2016 2Q

Wireless Communication Engineering

#4 Digital Modulation and Pulse Shaping

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Course Schedule (1)

	Date	Text	Contents
#1	June 17	1, 7	Introduction to wireless communication systems
#2	June 17	2, 5, etc	Link budget design of wireless access
#3	June 24		Up/down conversion and equivalent baseband system
#4	June 24	3.3, 3.4	Digital modulation and pulse shaping
	July 1		No class
#5	July 8	3.5	Demodulation and detection error due to noise
#6	July 8	4.4	Channel fading and diversity combining

From Previous Lecture

- Equivalent baseband system

$$y_{\mathrm{B}}(t) = \int h_{\mathrm{B}}(\tau) s_{\mathrm{B}}(t-\tau)\mathrm{d}\tau$$

$$y_{\mathrm{B}}(t) = y_{\mathrm{R}}(t) e^{-j2\pi f_0 t} \qquad y_{\mathrm{R}}(t) = y(t) + j\,\mathrm{hilb}\big(y(t)\big)$$

- Power spectrum of transmit signal

$$S^{\mathrm{s}}(f) = \frac{1}{4}\left(S_{\mathrm{B}}^{\mathrm{s}}(f - f_0) + S_{\mathrm{B}}^{\mathrm{s}}(-f - f_0)\right)$$

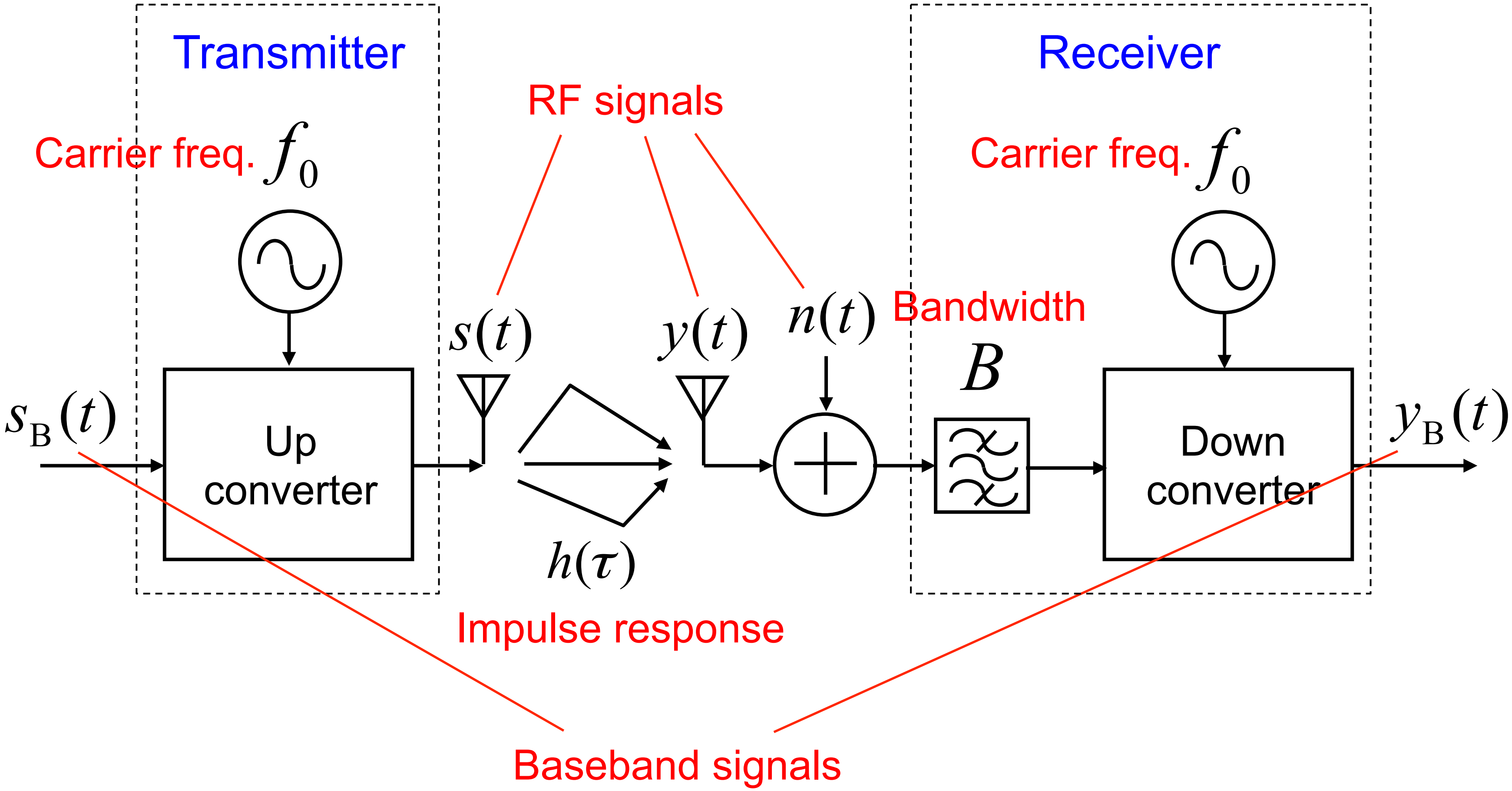
- Power spectrum of receive signal

$$S_{\mathrm{B}}^{\mathrm{y}}(f) = \left|H_{\mathrm{B}}(f)\right|^2 S_{\mathrm{B}}^{\mathrm{s}}(f)$$

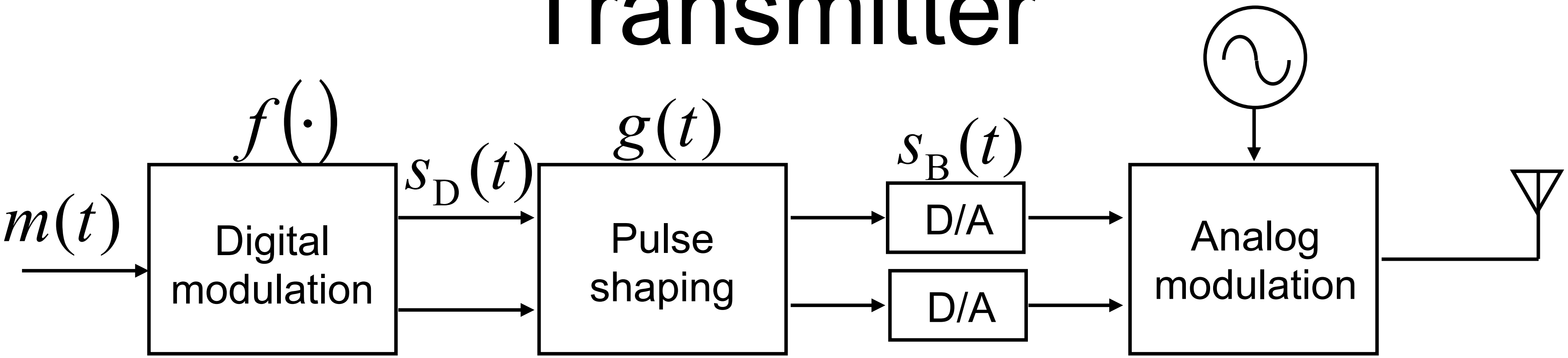
Contents

- Structure of transmitter
- Amplitude shift keying
- Phase shift keying
- Frequency shift keying
- Pulse shaping
- Experiments

