2016 2Q
Wireless Communication Engineering

#3 Up/Down Conversion and Equivalent Baseband System

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Course Schedule (1)

	Date	Text	Contents
#1	June 17	1, 7	Introduction to wireless communication systems
#2	June 17	2, 5, etc	Link budget design of wireless access
#3	June 24		Up/down conversion and equivalent baseband system
#4	June 24	3.3, 3.4	Digital modulation and pulse shaping
	July 1		No class
#5	July 8	3.5	Demodulation and detection error due to noise
#6	July 8	4.4	Channel fading and diversity combining

From Previous Lecture

- Channel capacity

$$C = B\log_2(1+\gamma) = \alpha \times f_0 \times R \ \text{[bps]}$$

- Friis propagation model

$$P_\mathrm{r} = \left(\frac{\lambda_0}{4\pi d}\right)^2 G_\mathrm{r} G_\mathrm{t} P_\mathrm{t} \qquad \gamma = \left(\frac{\lambda_0}{4\pi \mathrm{d}}\right)^2 \cdot \frac{G_\mathrm{r} G_\mathrm{t} P_\mathrm{t}}{P_\mathrm{n}}$$

- User rate and multiple access

$$C_\mathrm{UE} = \frac{B\log_2(1+\gamma)}{N_\mathrm{UE}} = \frac{B\log_2(1+\gamma)}{\pi d_0^2 \eta}$$

- Design of wireless access systems

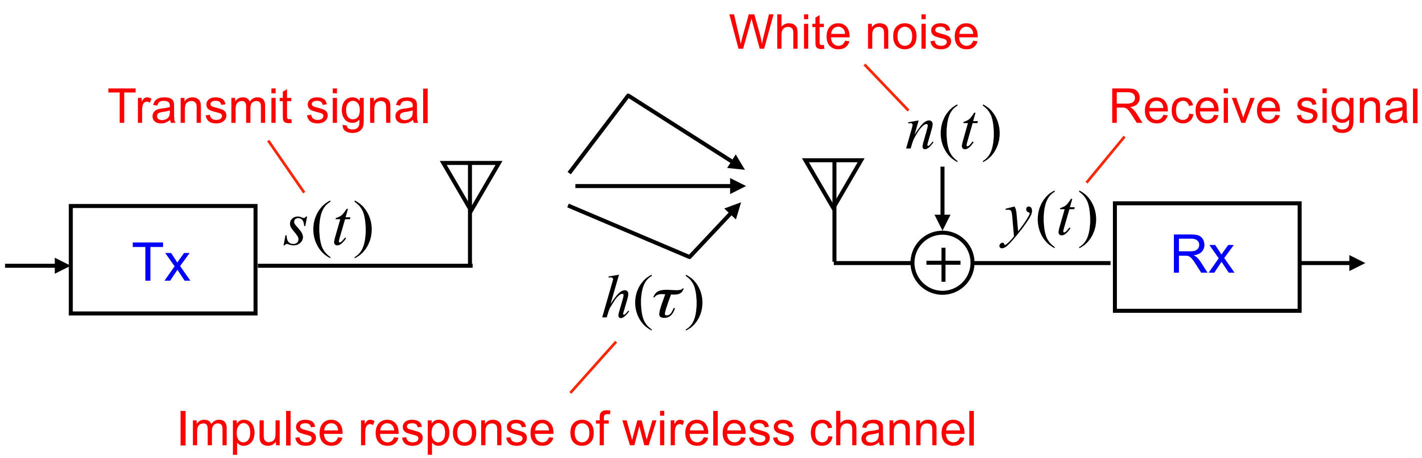
$$C_\mathrm{UE0}^\mathrm{req} \longrightarrow N_\mathrm{UE}, C \longrightarrow d_0, B, R \longrightarrow f_0, P_\mathrm{t}, G_\mathrm{t}$$

Contents

- Transmit signal (up conversion)
- Receive signal (down conversion)
- Equivalent baseband signal
- Auto-correlation & power spectrum
- Frequency domain analysis
- White noise

