

低雑音増幅回路

通信機器システムの要

雑音の評価

雑音の定式化

瞬時値での評価→困難(不可)



統計的な評価

$$\text{2乗平均値} : \overline{v_n^2} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} v_n^2(t) dt$$

複数の雑音源の取り扱い

$$\begin{aligned} & \lim_{T \rightarrow \infty} \left[\frac{1}{T} \int_{-T/2}^{T/2} \{v_{n1}(t) + v_{n2}(t)\}^2 dt \right] \\ &= \lim_{T \rightarrow \infty} \left[\frac{1}{T} \left\{ \int_{-T/2}^{T/2} v_{n1}^2(t) + 2v_{n1}(t)v_{n2}(t) + v_{n2}^2(t) dt \right\} \right] \\ &= \overline{v_{n1}^2} + \lim_{T \rightarrow \infty} \frac{2}{T} \int_{-T/2}^{T/2} \underline{\underline{v_{n1}(t)v_{n2}(t) dt}} + \overline{v_{n2}^2} \end{aligned}$$

$v_{n1}(t)$ と $v_{n2}(t)$ が全く独立：無相関

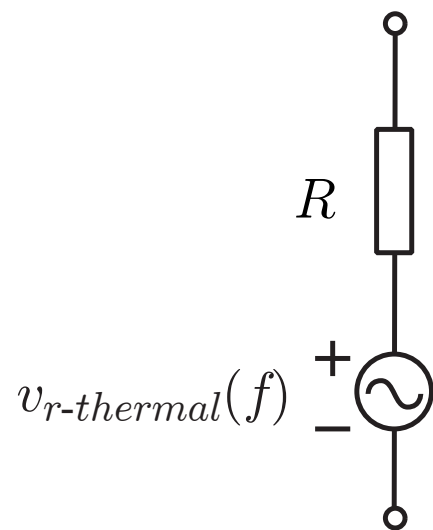
$$\int_0^\tau v_{n1}(t)v_{n2}(t)dt=0$$

雑音源を考慮した回路素子モデル

熱雑音, $1/f$ 雑音, ショット雑音

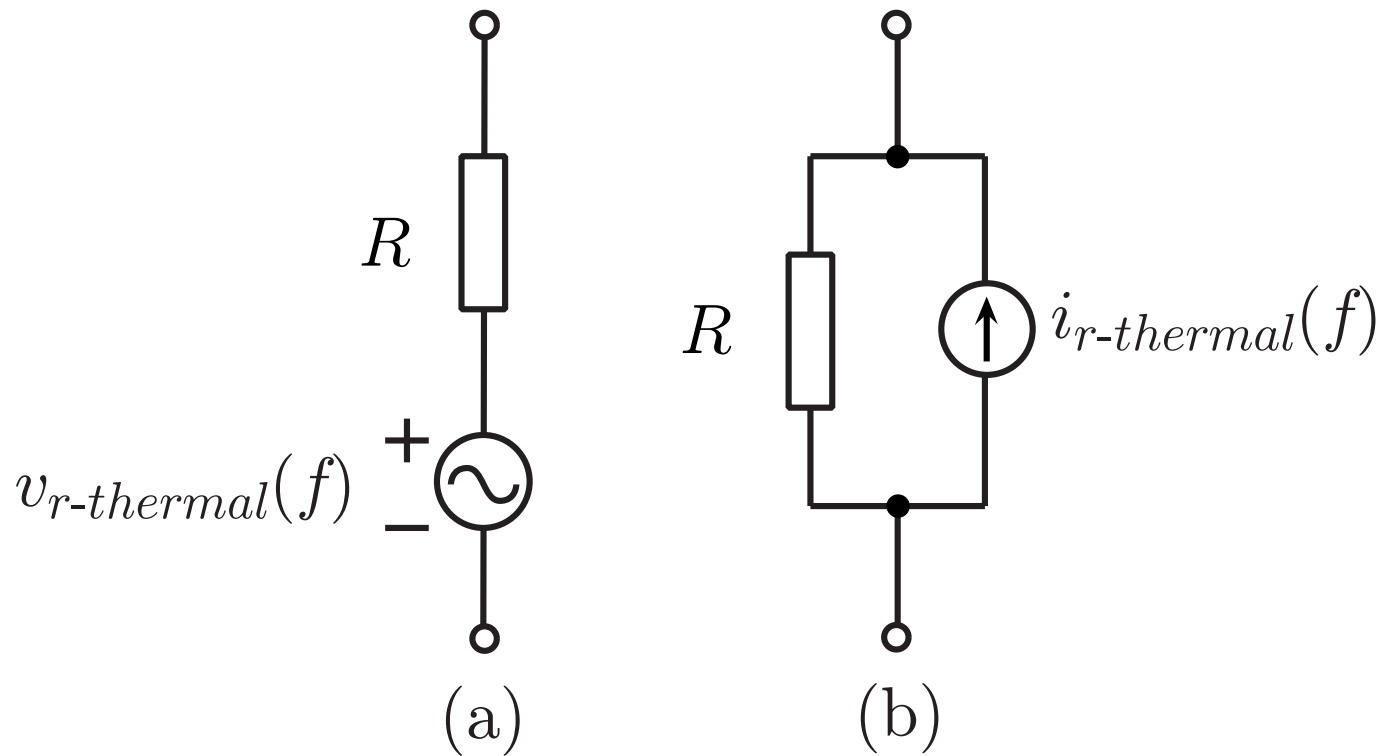
熱雑音: キャリアが移動する際のランダムな動きによって
生じる雑音

抵抗の熱雑音



$$\overline{v_{r-thermal}^2(f)} = 4kTR [\text{V}^2/\text{Hz}]$$

問 「電源の等価性」の定理を用いて
下記に示す抵抗の等価回路の雑音を表している電流源の値を求めよ.