Transceiver System



Transmitter



Message signal

Message sequence

Modulation order & bit rate

$$m(t) = \sum_n a_n \delta(t - nT_s) \qquad a_n = \begin{cases} 1, & \text{if symbol } 1 \\ 0, & \text{if symbol } 0 \end{cases} \longrightarrow \begin{array}{l} M = 2 \\ H = \log_2 M = 1 \end{array}$$

Symbol period

Digital modulation

$$s_D(t) = s_{DI}(t) + j s_{DQ}(t) = f(m(t))$$

Mapping function

- Amplitude
- Phase
- Frequency

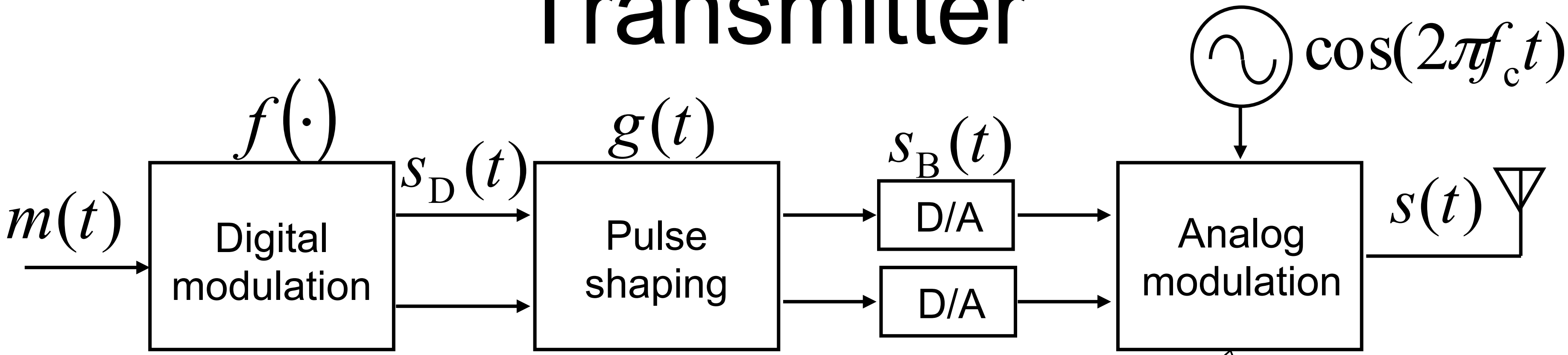
Pulse shaping

$$s_B(t) = s_{BI}(t) + j s_{BQ}(t) = \int g(\tau) s_D(t - \tau) \mathrm{d}\tau$$

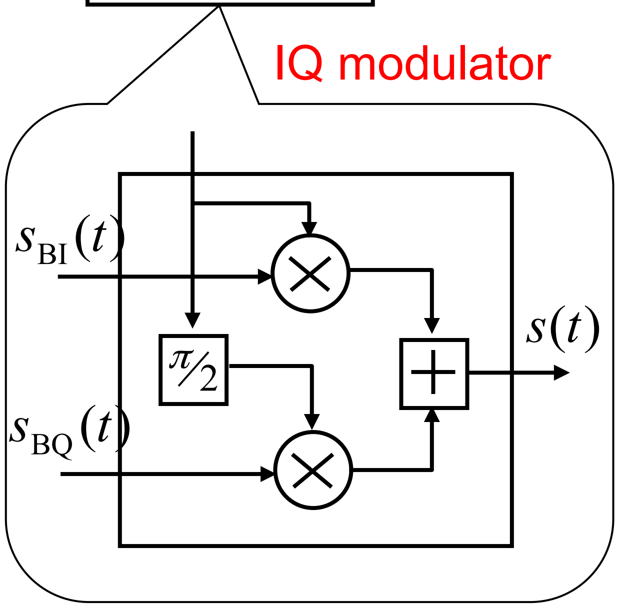
Pulse shape

- Rectangular
- Nyquist
- Gaussian

Transmitter



IQ modulator



Analog modulation

Carrier freq.

$$s(t) = \underbrace{s_{BI}(t)\cos(2\pi f_c t)}_{\text{In-phase (I)}} - \underbrace{s_{BQ}(t)\sin(2\pi f_c t)}_{\text{Quadrature (Q)}}$$

$$= \mathrm{Re}\left[\underbrace{\left(s_{BI}(t) + js_{BQ}(t)\right)}_{\text{Complex baseband signal}}\left(\cos(2\pi f_c t) + j\sin(2\pi f_c t)\right)\right]$$

$$= \mathrm{Re}\left[r(t)\exp\left(j\theta(t)\right)\exp\left(j2\pi f_c t\right)\right] = \underbrace{r(t)}_{\text{Amplitude}}\cos(2\pi \underbrace{f_c}_{\text{Frequency}} t + \underbrace{\theta(t)}_{\text{Phase}})$$

Design of Modulation & Pulse Shaping

Carrier freq. f_{c}	$\Leftrightarrow$	Coverage $d_0 = \dfrac{c}{4\pi f_0^{3/2}}\sqrt{\dfrac{P_{\mathrm{t}}}{\alpha N_0 \gamma_0}}$
Symbol period T_{s}	$\Leftrightarrow$	Bandwidth $B \propto 1/T_{\mathrm{s}}$
Modulation order M	$\Leftrightarrow$	Data rate $\dfrac{\log_2 M}{T_{\mathrm{s}}}$
Modulation $f(\cdot)$	$\Leftrightarrow$	Power efficiency, complexity, error rate p_{e}
Pulse shape $g(t)$	$\Leftrightarrow$	Power leakage $S_{\mathrm{B}}^{s}(t) = \lvert g(f)\rvert^2 S_{\mathrm{D}}^{s}(t)$

Amplitude Shift Keying (ASK)

Digital modulation

$$s_{\mathrm{D}}(t) = s_{\mathrm{DI}}(t) = m(t)$$

Pulse shaping (rectangular)

$$s_{\mathrm{B}}(t) = \int g_{\mathrm{rect}}(\tau) s_{\mathrm{D}}(t-\tau)\mathrm{d}\tau$$
$$= \sum_n a_n g_{\mathrm{rect}}(t - nT_{\mathrm{s}})$$