System Model

- System model



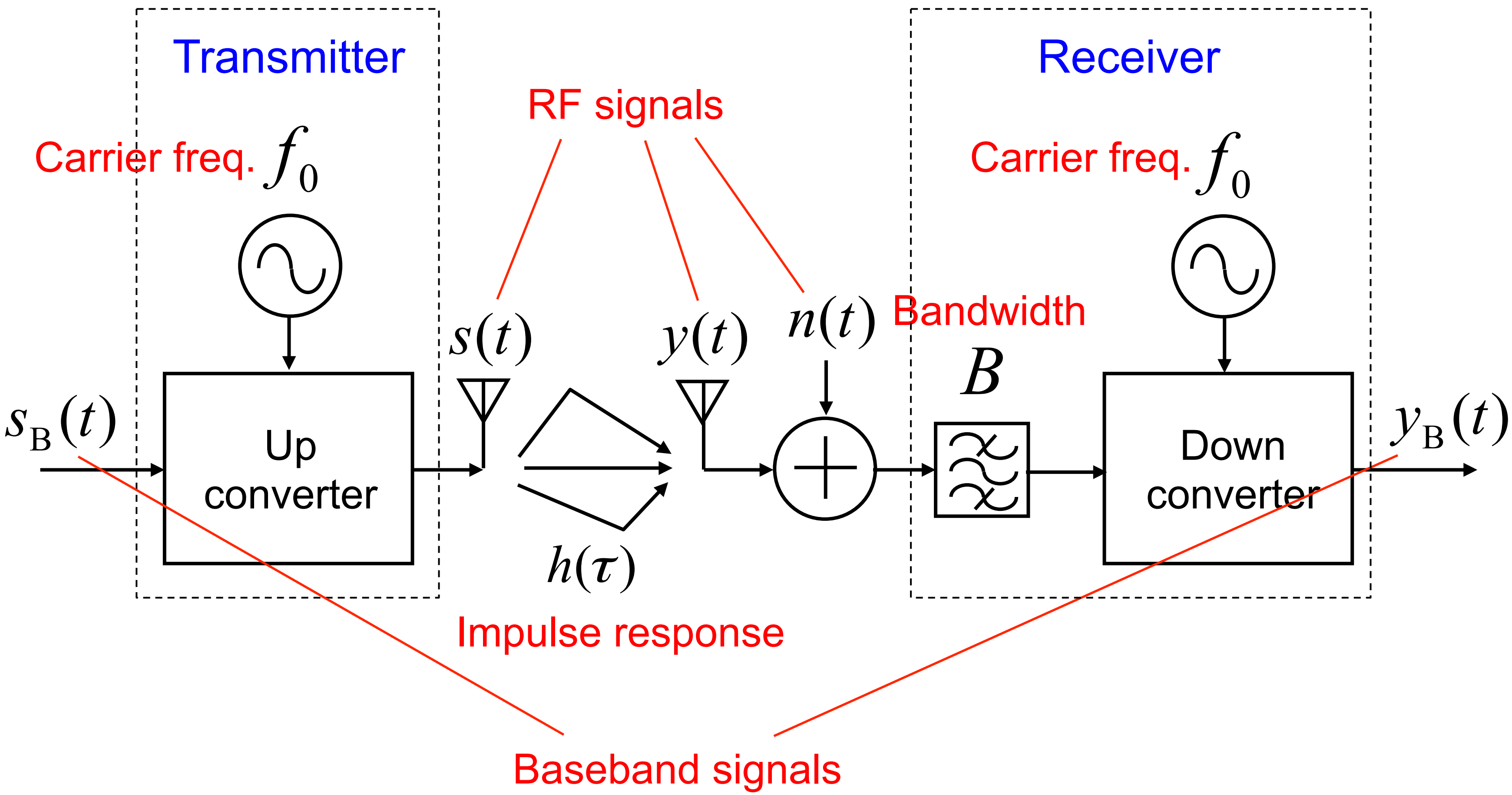
- Receive signal model

$$y(t) = \int h(\tau) s(t-\tau) \mathrm{d}\tau + n(t)$$

Assuming delay spread
(frequency selective fading)

Carrier freq. f_0 & bandwidth B

Carrier Frequency and Modem



Transmit Signal (Time Domain)

- Baseband & RF analytic signal

Baseband transmit signal: $s_{\mathrm{B}}(t) = s_{\mathrm{BR}}(t) + j s_{\mathrm{BI}}(t)$