$$v_{r-thermal}^2(f) = 4kTR [\text{V}^2/\text{Hz}]$$

MOSTランジスタの熱雑音

キャリアがチャネルを通過する際に発生

$$\overline{v_{mos-thermal}^2} = 4kT\gamma_n \frac{1}{g_m}$$

$$\overline{i_{mos-thermal}^2} = 4kT\gamma_n g_m$$

$$\gamma_n \approx \frac{2}{3} \quad (\text{最小線幅が短くなると増加})$$

バイポーラトランジスタの熱雑音

$$\text{ベース広がり抵抗} : \overline{v_{b-thermal}(f)^2} = 4kTr_b$$

エミッタ抵抗, コレクタ・エミッタ間抵抗: 仮想の抵抗



熱雑音の発生無し

スペクトラム強度が一定: 白色雑音

1/f雑音: シリコンの汚れや結晶欠陥によりキャリアが捕らわれたり, 捕らわれたキャリアが離されたりを繰り返すというランダムな過程によって生じる雑音

直流電流がないと発生しない

$$\overline{v_{mos-1/f}^2(f)} = \frac{\alpha_{1/f}}{C_{OX}WLf}$$

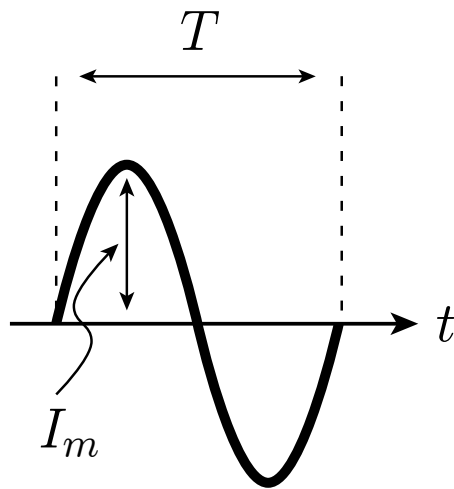
$$\overline{i_{b-1/f}^2(f)} = \frac{K_{1/f}I_B^a}{f}$$

ショット雑音:

電流=キャリアによる電流パルスの和の平均値
電流の平均値からの揺らぎ

$$\overline{i_{b-shot}^2(f)} = 2qI_B$$

$$\overline{i_{c-shot}^2(f)} = 2qI_C$$



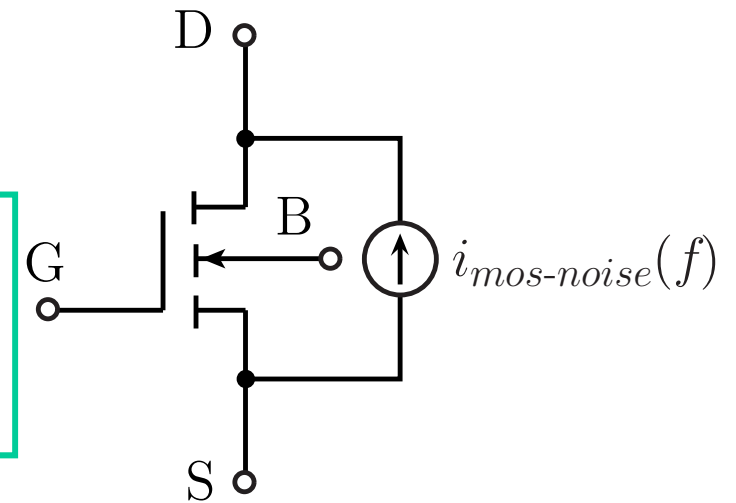
$$Q = \int_0^{T/2} I_m \sin(2\pi \frac{t}{T}) dt = \frac{I_m T}{\pi}$$

$$1\text{nA}@1\text{GHz} \rightarrow 1.6 \times 10^{-19} \text{ C} \times 2 \text{ 個}$$

雑音源を含むトランジスタモデル

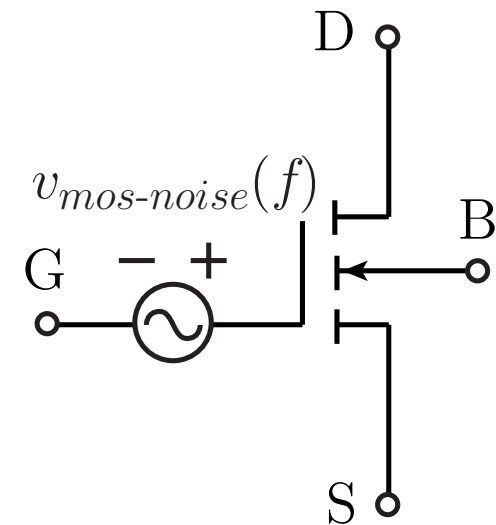
MOSTランジスタモデル

$$\overline{i_{mos-noise}^2(f)} = 4kT\gamma_n g_m + \frac{\alpha_{1/f} g_m^2}{C_{OX}WLf}$$



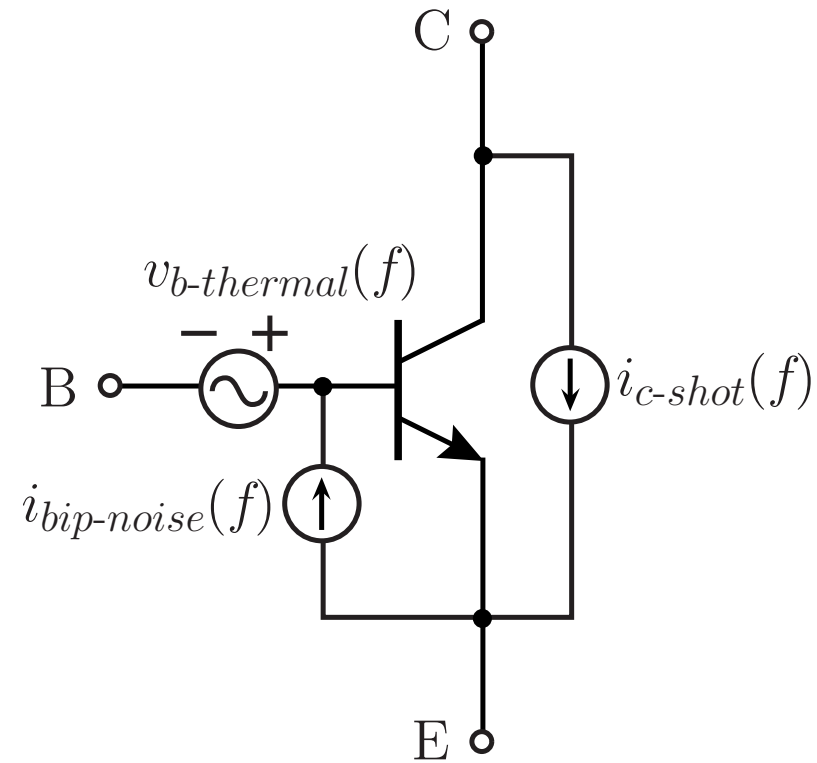
(a)

$$\overline{v_{mos-noise}^2(f)} = 4kT\gamma_n \frac{1}{g_m} + \frac{\alpha_{1/f}}{C_{OX}WLf}$$