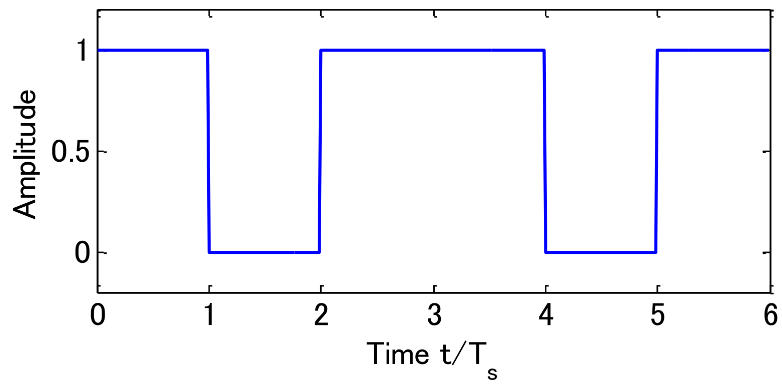
Analog modulation

$$s(t) = s_{\mathrm{B}}(t)\cos(2\pi f_{\mathrm{c}} t)$$
$$= \begin{cases} \cos(2\pi f_{\mathrm{c}} t), & \text{if } a_n = 1 \\ 0, & \text{if } a_n = 0 \end{cases}$$

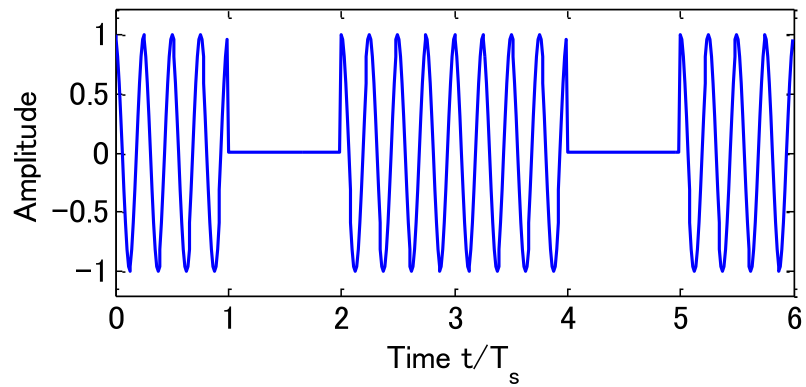
Main features

Error rate = Fair, Power efficiency = Fair, Complexity = Excellent

Message sequence



RF signal



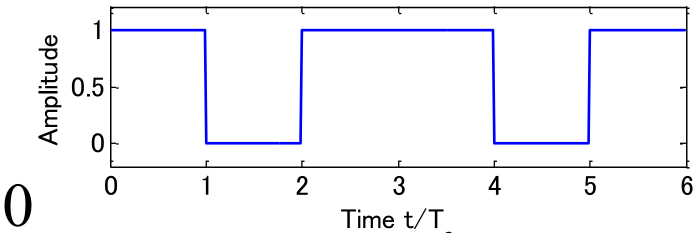
Binary Phase Shift Keying (BPSK)

Digital modulation

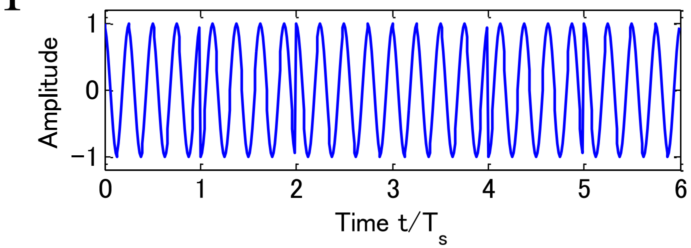
$$s_{\mathrm{D}}(t) = \exp\left(j\pi m(t)\right) = \cos(\pi m(t)) + j\sin(\pi m(t)) = \sum_n a_{2n}\delta\left(t - nT_{\mathrm{s}}\right) \quad a_{2n} = \begin{cases} 1, & \text{if } a_n = 0 \\ -1, & \text{if } a_n = 1 \end{cases}$$

NRZ signal

Message sequence



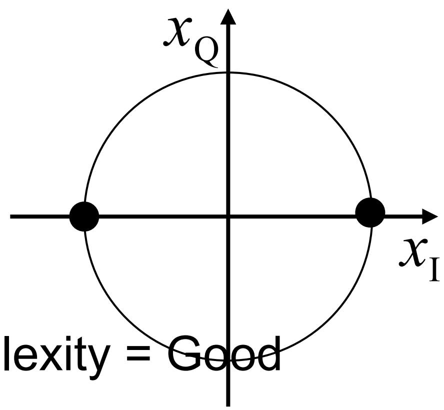
RF signal



Pulse shaping (rectangular)

$$s_{\mathrm{B}}(t) = \sum_n a_{2n} g_{\mathrm{rect}}\left(t - nT_{\mathrm{s}}\right)$$

Constellation



Analog modulation

$$s(t) = s_{\mathrm{B}}(t)\cos(2\pi f_{\mathrm{c}} t) = \begin{cases} \cos(2\pi f_{\mathrm{c}} t), & \text{if } a_n = 0 \\ -\cos(2\pi f_{\mathrm{c}} t), & \text{if } a_n = 1 \end{cases}$$

Main features

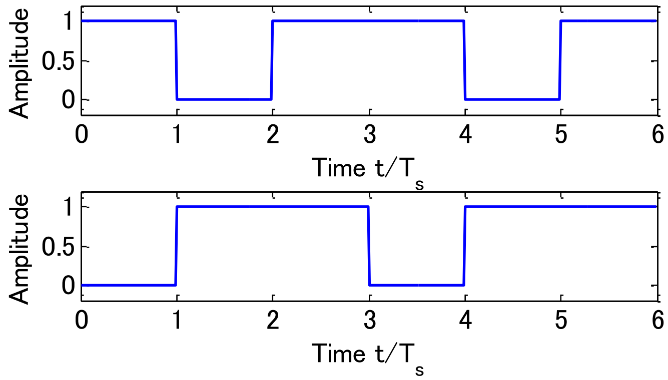
Error rate = Excellent, Power efficiency = Good, Complexity = Good

Quadrature Phase Shift Keying (QPSK)

Message signal

$M = 4 \longrightarrow$ Two binary sequences $m_{\mathrm{I}}(t), m_{\mathrm{Q}}(t)$