In-phase Quadrature

RF analytic signal: $s_{\mathrm{R}}(t) = s_{\mathrm{B}}(t) e^{j2\pi f_0 t}$

Up conversion (BB → RF)

- Transmit signal

$$
\begin{aligned}
s(t) &= \mathrm{Re}\left[s_R(t)\right] \\
&= s_{\mathrm{BR}}(t)\cos 2\pi f_0 t - s_{\mathrm{BI}}(t)\sin 2\pi f_0 t \\
&= \sqrt{s_{\mathrm{BR}}^2(t) + s_{\mathrm{BI}}^2(t)}\cos\left(2\pi f_0 t + \angle s_{\mathrm{BI}}/s_{\mathrm{BR}}\right)
\end{aligned}
$$

Receive Signal (Time Domain)

- Receive signal

$$y(t) = \int h(\tau) s(t-\tau) \mathrm{d}\tau$$
$$= \mathrm{Re}\left[\int h(\tau) s_{\mathrm{R}}(t-\tau) \mathrm{d}\tau \right]$$
$$= \mathrm{Re}\left[\int h(\tau) s_{\mathrm{B}}(t-\tau) e^{j2\pi f_0 (t-\tau)} \mathrm{d}\tau \right]$$

- RF analytic & baseband (BB) receive signal

Analytic receive signal: $y_{\mathrm{R}}(t) = y(t) + j\,\mathrm{hilb}\left(y(t)\right)$

Hilbert transformation

Baseband receive signal: $y_{\mathrm{B}}(t) = y_{\mathrm{R}}(t) e^{-j2\pi f_0 t}$

Down conversion (RF → BB)

Equivalent Baseband Signal

- RF and baseband (BB) receive signal

$$y(t) = \mathrm{Re}\left[\int h(\tau) s_\mathrm{B}(t-\tau) e^{j2\pi f_0 (t-\tau)} \mathrm{d}\tau\right]$$

$$y_\mathrm{B}(t) = y_\mathrm{R}(t) e^{-j2\pi f_0 t} = \left(y(t) + j\,\mathrm{hilb}\left(y(t)\right)\right) e^{-j2\pi f_0 t}$$

- Equivalent baseband system

$$y_\mathrm{B}(t) = \int h(\tau) s_\mathrm{B}(t-\tau) e^{-j2\pi f_0 \tau} \mathrm{d}\tau = \int h_\mathrm{B}(\tau) s_\mathrm{B}(t-\tau) \mathrm{d}\tau$$

Separation from carrier freq.

Equivalent baseband impulse response: $h_\mathrm{B}(\tau) = h(\tau) e^{-j2\pi f_0 \tau}$

Phase rotation depending on carrier freq.

Auto-correlation & Power Spectrum

- Auto-correlation

$$R_{\mathrm{B}}^{\mathrm{s}}\left(\tau\right) = \mathrm{E}\left[s_{\mathrm{B}}^{*}(t)s_{\mathrm{B}}(t+\tau)\right]$$