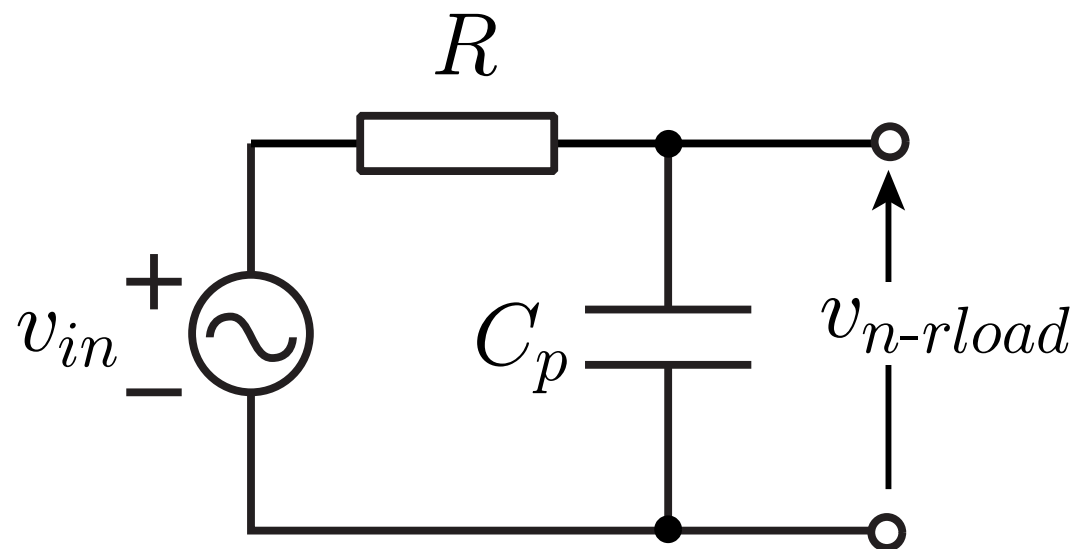
(b)

バイポーラトランジスタモデル



$$\overline{i_{bip-noise}^2(f)} = \frac{K_{1/f} I_B^a}{f} + 2qI_B$$

雑音解析の例(抵抗が発生する雑音)

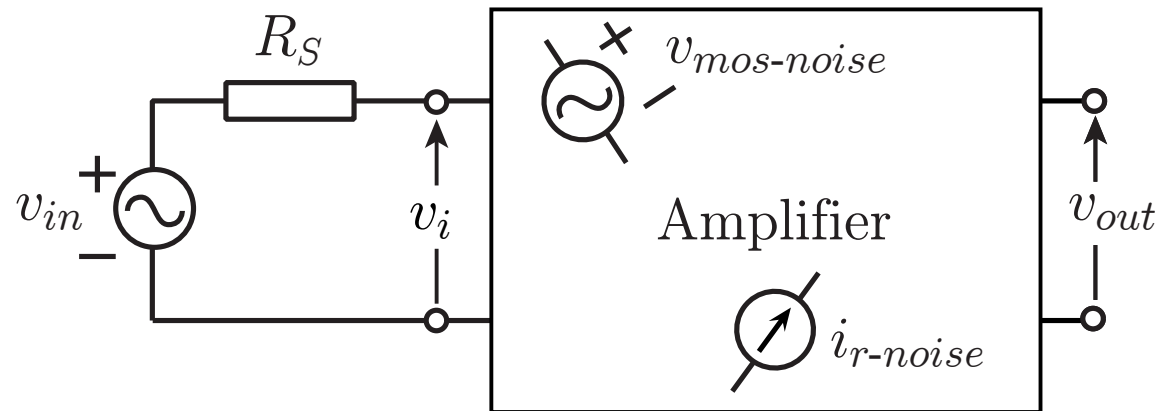


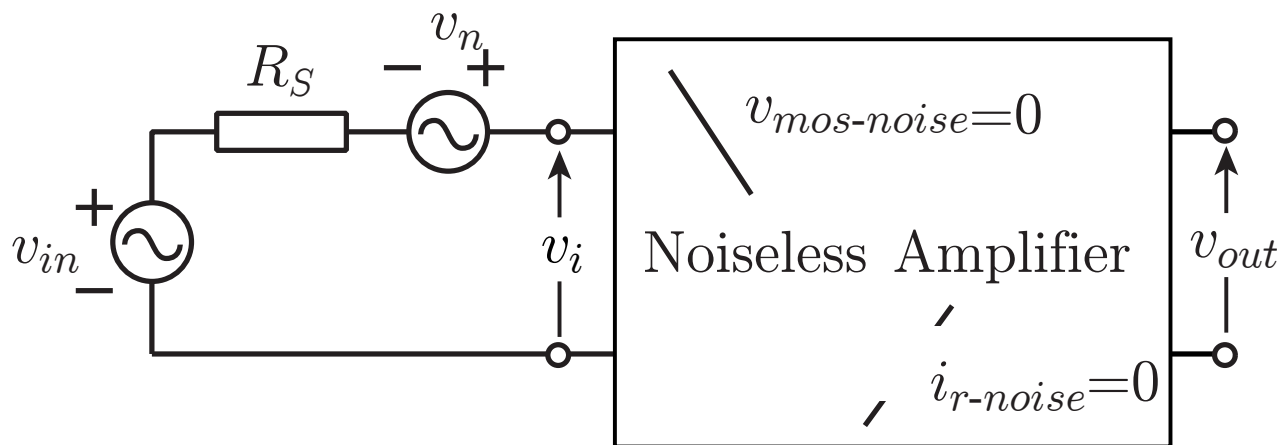
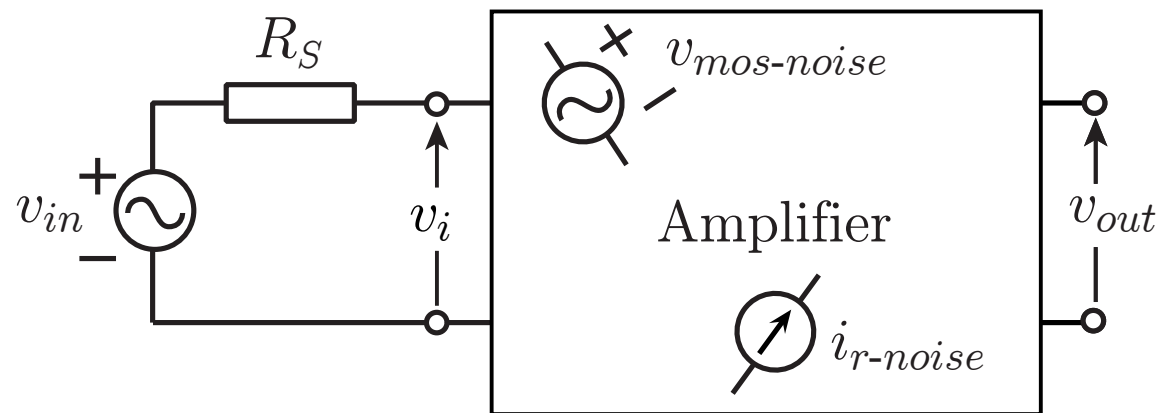
問 1 上の回路の伝達関数 $T_{rload}(s) = \frac{v_{n-rload}}{v_{in}}$ を求めよ.

