Neutron Transport Theory Lecture Note (2)
-Multiplication Factor and Criticality-

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2. Nuclear Fission Chain Reactions

2.1 Nuclear fission

(1)Mass defect and binding energy

The direct determination of nuclear masses has shown that the actual mass is always less than the sum of the masses of the constituent nucleons.

The difference is called mass defect.

The energy equivalent of the mass defect is called the binding energy.

$$\Delta E = \Delta mc^2$$

The energy should be obtained by the composition (or fusion) of lightest nuclei or by splitting (or fission) of those of high mass number.

(2)Nuclear Fission Reactions

- Nuclear fission by neutron capture

  Ex. $^{1}_{0}n + ^{235}_{92}U \rightarrow (^{236}_{92}U)^{*} \rightarrow$ fission products + neutrons

- Fissile nuclides : ^{235}U、^{233}U、^{239}Pu、^{241}Pu

- Fertile : ^{238}U、^{232}Th

  (Isotopes that can be transmuted into fissile nuclides via neutron captures.)

- Decay heat : The energy released in radioactive decay reactions of fission products

- Fission neutrons
  - prompt neutrons : appear instantaneously
  - delayed neutrons (less than 1%) : appear by the decay of fission products

- Average number of fission neutrons $\nu \cong 2.4$ (in ^{235}U)

- Average fission neutron energy : ≅ 2MeV

(3)η value

Definition

$\eta \equiv$ (Average number of neutrons produced per neutron absorbed in fuel)

For a fuel composed of a single fissile isotope

$$\eta = \frac{\nu\sigma_f}{\sigma_a}$$

For a fuel that contains a mixture of isotopes

$$\eta = \frac{\sum_j \nu_j \Sigma_f^j}{\sum_j \Sigma_a^j}$$ j : index of nuclide

2.2 Multiplication Factor and Criticality

(1)Multiplication factor

Definition

$k \equiv$ Multiplication factor

$$\equiv \frac{\text{Number of neutrons in one generation}}{\text{Number of neutrons in preceding generation}}$$

$k<1$ subcritical

$k=1$ critical

$k>1$ supercritical

(2)Criticality in thermal reactors

Thermal reactor : Fast fission neutrons are slowed down by low mass number materials. The low energy neutrons (thermal neutrons) occur the next fissions.

(Fission cross section is the largest in low energy, so it is easy to maintain fission reaction chains.)

Definition

Thermal utilization f : Conditional probability that if neutron is absorbed, it will be absorbed in the fuel

Fast fission factor ε：

$$\varepsilon \equiv \frac{\text{Total number of fission neutrons(from both fast and thermal fission)}}{\text{Number of fission neutrons from thermal fissions}}$$

Resonance escape probability p：

$$p \equiv \left[\begin{array}{c}\text{Fraction of fission neutrons that manage} \\ \text{to slow down from fission to thermal energies} \\ \text{without being absorbed.}\end{array}\right]$$

- Multiplication factor in infinite medium（infinite multiplication factor） k_∞

$$k_\infty = \frac{\text{npf}\eta\varepsilon}{\eta} = \text{pf}\eta\varepsilon$$ （four-factor formula）

- Multiplication factor in finite medium

$P_{FNL} \equiv$ [Probability that fast neutron will not leak out (fast nonleakage)]

$P_{TNL} \equiv$
[Probability that thermal neutron will not leak out (thermal nonleakage)]

$$k_{eff} = \text{pf}\eta\varepsilon P_{FNL} P_{TNL}$$ （six-factor formula）

k_{eff}：Effective multiplication factor