- Power spectrum

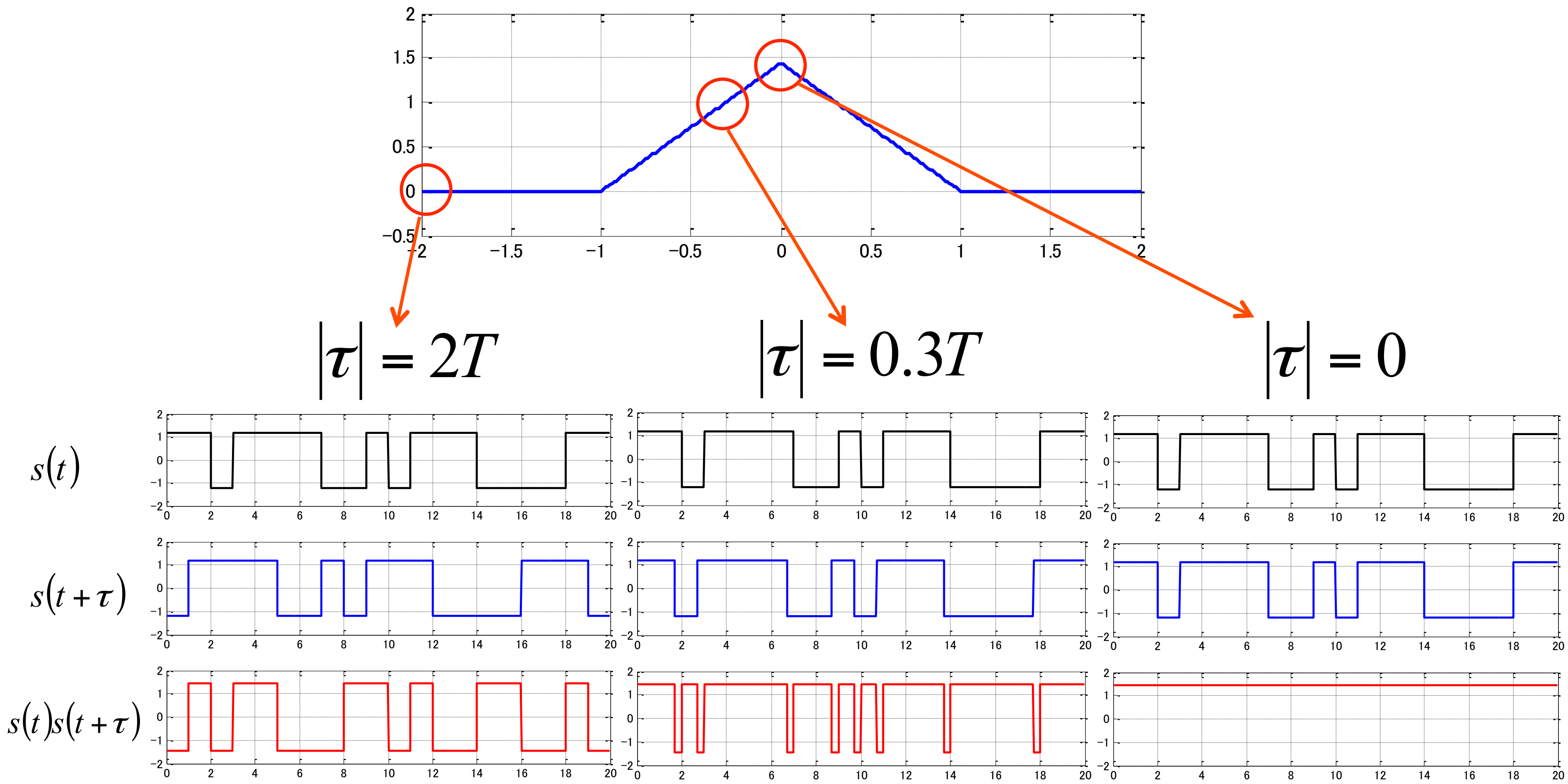
$$S_{\mathrm{B}}^{\mathrm{s}}\left(f\right) = \int_{-\infty}^{\infty} R_{\mathrm{B}}^{\mathrm{s}}\left(\tau\right)e^{-j2\pi f\tau}\,\mathrm{d}\tau$$

- (Transmit) power

$$P_{\mathrm{B}}^{\mathrm{s}} = R_{\mathrm{B}}^{\mathrm{s}}\left(0\right) = \int_{-\infty}^{\infty} S_{\mathrm{B}}^{\mathrm{s}}\left(f\right)\mathrm{d}f$$

Auto-correlation of Rectangular Random Pulse

$$R_s(\tau) = \mathrm{E}[s(t+\tau)s(t)]$$



Power Spectrum of Rectangular Random Pulse

$$R_t(\tau) = \begin{cases} 1 - \dfrac{|\tau|}{T}, & |\tau| < T \\ 0, & |\tau| \geq T \end{cases}$$