問 2 $\overline{v_{n-rload}^2} = \int_0^\infty |T_{rload}(j\omega)|^2 \frac{4kTR}{2\pi} d\omega$ を求めよ.

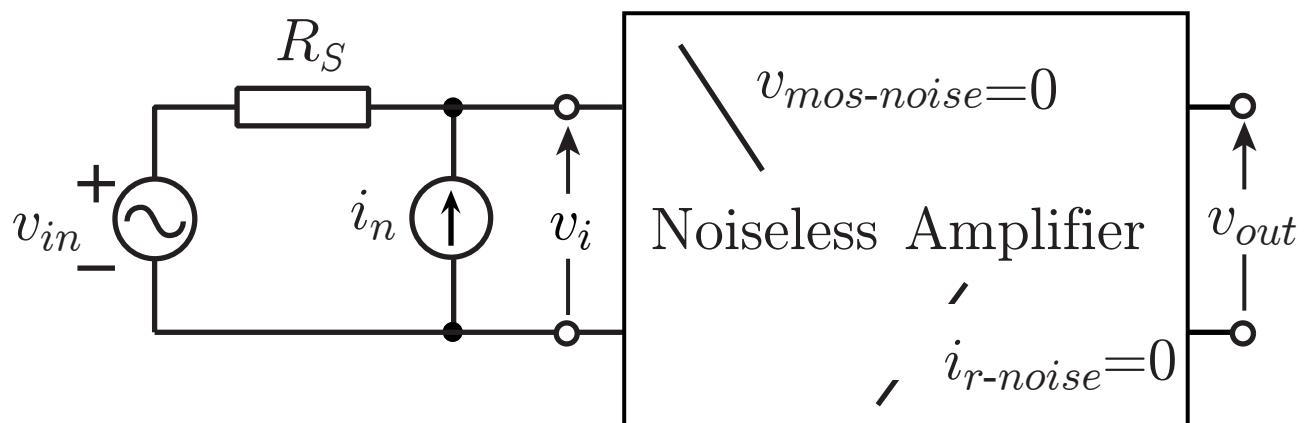
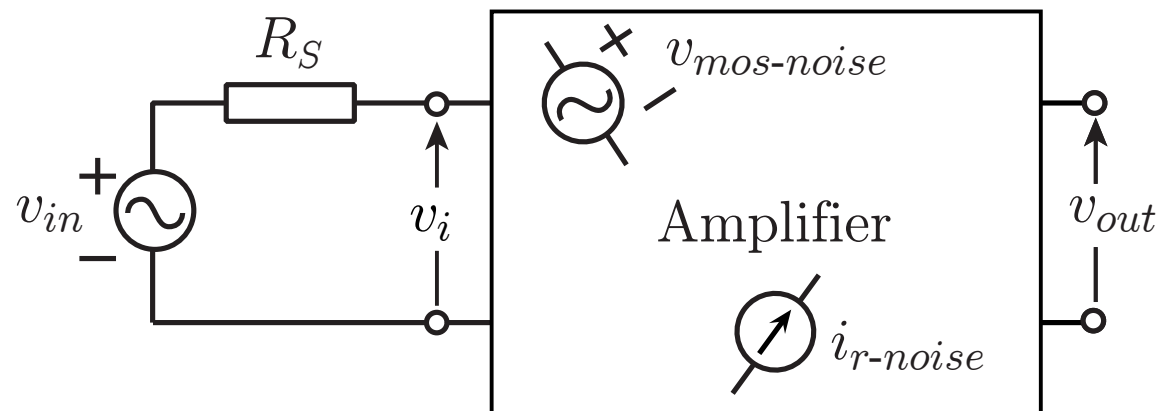
増幅回路と雑音

