

Neutron Transport Theory Lecture Note (1)

-Nuclear Reactions and Nuclear Cross Section

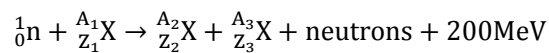
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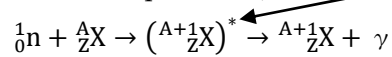
1. Nuclear Reactions and Nuclear Cross Sections

1.1 Reactions between Neutrons and nuclei

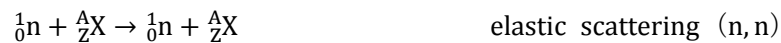
- Nuclear fission (n, fission) :



- Radioactive capture (n, γ) : excited state



- Scattering (n,n) or (n,n') :



1.2 Microscopic Cross Sections

The probability that a neutron-nuclear reaction will occur is characterized by a quantity called a nuclear cross section.

(1) Definition

Consider a beam of neutrons travelling with the same speed and direction and a sufficiently thin target (one atomic layer thick).

The rate R at which reactions occur per unit area on the target:

$$R = \sigma \cdot I \cdot N_A \quad [\text{cm}^{-2} \cdot \text{s}^{-1}]$$

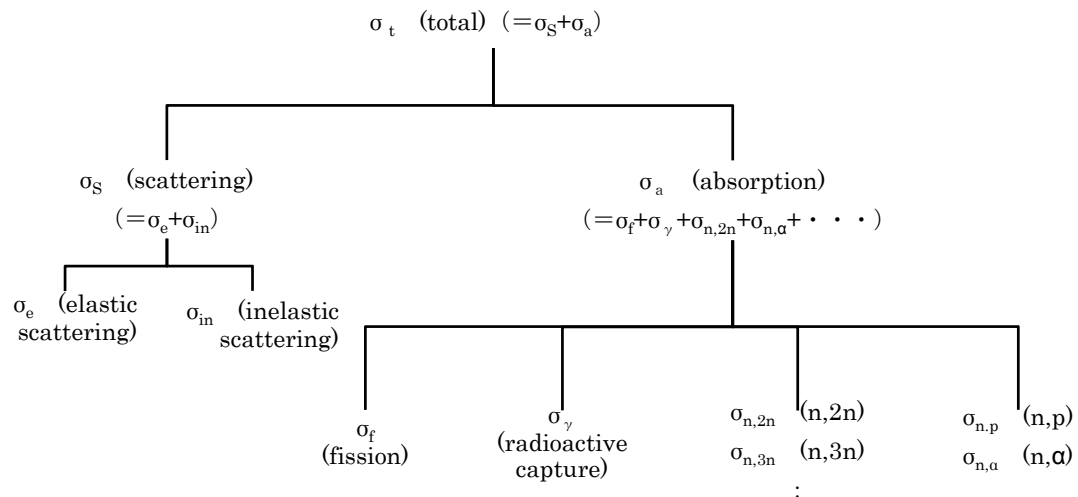
The microscopic cross section σ is defined as

$$\begin{aligned} \sigma &= \frac{(R/N_A)}{I} [\text{cm}^2] \\ &= \frac{(\text{Number of reactions/nucleus/sec})}{(\text{Number of incident neutrons/cm}^2/\text{sec})} \end{aligned}$$

The unit : b (barn)

$$1\text{b} = 10^{-24}\text{cm}^2$$

(2) Type of neutron cross sections



Microscopic cross sections depend on the nucleus, reactions and incident neutron speed (energy)

1.3 Macroscopic Cross Sections

(1) Definition

Consider a monoenergetic neutron beam incident on the surface of a target of arbitrary thickness.

The total reaction rate per unit area in dx :

$$dR = \sigma_t I N dx$$

The dR can be equal to the decrease in beam intensity between x and $x+dx$ (with the prescription that any type of interactions will remove an incident neutron)

$$-dI(x) = \sigma_t I N dx$$

$$\therefore \frac{dI}{dx} = -N \sigma_t I(x)$$

$$\therefore I(x) = I_0 \exp(-\sigma_t N x)$$

The definition of total macroscopic cross section

$$\Sigma_t \equiv \sigma_t N [\text{cm}^{-1}]$$

(2) Neutron mean free path and collision frequency

(Probability that a neutron moves a distance x without any interaction)

$$= \frac{I_0 \exp(-\Sigma_t x)}{I_0} = \exp(-\Sigma_t x)$$

So

$$\begin{aligned} & \text{(probability that a neutron has its first interaction in } dx) \\ &= \exp(-\Sigma_t x) \cdot \Sigma_t dx \equiv p(x) dx \end{aligned}$$

We can calculate the average distance a neutron travels before interacting with a nucleus in the sample.

$$\begin{aligned} \bar{x} &\equiv \int_0^\infty x p(x) dx = \Sigma_t \int_0^\infty x \exp(-\Sigma_t x) dx \\ &= \frac{1}{\Sigma_t} \quad \text{(neutron mean free path)} \end{aligned}$$

Σ_t is the probability per unit path length that a neutron will undergo a reaction, so

$$\begin{aligned} & \text{(The collision frequency)} = v \Sigma_t [\text{s}^{-1}] \\ & \text{where } v \text{ is the neutron speed.} \end{aligned}$$

(3) Macroscopic cross sections for specific reaction

ex. macroscopic fission cross section

$$\Sigma_f = \sigma_f N$$

macroscopic absorption cross section

$$\Sigma_a = \sigma_a N$$

(4) Macroscopic cross sections for mixture

ex. The total cross section of homogenized mixture of three different species of nuclide X, Y, and Z:

$$\Sigma_t = N_X \sigma_t^X + N_Y \sigma_t^Y + N_Z \sigma_t^Z$$