$$\parallel$$

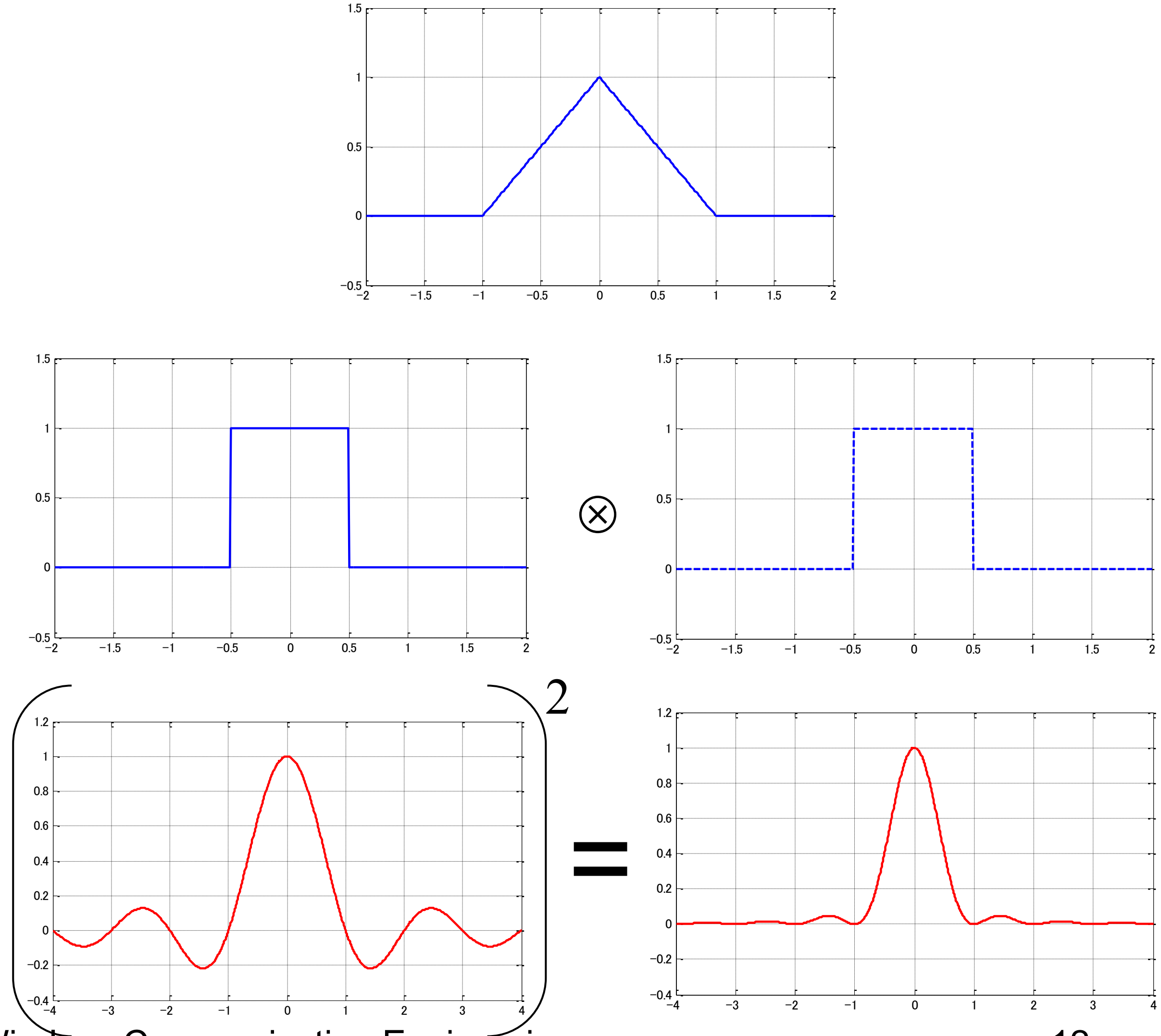
$$R_t(\tau) = \int_{-\infty}^{\infty} R_r(t) R_r(\tau - t) dt$$

$$R_r(\tau) = \begin{cases} 1, & |\tau| \leq T/2 \\ 0, & |\tau| > T/2 \end{cases}$$

⇕

$$S_t(f) = S_r(f) S_r(f)$$

$$S_r(f) = T \operatorname{sinc}(fT)$$



Analytic Signal (Freq. Domain)

- Auto-correlation of analytic signal

$$R_{\mathrm{R}}^{\mathrm{s}}(\tau)=\mathrm{E}\left[s_{\mathrm{R}}^{*}(t)s_{\mathrm{R}}(t+\tau)\right]$$

$$=\mathrm{E}\left[s_{\mathrm{B}}^{*}(t)s_{\mathrm{B}}(t+\tau)\right]e^{j2\pi f_0\tau}=R_{\mathrm{B}}^{\mathrm{s}}(\tau)e^{j2\pi f_0\tau}$$