増幅回路内部で発生する
雑音の等価表現

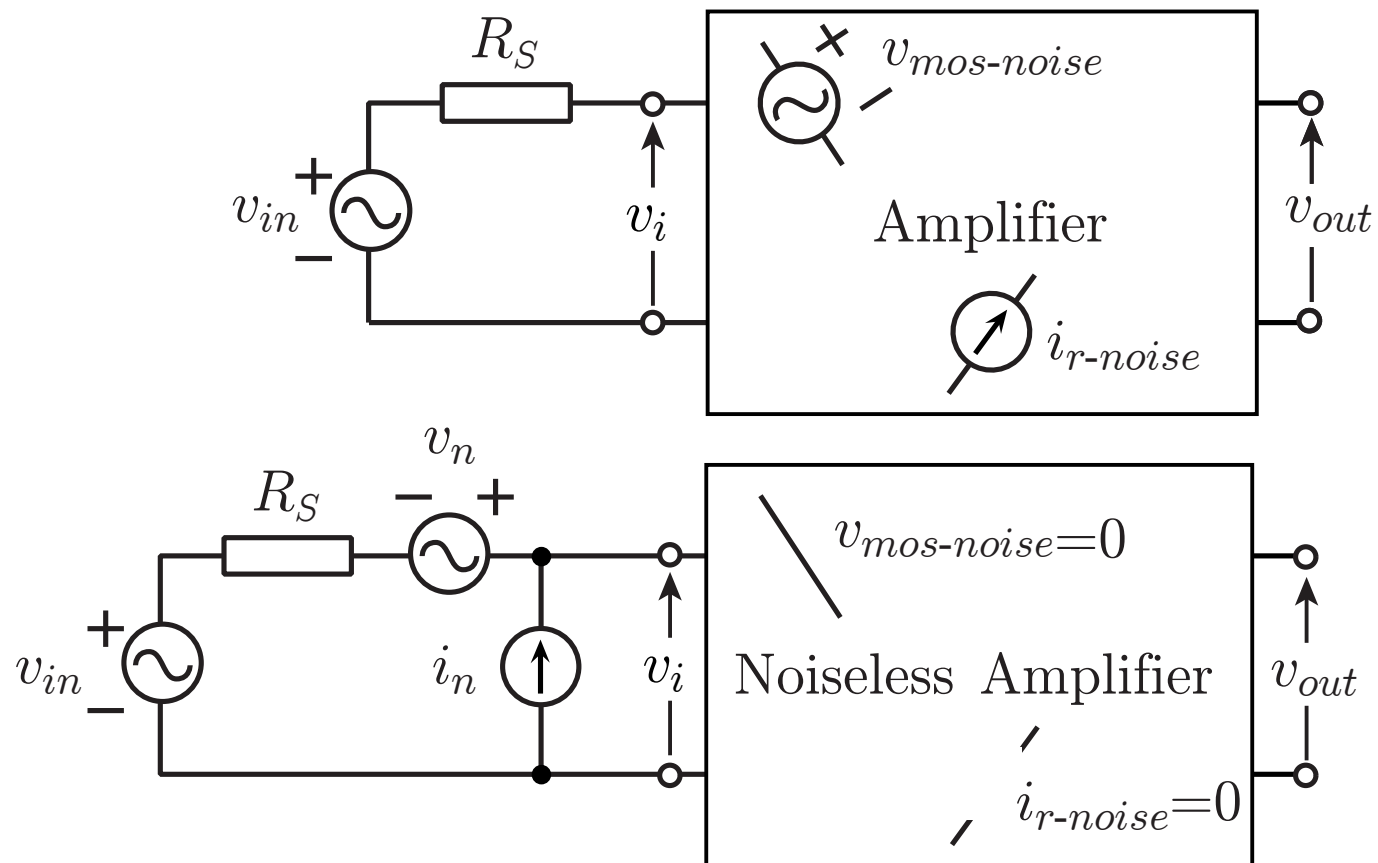




$R_S=\infty$ のとき矛盾



$R_S=0$ のとき矛盾



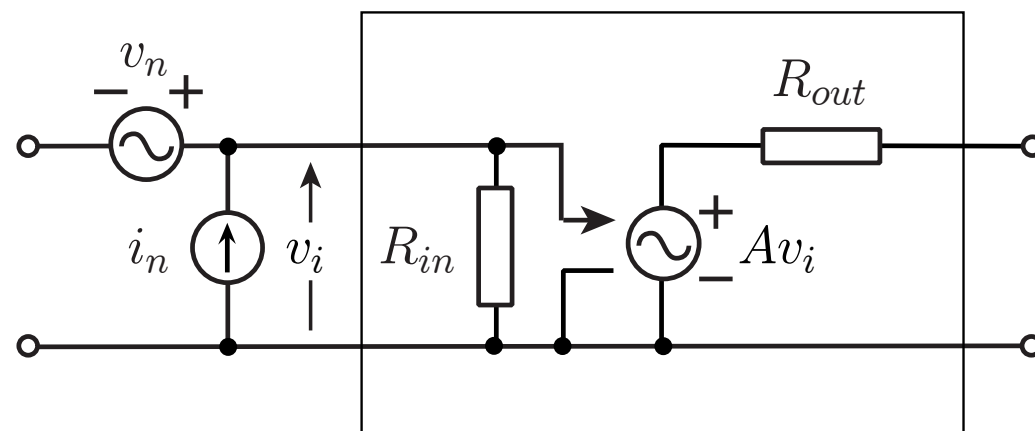
任意の R_S について矛盾無し

導出方法: テブナンの定理を拡張

雑音係数と雑音指数

増幅回路のモデル

Noiseless Amplifier



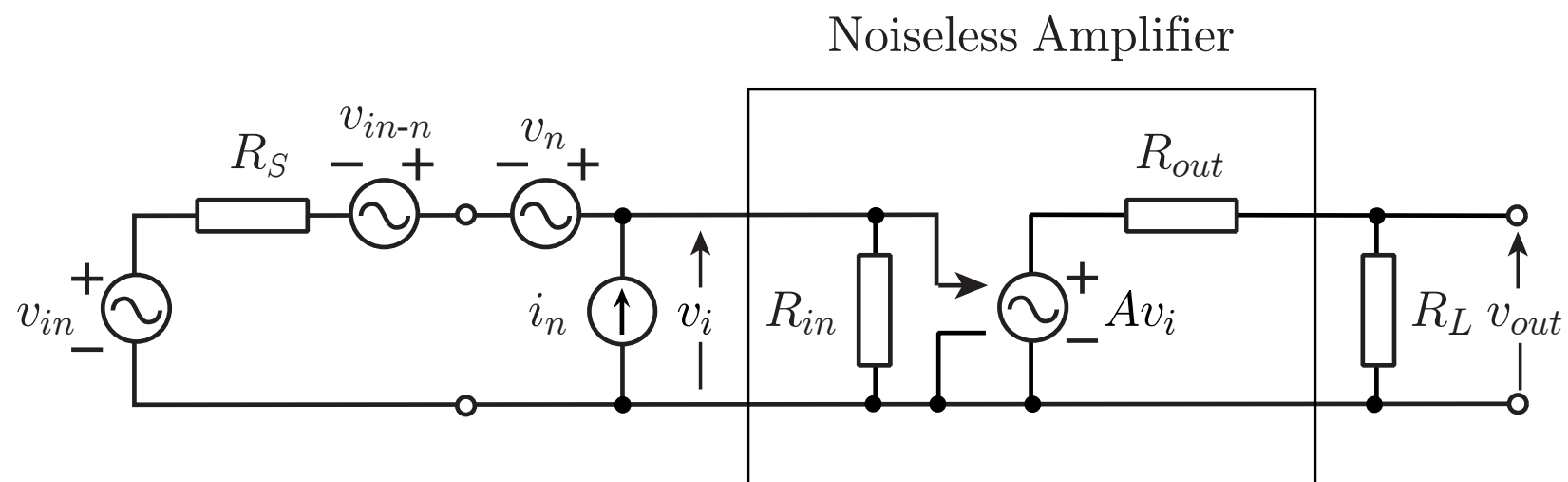
A : 増幅回路の電圧利得

R_{in} : 増幅回路の入力インピーダンス

R_{out} : 増幅回路の出力インピーダンス

v_n : 増幅回路の入力換算雑音電圧

i_n : 増幅回路の入力換算雑音電流

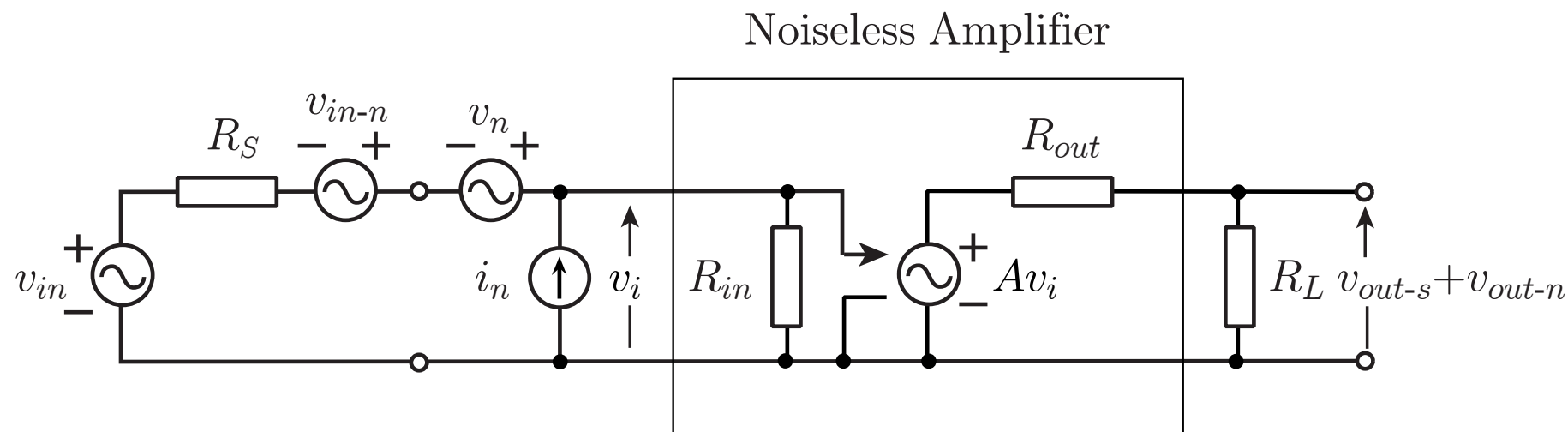