Message sequence



Digital modulation & pulse shaping

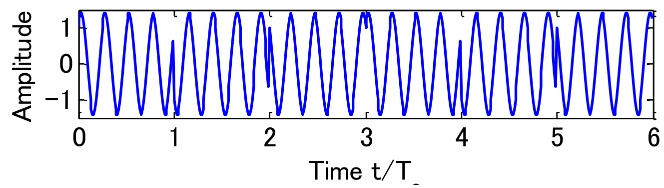
$$s_{\mathrm{D}}(t) = \exp\left(j\pi m_{\mathrm{I}}(t)\right) + \exp\left(j(\pi m_{\mathrm{Q}}(t) + \pi/2)\right)$$

$$= \cos\left(\pi m_{\mathrm{I}}(t)\right) + j\cos\left(\pi m_{\mathrm{Q}}(t)\right)$$

$$= \sum_n a_{2\mathrm{I}n}\delta(t - nT_{\mathrm{s}}) + j\sum_n a_{2\mathrm{Q}n}\delta(t - nT_{\mathrm{s}})$$

$$s_{\mathrm{B}}(t) = \sum_n a_{2\mathrm{I}n} g_{\mathrm{rect}}(t - nT_{\mathrm{s}}) + j\sum_n a_{2\mathrm{Q}n} g_{\mathrm{rect}}(t - nT_{\mathrm{s}})$$

RF signal

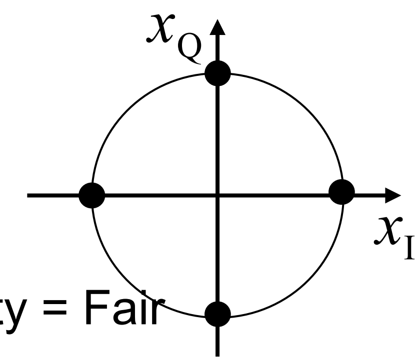


Analog modulation

$$s(t) = s_{\mathrm{BI}}(t)\cos(2\pi f_{\mathrm{c}} t) - s_{\mathrm{BQ}}(t)\sin(2\pi f_{\mathrm{c}} t)$$

$$= \sqrt{2}\cos(2\pi f_{\mathrm{c}} t + \mathrm{atan}(s_{\mathrm{BI}}(t), s_{\mathrm{BQ}}(t)))$$

Constellation



Main features

Rate = Good, Error = Excellent, Power = Good, Complexity = Fair

Quadrature Amplitude Modulation (QAM)

Message signal

$M = 16 \longrightarrow$ Two 4-level sequences $m_{\mathrm{I}}(t), m_{\mathrm{Q}}(t)$

Digital modulation & pulse shaping

$$s_{\mathrm{D}}(t) = s_{\mathrm{DI}}(t) + j s_{\mathrm{DQ}}(t)$$

$$s_{\mathrm{DI}}(t) = 2m_{\mathrm{I}}(t) - \sqrt{M} + 1 = \sum_n a_{4\mathrm{I}n}\delta(t - nT_{\mathrm{s}})$$

$$s_{\mathrm{DQ}}(t) = 2m_{\mathrm{Q}}(t) - \sqrt{M} + 1 = \sum_n a_{4\mathrm{Q}n}\delta(t - nT_{\mathrm{s}})$$

$$s_{\mathrm{B}}(t) = \sum_n a_{4\mathrm{I}n} g_{\mathrm{rect}}(t - nT_{\mathrm{s}}) + j\sum_n a_{4\mathrm{Q}n} g_{\mathrm{rect}}(t - nT_{\mathrm{s}})$$

Analog modulation

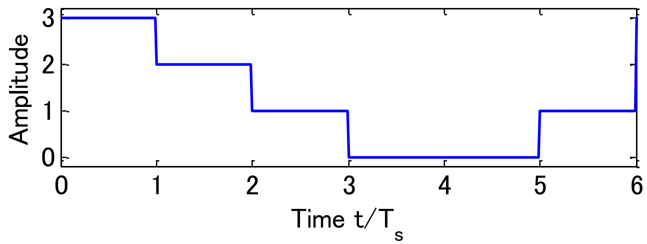
$$s(t) = s_{\mathrm{BI}}(t)\cos(2\pi f_{\mathrm{c}} t) - s_{\mathrm{BQ}}(t)\sin(2\pi f_{\mathrm{c}} t)$$

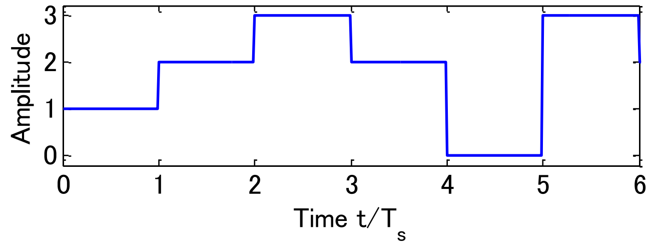
$$= \sqrt{s_{\mathrm{BI}}^2(t) + s_{\mathrm{BQ}}^2(t)}\,\cos(2\pi f_{\mathrm{c}} t + \mathrm{atan}(s_{\mathrm{BI}}(t), s_{\mathrm{BQ}}(t)))$$

Main features

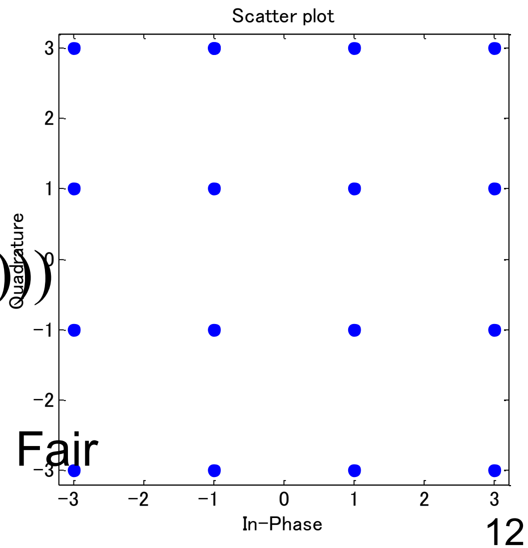
Rate = Excellent, Error = Fair, Power, Fair, Complexity = Fair

Message sequence





Constellation $M = 16$

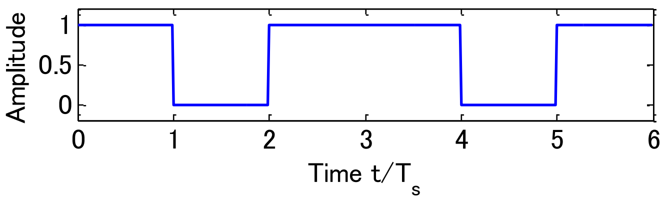


Binary Freq. Shift Keying (BFSK)

BPSK modulation

$$\tilde{s}_{\mathrm{D}}(t) = \sum_n a_{2n} g_{\mathrm{rect}}(t - nT_{\mathrm{s}})$$

