mathbf{w}^H\mathbf{h}s + \mathbf{w}^H\mathbf{n}$$



- Performance of MRC diversity

$$\gamma_{\mathrm{opt}} = \frac{|\mathbf{h}|^2 P}{\sigma^2} = \frac{\sum_{i=1}^{M} |h_i|^2 P}{\sigma^2} = \sum_{i=1}^{M} \gamma_i \quad \Rightarrow \quad f(\gamma) = \frac{1}{(M-1)!\bar{\gamma}^M}\gamma^{M-1}\exp\left(-\frac{\gamma}{\bar{\gamma}}\right)$$

Demo


Diversitymodel *
ファイル(F) 編集(E) 表示(V) シミュレーション(S) 書式(O) ツール(T) ヘルプ(H)
inf
Antenna #1
Antenna #2
MRC diversity
Repeating Sequence Stair
Buffer1
Goto1
Goto
Bernoulli Binary Generator
Rectangular QAM Modulator Baseband
Matrix Concatenate
Multipath Rayleigh Fading Channel
AWGN Channel
Multipath Rayleigh Fading Channel1
AWGN Channel1
Selector
Selector1
From1
From3
Channel Estimation
Channel Estimation1
Maximum Ratio Combination
Discrete-Time Scatter Plot Scope2
Discrete-Time Scatter Plot Scope1
Discrete-Time Scatter Plot Scope3
Discrete-Time Scatter Plot Scope
Divide1
Divide3
Rectangular QAM Demodulator Baseband
Rectangular QAM Demodulator Baseband1
From
From2
Logical Operator
Logical Operator1
Buffer
Buffer2
Scope1
Abs
Math Function
Math Function1
Gain
Gain1
MinMax
Buffer4
Buffer3
Scope2
Scope
レディ
80%
FixedStepDiscrete

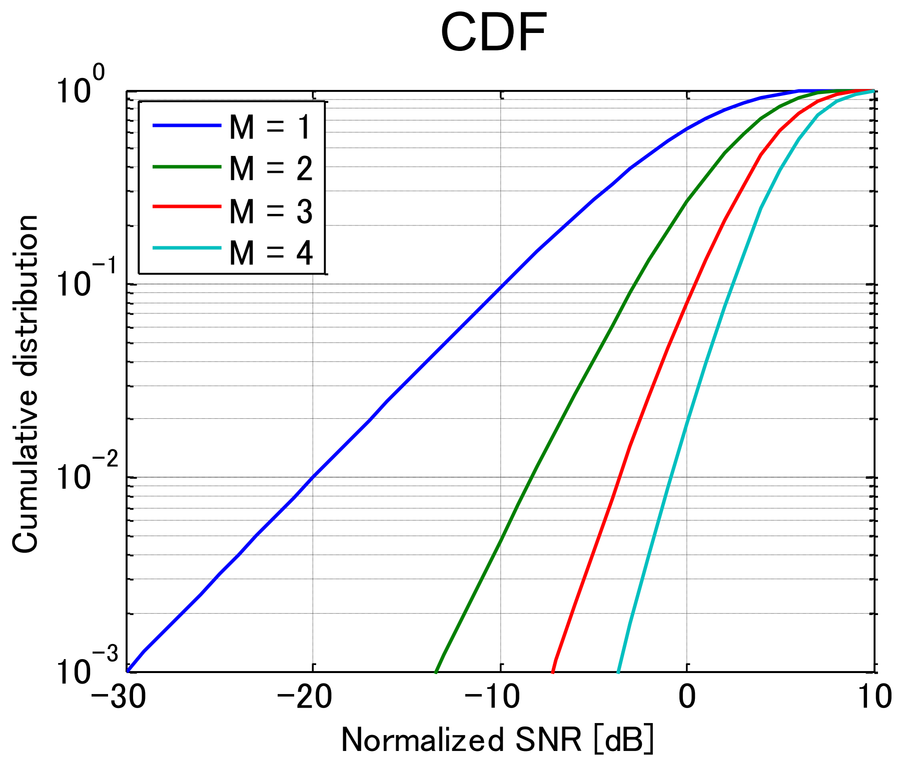
Diversity Order

PDF of SNR after MRC

$$f(\gamma) = \frac{1}{(M-1)!\bar{\gamma}^M}\gamma^{M-1}\exp\left(-\frac{\gamma}{\bar{\gamma}}\right)$$

CDF of SNR after MRC

$$\tilde{f}(\gamma) = 1 - \exp\left(-\frac{\gamma}{\bar{\gamma}}\right)\sum_{i=1}^{M}\frac{\left(\gamma/\bar{\gamma}\right)^{i-1}}{(i-1)!}$$



1 antenna

$$\tilde{f}(\gamma) = 1 - \exp\left(-\frac{\gamma}{\bar{\gamma}}\right) \cong \frac{\gamma}{\bar{\gamma}} + O\left(\gamma^2\right)$$

2 antenna MRC

Diversity order

$$\tilde{f}(\gamma) = 1 - \exp\left(-\frac{\gamma}{\bar{\gamma}}\right) - \frac{\gamma}{\bar{\gamma}}\exp\left(-\frac{\gamma}{\bar{\gamma}}\right) \cong \frac{1}{2}\left(\frac{\gamma}{\bar{\gamma}}\right)^2 + O\left(\gamma^3\right)$$