Up conversion (BB → RF)

- Power spectrum of analytic signal

$$S_{\mathrm{R}}^{\mathrm{s}}(f)=\int_{-\infty}^{\infty}R_{\mathrm{R}}^{\mathrm{s}}(\tau)e^{-j2\pi f\tau}\,\mathrm{d}\tau=S_{\mathrm{B}}^{\mathrm{s}}(f-f_0)$$

Frequency conversion

Transmit Signal (Freq. Domain)

- Auto-correlation of transmit signal

$$s(t) = \mathrm{Re}\left[s_{\mathrm{R}}(t)\right] = \frac{1}{2}\left(s_{\mathrm{R}}(t) + s_{\mathrm{R}}^{*}(t)\right)$$

$$R^{\mathrm{s}}(\tau) = \mathrm{E}\left[s^{*}(t)s\ (t+\tau)\right]$$

Assuming independency between in-phase & quadrature signals

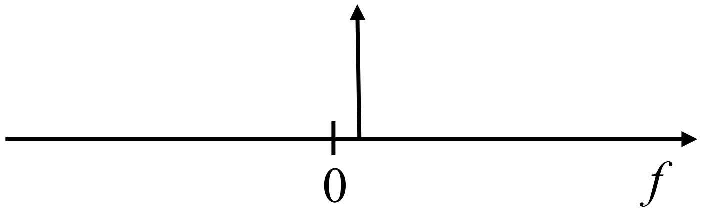
$$= \frac{1}{4}\left(R_{\mathrm{R}}^{\mathrm{s}}(\tau) + R_{\mathrm{R}}^{\mathrm{s}*}(\tau)\right) = \frac{1}{4}\left(R_{\mathrm{B}}^{\mathrm{s}}(\tau)e^{j2\pi f_0\tau} + R_{\mathrm{B}}^{\mathrm{s}*}(\tau)e^{-j2\pi f_0\tau}\right)$$

- Power spectrum of transmit signal

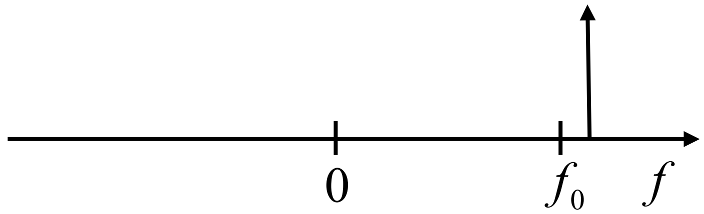
$$S^{\mathrm{s}}(f) = \int_{-\infty}^{\infty} R^{\mathrm{s}}(\tau)e^{-j2\pi f\tau}\,\mathrm{d}\tau = \frac{1}{4}\left(S_{\mathrm{B}}^{\mathrm{s}}(f - f_0) + S_{\mathrm{B}}^{\mathrm{s}}(-f - f_0)\right)$$

Positive freq. Negative freq.

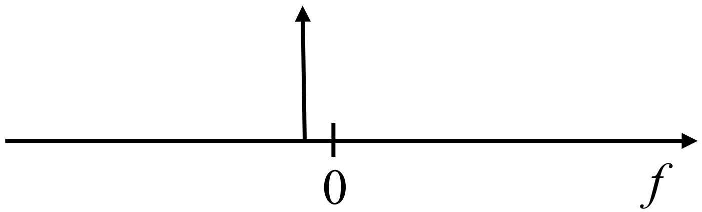
Example of Transmit Spectrum



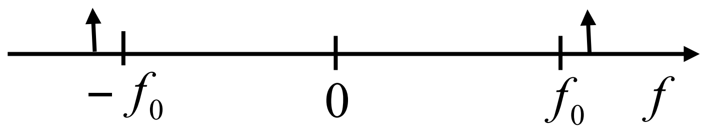
$$S_{\mathrm{B}}^{\mathrm{s}}(f)$$



$$S_{\mathrm{R}}^{\mathrm{s}}(f) = S_{\mathrm{B}}^{\mathrm{s}}(f - f_0)$$



$$S_{\mathrm{B}}^{\mathrm{s}}(-f)$$



$$S^{\mathrm{s}}(f) = \frac{1}{4}\left(S_{\mathrm{B}}^{\mathrm{s}}(f - f_0) + S_{\mathrm{B}}^{\mathrm{s}}(-f - f_0)\right)$$