Message sequence



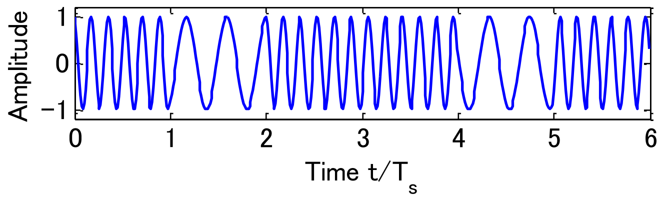
BFSK modulation

$$s_{\mathrm{B}}(t) = \cos(\theta(t)) + j\sin(\theta(t))$$

$$\theta(t) = \pi\Delta f \int_{-\infty}^{t} \tilde{s}_{\mathrm{D}}(\tau)\mathrm{d}\tau$$

$$\frac{1}{2\pi}\frac{\mathrm{d}\theta(t)}{\mathrm{d}t} = \Delta f \big/ 2 \, \tilde{s}_{\mathrm{D}}(t)$$

RF signal



Analog modulation

$$s(t) = \cos(\theta(t))\cos(2\pi f_{\mathrm{c}} t) - \sin(\theta(t))\sin(2\pi f_{\mathrm{c}} t)$$

$$= \cos(2\pi f_{\mathrm{c}} t + \theta(t)) = \cos(2\pi(f_{\mathrm{c}} + \Delta f \big/ 2 \, \tilde{x}(t))t)$$

Main features

Rate = Fair, Error = Fair, Power = Good, Complexity = Good

Minimum Shift Keying (MSK)

Differential representation of phase

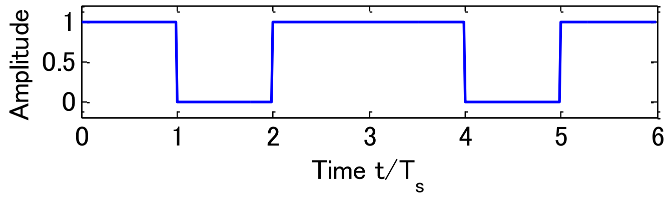
$$\theta(nT_s) = \pi\Delta f \int_{-\infty}^{nT_s} \tilde{s}_D(\tau)d\tau$$

$$= \pi\Delta f \int_{(n-1)T_s}^{nT_s} \tilde{s}_D(\tau)d\tau + \theta((n-1)T_s)$$

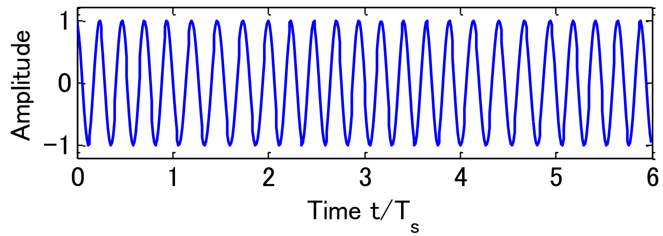
$$= \underline{\pi\Delta f T_s} a_{2n} + \theta((n-1)T_s)$$

Modulation order (BT factor)

Message sequence



RF signal

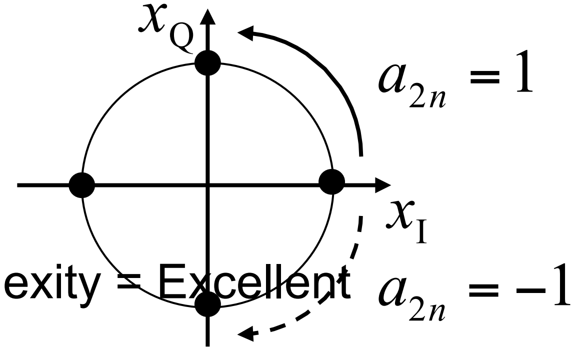


Condition on constant phase difference

$$\pi\Delta f T a_{2n} = k\frac{\pi}{2} \quad \longrightarrow \quad \Delta f T = \frac{k}{2}$$

$$\Delta f T = \frac{1}{2} \quad \longrightarrow \quad \text{MSK}$$

Constellation

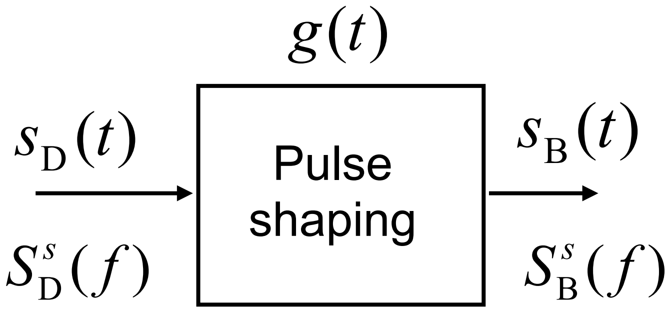


Main features

Rate = Fair, Error = Excellent, Power = Excellent, Complexity = Excellent

Pulse Shaping Filter

Pulse shaping filter

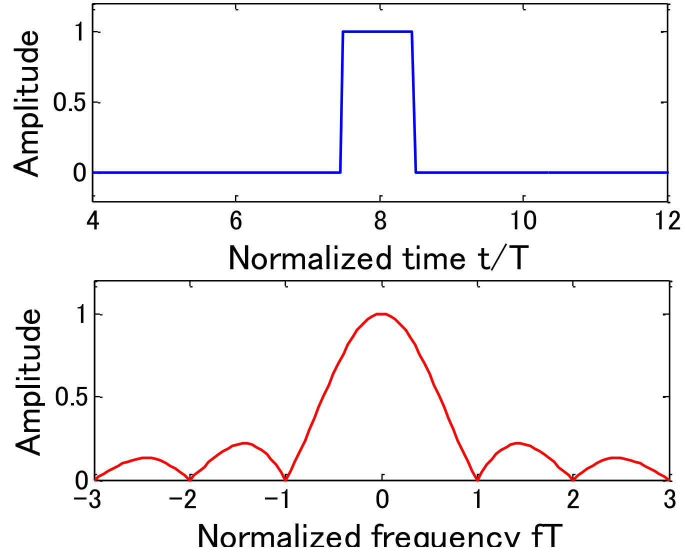


$$s_{\mathrm{B}}(t) = \int g(\tau) s_{\mathrm{D}}(t-\tau)\,\mathrm{d}\tau$$

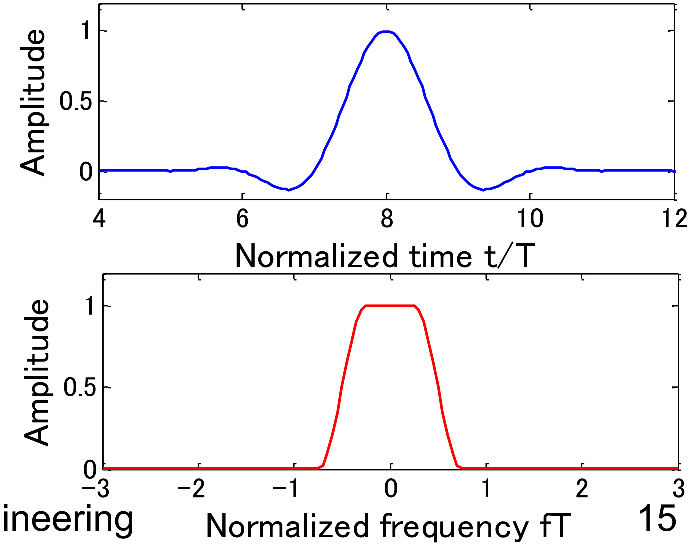
$$S_{\mathrm{B}}^{s}(f) = \left|G(f)\right|^2 S_{\mathrm{D}}^{s}(f)$$