v_{in} : 入力電圧信号

R_S : 信号源抵抗

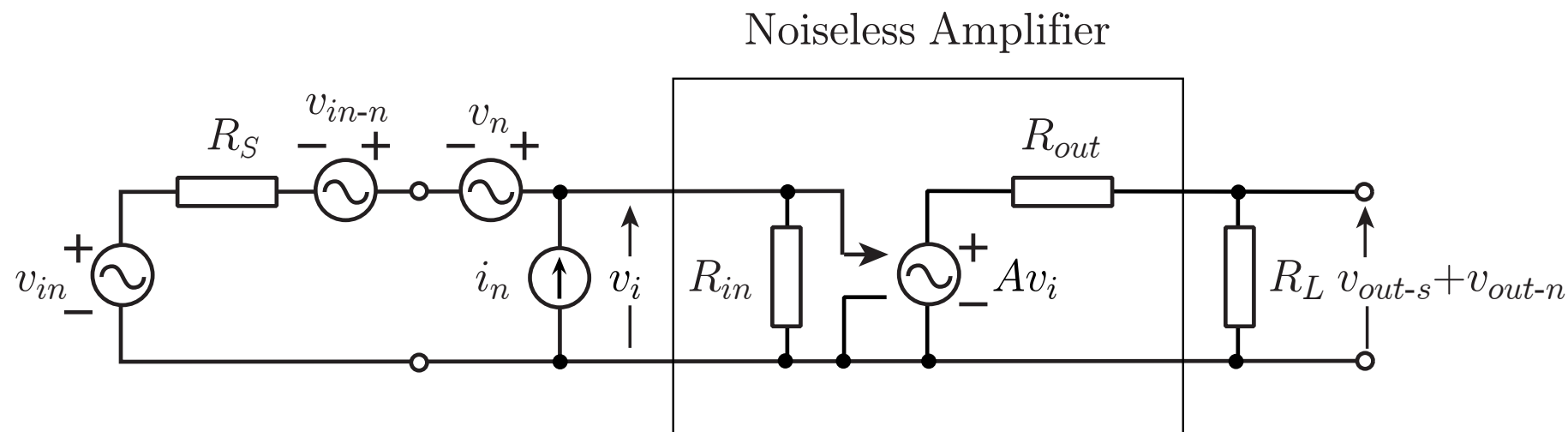
v_{in-n} : 入力信号に加わる雑音電圧

R_L : 負荷抵抗



入力における信号対雑音比 : $SNR_{in} = \frac{\overline{v_{in}^2}}{\overline{v_{in-n}^2}}$

出力における信号対雑音比 : $SNR_{out} = \frac{\overline{v_{out-s}^2}}{\overline{v_{out-n}^2}}$



雑音係数 : $F = \frac{SNR_{in}}{SNR_{out}}$

雑音指数 : $NF = 10 \log F$

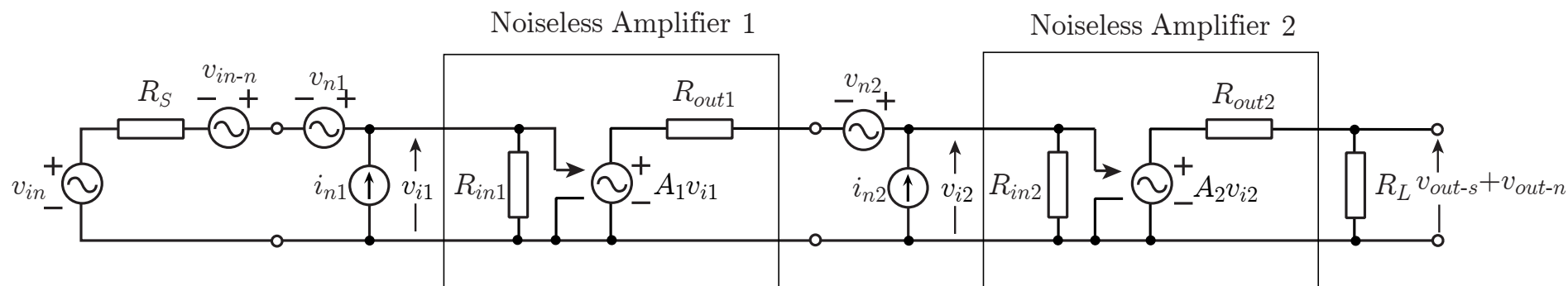
問 上の増幅回路の雑音係数を求めよ.