Receive Signal (Freq. Domain)

- Auto-correlation of receive signal

$$y_{\mathrm{B}}(t) = \int h_{\mathrm{B}}(\tau) s_{\mathrm{B}}(t-\tau)\,\mathrm{d}\tau$$

$$R_{\mathrm{B}}^{\mathrm{y}}(\tau) = \mathrm{E}\left[y_{\mathrm{B}}^{*}(t)\, y_{\mathrm{B}}(t+\tau)\right]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_{\mathrm{B}}^{*}(\tau_1) h_{\mathrm{B}}(\tau_2) R_B^{s}(\tau-\tau_1+\tau_2)\,\mathrm{d}\tau_1\,\mathrm{d}\tau_2$$

Double convolution

- Power spectrum of receive signal

$$S_{\mathrm{B}}^{\mathrm{y}}(f) = \int_{-\infty}^{\infty} R_{\mathrm{B}}^{\mathrm{y}}(\tau) e^{-j2\pi f\tau}\,\mathrm{d}\tau = \left|H_{\mathrm{B}}(f)\right|^2 S_{\mathrm{B}}^{\mathrm{s}}(f)$$

Feature of double convolution

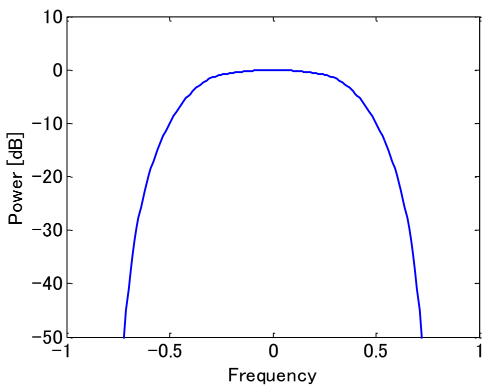
$$H_{\mathrm{B}}(f) = \int_{-\infty}^{\infty} h_{\mathrm{B}}(\tau) e^{-j2\pi f\tau}\,\mathrm{d}\tau$$

Example of Receive Spectrum

$$S_{\mathrm{B}}^{\mathrm{y}}(f) = |H_{\mathrm{B}}(f)|^2 S_{\mathrm{B}}^{\mathrm{s}}(f)$$

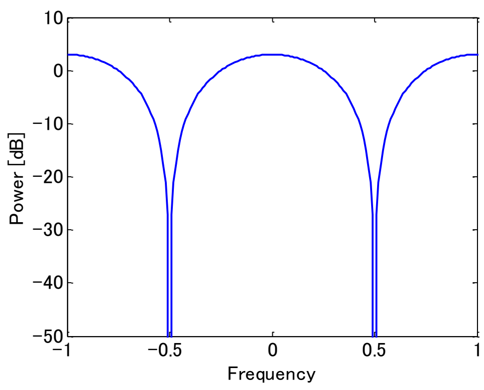
$S_{\mathrm{B}}^{\mathrm{s}}(f)$

Transmit spectrum



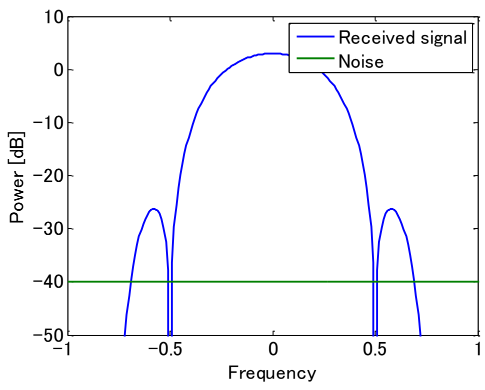
$|H_{\mathrm{B}}(f)|^2$

Channel response



$S_{\mathrm{B}}^{\mathrm{y}}(f)$

Receive spectrum



White Noise

- Receive signal

White noise

$$y(t) = \int h(\tau)s(t-\tau)\mathrm{d}\tau + n(t)$$

Transmit (depends on frequency & bandwidth)