Trade-off between
bandwidth & inter-symbol interference

Rectangular pulse



Nyquist pulse



Nyquist Pulse

Frequency response

$$G(f) =$$

$$= \begin{cases} 1 & 0 \le |fT_s| < \dfrac{1-\alpha}{2} \\ \dfrac{1}{2}\left(1 - \sin\left(\dfrac{\pi}{2\alpha}(2fT_s - 1)\right)\right) & \dfrac{1-\alpha}{2} \le |fT_s| < \dfrac{1+\alpha}{2} \\ 0 & \dfrac{1+\alpha}{2} \le |fT_s| \le 1 \end{cases}$$

Role-off factor

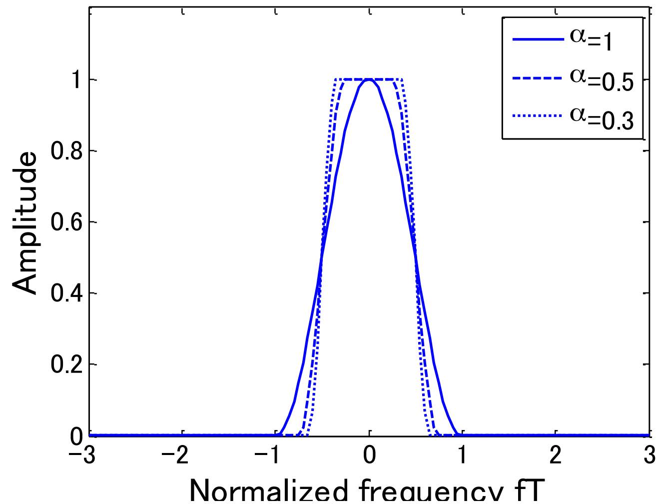
Impulse response

$$g(t) = \frac{\sin\left(\dfrac{\pi t}{T_s}\right)}{\dfrac{\pi t}{T_s}} \frac{\cos\left(\dfrac{\alpha \pi t}{T_s}\right)}{1 - \left(\dfrac{2\alpha t}{T_s}\right)^2}$$

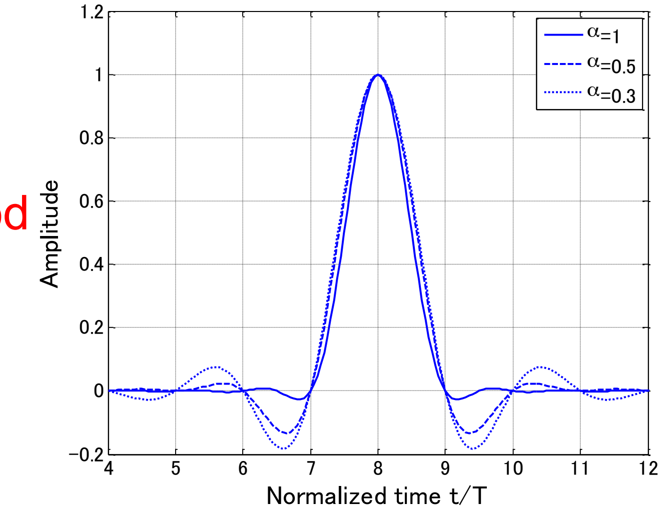
Zero cross every symbol period

No inter-symbol interference

Frequency response



Impulse response



Gaussian Pulse

Frequency response

$$G(f) = \exp\left(-\left(\frac{f}{f_0}\right)^2\right)$$

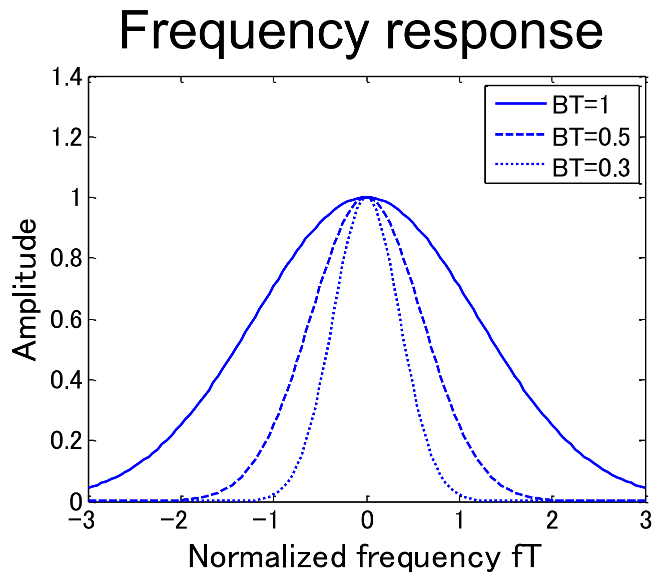
$$f_0 = \frac{B}{\sqrt{2\ln 2}}$$ — 3dB bandwidth

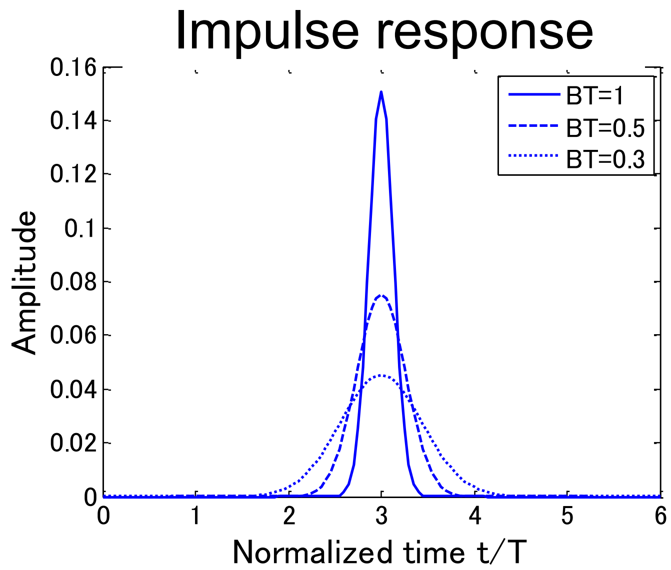
$$G(f) = \exp\left(-2\ln 2\left(\frac{f}{B}\right)^2\right)$$