$$SNR_{out} = \frac{\overline{v_{in}^2}}{(v_n + R_S i_n)^2 + \overline{v_{in-n}^2}} \quad SNR_{in} = \frac{\overline{v_{in}^2}}{\overline{v_{in-n}^2}}$$

$$F = \frac{(v_n + R_S i_n)^2 + \overline{v_{in-n}^2}}{\overline{v_{in-n}^2}} \\ = 1 + \frac{(v_n + R_S i_n)^2}{\overline{v_{in-n}^2}}$$

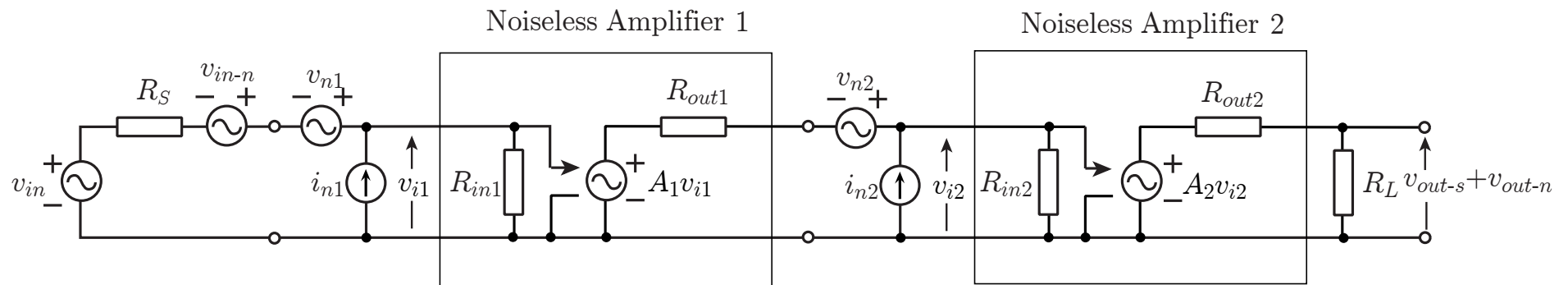
信号対雑音比は必ず劣化

縦続接続型システムの評価



$$F_{total} = \frac{\frac{v_{in-n}^2}{v_{in}^2}}{\frac{(A_{total} v_{in})^2}{v_{out-n}^2}} = \frac{v_{out-n}^2}{A_{total}^2 v_{in-n}^2}$$

$$A_{total} = \frac{R_{in1}}{R_S + R_{in1}} \times A_1 \times \frac{R_{in2}}{R_{out1} + R_{in2}} \times A_2 \times \frac{R_L}{R_{out2} + R_L}$$

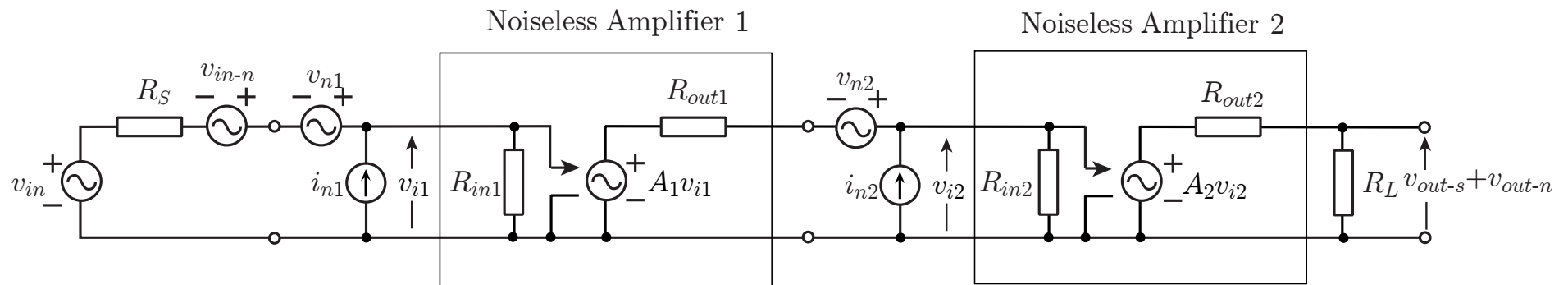


$$\overline{v_{in1-n}^2} = \left(\frac{R_{in1}}{R_S + R_{in1}} v_{n1} + (R_S // R_{in1}) i_{n1} \right)^2$$

$$+ \left(\frac{R_{in1}}{R_S + R_{in1}} \right)^2 \overline{v_{in-n}^2}$$

$$\overline{v_{in2-n}^2} = \left\{ \frac{R_{in2}}{R_{out1} + R_{in2}} v_{n2} + (R_{out1} // R_{in2}) i_{n2} \right\}^2$$

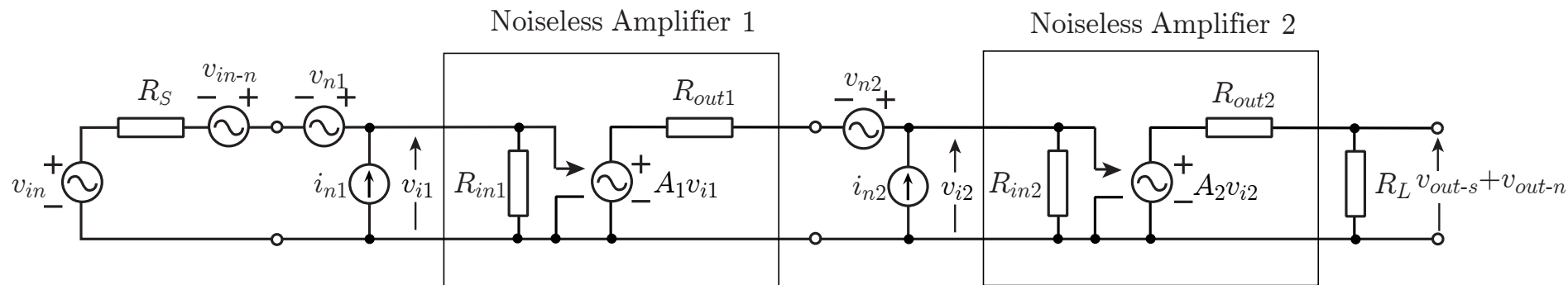
$$+ \left(\frac{A_1 R_{in2}}{R_{out1} + R_{in2}} \right)^2 \overline{v_{in1-n}^2}$$



$$F_{total} = \frac{\overline{v_{out-n}^2}}{A_{total}^2 \overline{v_{in-n}^2}}$$

$$A_{total} = \left(\frac{A_1 R_{in1}}{R_S + R_{in1}} \right) \left(\frac{A_2 R_{in2}}{R_{out1} + R_{in2}} \right) \left(\frac{R_L}{R_{out2} + R_L} \right)$$