- Auto-correlation of white noise

$$R^{\mathrm{n}}(\tau) = \mathrm{E}\left[n^*(t)n\ (t+\tau)\right] = \frac{N_0}{2}\delta(0)$$

- Power spectrum of white noise

$$S^{\mathrm{n}}(f) = \int_{-\infty}^{\infty} R^{\mathrm{n}}(\tau)e^{-j2\pi f\tau}\,\mathrm{d}\tau = \frac{N_0}{2}$$

Equivalent Baseband Noise

- Noise in equivalent baseband system

$$n_{\mathrm{R}}(t) = n(t) + j\,\mathrm{hilb}\big(n(t)\big)$$

$$n_{\mathrm{B}}(t) = n_{\mathrm{R}}(t)e^{-j2\pi f_0 t}$$

- Power spectrum and auto-correlation

$$S_{\mathrm{B}}^{\mathrm{n}}(f) = \begin{cases} N_0, & -\dfrac{B}{2} \le f \le \dfrac{B}{2} \\ 0, & \text{otherwise} \end{cases}$$

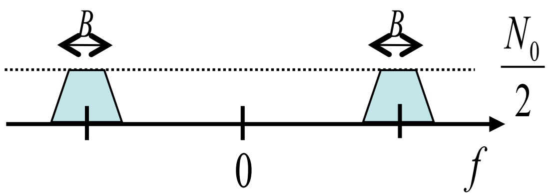
$$R_{\mathrm{B}}^{\mathrm{n}}(\tau) = \int_{-\infty}^{\infty} S_{\mathrm{B}}^{\mathrm{n}}(f)e^{j2\pi f\tau}\,\mathrm{d}\tau = N_0 B\,\mathrm{sinc}(\tau B)$$

Power Spectrum of Bandpath Noise

$$P_{\mathrm{n}} = N_0 B = kT_{\mathrm{emp}} B = \alpha N_0 f_0$$

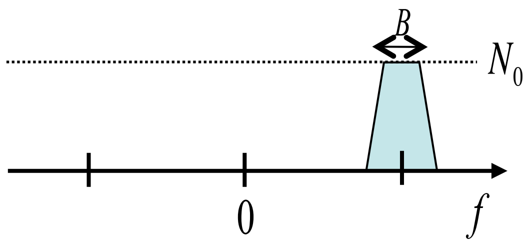
$S^{\mathrm{n}}(f)$

Bandpass noise



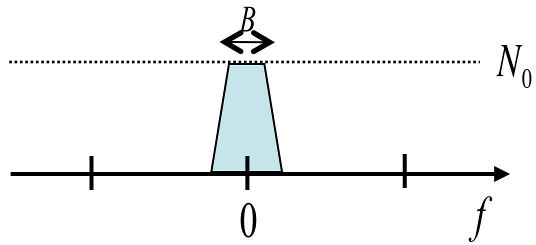
$S^{\mathrm{n}}_{\mathrm{R}}(f)$

Analytic signal of noise



$S^{\mathrm{n}}_{\mathrm{B}}(f)$

Baseband noise



Summary

- Equivalent baseband system

$$y_{\mathrm{B}}(t) = \int h_{\mathrm{B}}(\tau) s_{\mathrm{B}}(t-\tau)\mathrm{d}\tau$$

$$y_{\mathrm{B}}(t) = y_{\mathrm{R}}(t)e^{-j2\pi f_0 t} \qquad y_{\mathrm{R}}(t) = y(t) + j\,\mathrm{hilb}\big(y(t)\big)$$

- Power spectrum of transmit signal

$$S^{\mathrm{s}}(f) = \frac{1}{4}\left(S_{\mathrm{B}}^{\mathrm{s}}(f-f_0) + S_{\mathrm{B}}^{\mathrm{s}}(-f-f_0)\right)$$

- Power spectrum of receive signal

$$S_{\mathrm{B}}^{\mathrm{y}}(f) = \left|H_{\mathrm{B}}(f)\right|^2 S_{\mathrm{B}}^{\mathrm{s}}(f)$$