Impulse response

$$g(t) = \sqrt{\frac{\pi}{\ln 2}}B\exp\left(-\frac{\pi^2}{2\ln 2}Bt^2\right)$$

Sharpest response in both freq. & time





Summary

■ Digital modulation

$$s_{\mathrm{D}}(t) = s_{\mathrm{DI}}(t) + j s_{\mathrm{DQ}}(t) = f\big(m(t)\big)$$

- Amplitude
- Phase
- Frequency

Data rate, power efficiency, complexity, error rate

■ Pulse shaping (band limitation)

$$s_{\mathrm{B}}(t) = \int g(\tau) s_{\mathrm{D}}(t-\tau)\,\mathrm{d}\tau$$

- Rectangular
- Nyquist
- Gaussian

Bandwidth, error rate

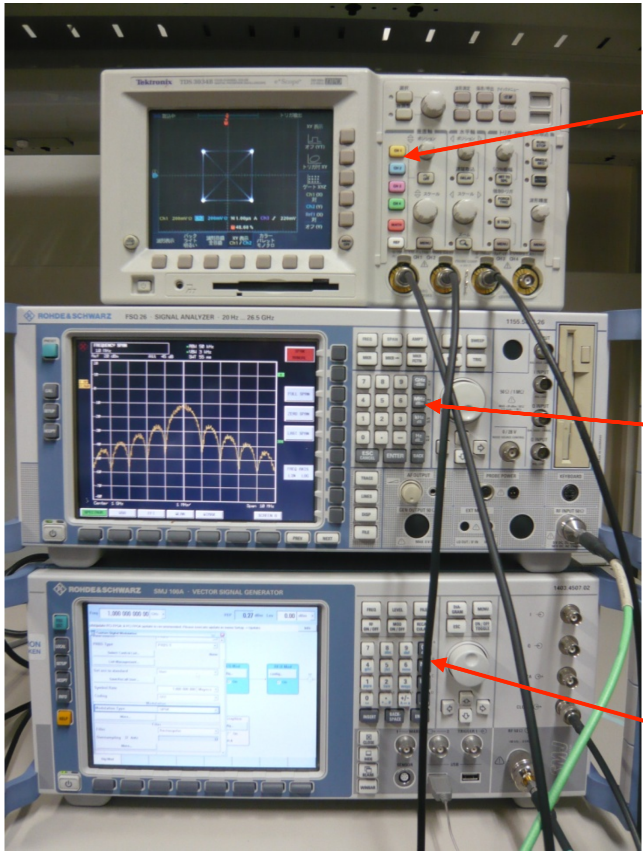
■ IQ analog modulation

$$s(t) = s_{\mathrm{BI}}(t)\cos(2\pi f_{\mathrm{c}} t) - s_{\mathrm{BQ}}(t)\sin(2\pi f_{\mathrm{c}} t)$$

Carrier frequency

Experiment

Set up



Osilloscope

- TDS 3034B (Tektoronix)
- Time domain analysis
- Constellation (X-Y) analysis

Spectrum analyzer

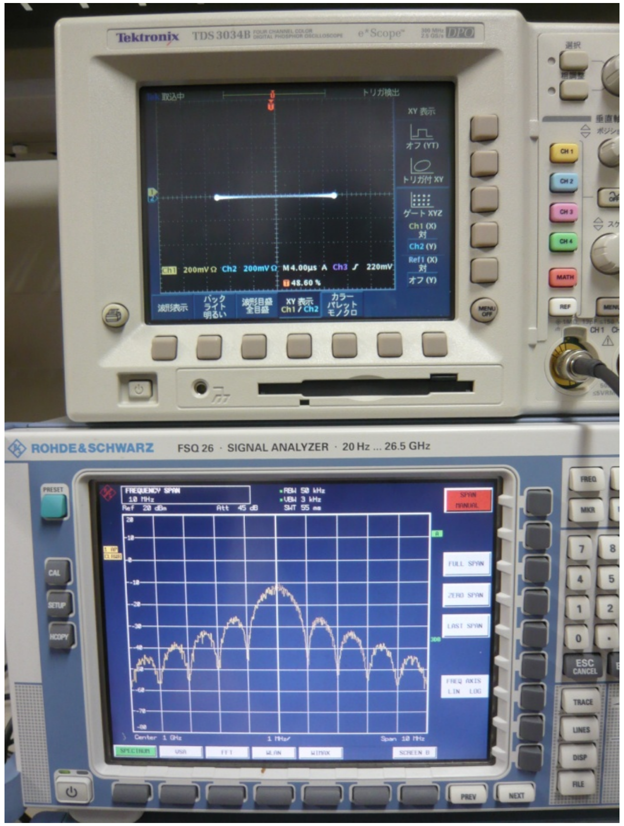
- FSQ 26（R&S）
- Spectrum analyss

Signal generator

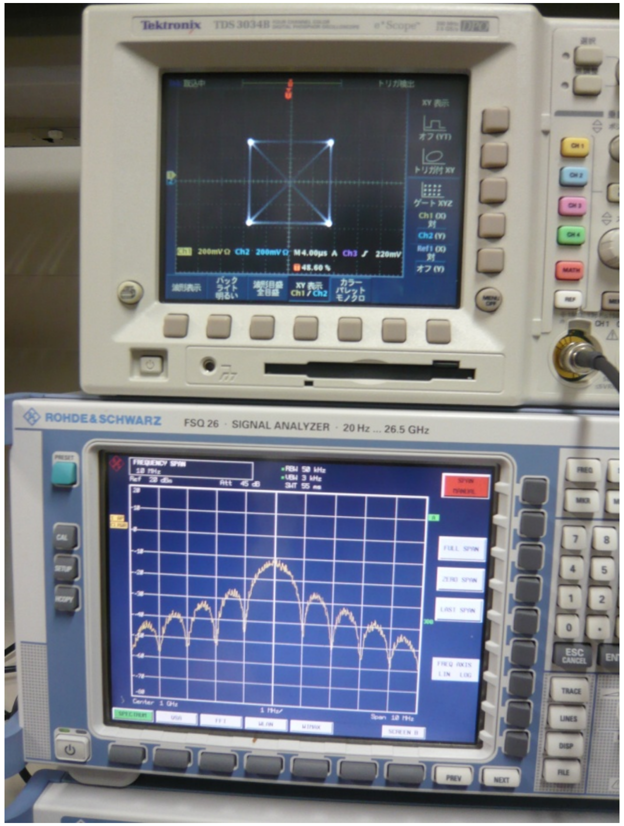
- SMJ 100A（R&S）
- Modulation, pulse shaping

BPSK(QPSK) + Rect. Pulse

BPSK

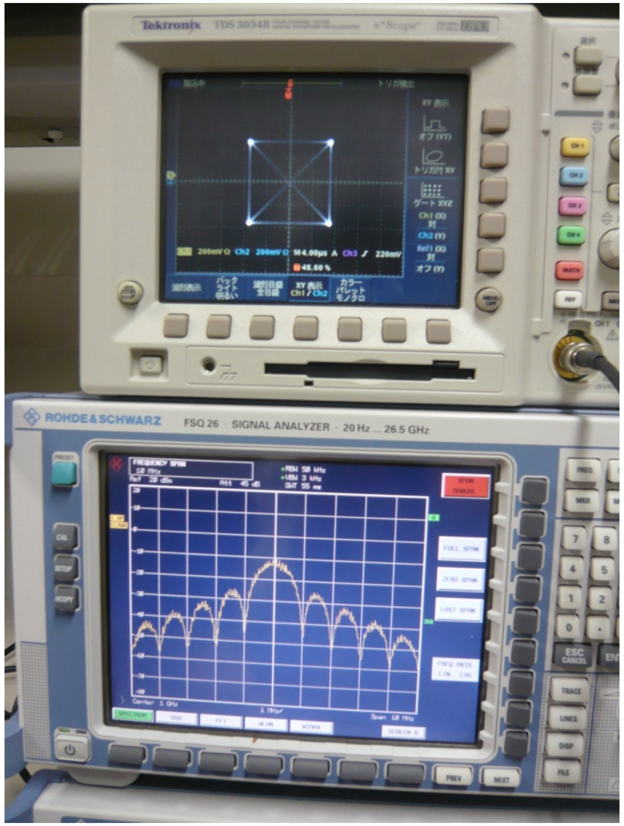


QPSK

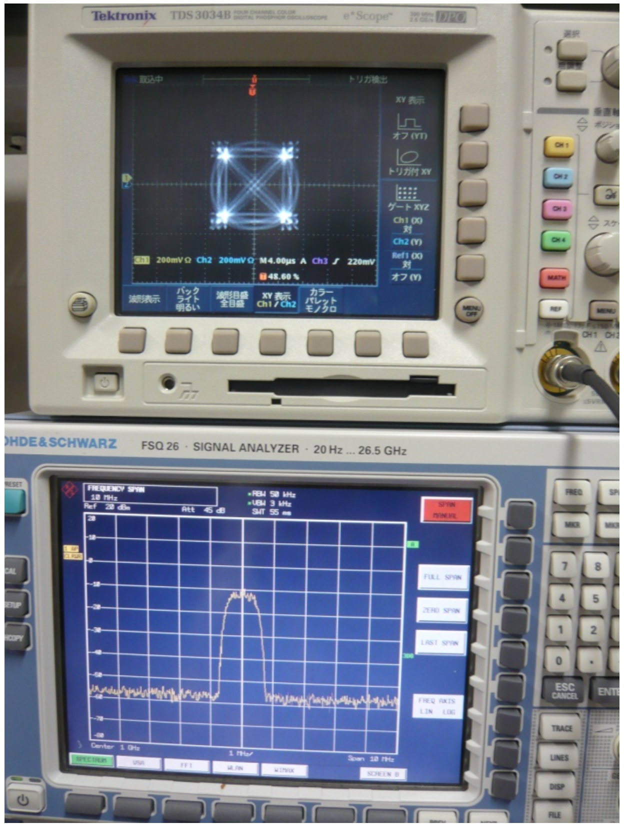


QPSK + Nyquist Pulse

Rectangular pulse

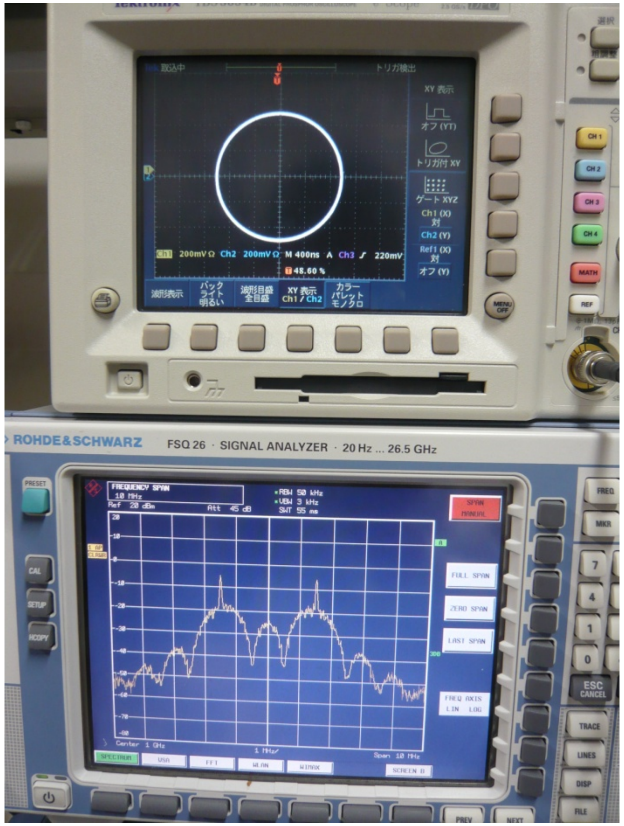


Nyquist pulse

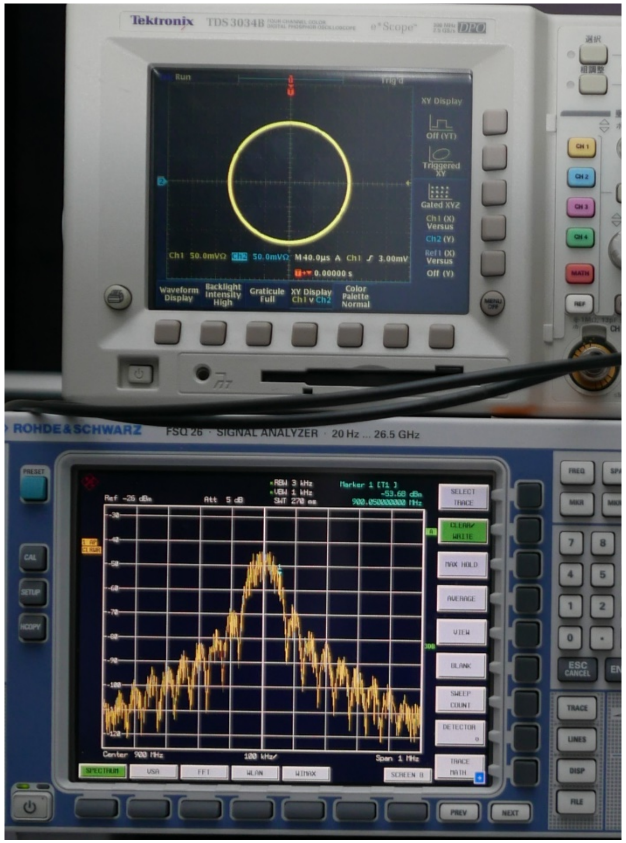


BFSK + Rect. Pulse

BFSK



MSK



MSK + Gaussian Pulse

Nyquist pulse, Gaussian pulse