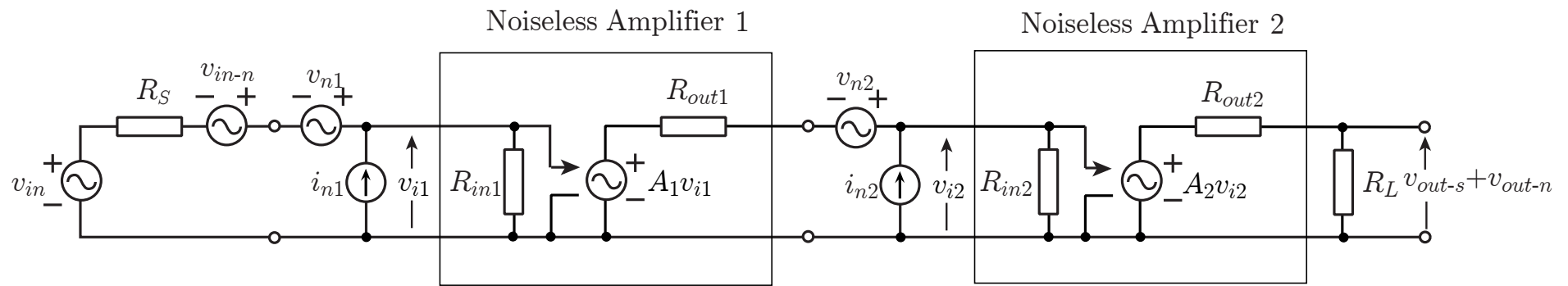
$$\overline{v_{out-n}^2} = \frac{A_2^2 R_L^2 \overline{v_{in2-n}^2}}{(R_{out2} + R_L)^2}$$



$$F_{total} = \frac{\overline{(v_{n1} + R_S i_{n1})^2} + \overline{v_{in-n}^2}}{\overline{v_{in-n}^2}} \quad \boxed{F = 1 + \frac{\overline{(v_n + R_S i_n)^2}}{\overline{v_{in-n}^2}}}$$

$$+ \frac{\overline{(v_{n2} + R_{out1} i_{n2})^2}}{A_1^2} \times \frac{1}{\left(\frac{R_{in1}}{R_S + R_{in1}} \right)^2 \overline{v_{in-n}^2}}$$

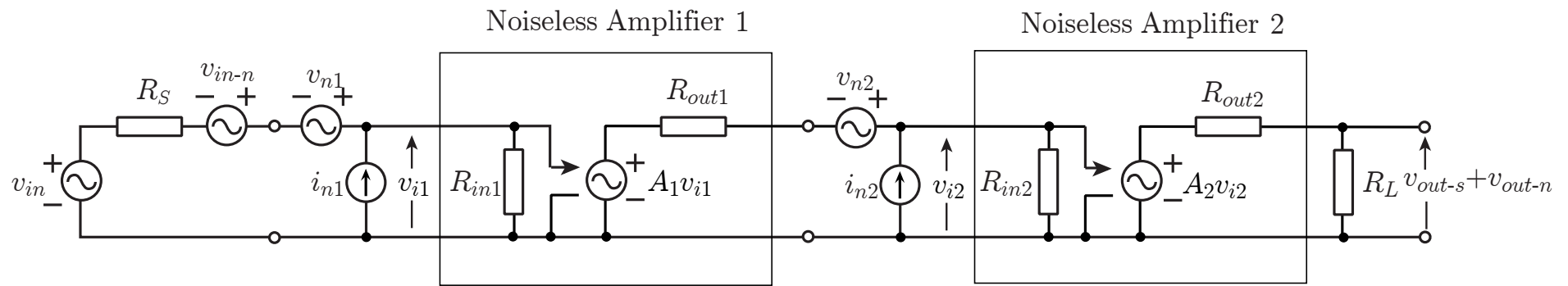
$$\boxed{\overline{v_{in-n}^2} = 4kTR_S: \text{抵抗 } R_S \text{ の熱雑音}}$$



$$F_{total} = F_1 + \frac{\overline{(v_{n2} + R_{out1}i_{n2})^2}}{A_1^2} \times \frac{1}{\left(\frac{R_{in1}}{R_S + R_{in1}}\right)^2 4kTR_S}$$

$$= F_1 + \frac{\overline{(v_{n2} + R_{out1}i_{n2})^2}}{4kTR_{out1}} \times \frac{1}{A_1^2 \frac{R_S}{R_{out}} \left(\frac{R_{in1}}{R_S + R_{in1}}\right)^2}$$

$$F = 1 + \frac{\overline{(v_n + R_S i_n)^2}}{v_{in-n}^2}$$



有効電力利得 = $\frac{\text{出力から取り出し可能な電力の最大値}}{\text{入力から取り出し可能な電力の最大値}}$

$$A_{p1} = \frac{\left(A_1 \frac{R_{in1}}{R_S + R_{in1}} v_{in} \right)^2 \frac{4R_{out1}}{v_{in}^2}}{4R_S} = \left(A_1 \frac{R_{in1}}{R_S + R_{in1}} \right)^2 \frac{R_S}{R_{out1}}$$

$$F_{total} = F_1 + \frac{F_{2,R_{out1}} - 1}{A_{p1}}$$

$$F_{total} = F_1 + \frac{F_{2,R_{out1}} - 1}{A_{p1}}$$

一般化 (Friisの式, フリスの式)

$$F_{total} = 1 + (F_1 - 1) + \frac{F_{2,R_{out1}} - 1}{A_{p1}} + \frac{F_{3,R_{out2}} - 1}{A_{p1}A_{p2}} + \dots + \frac{F_{m,R_{outm-1}} - 1}{A_{p1}A_{p2}\dots A_{pm-1}}$$

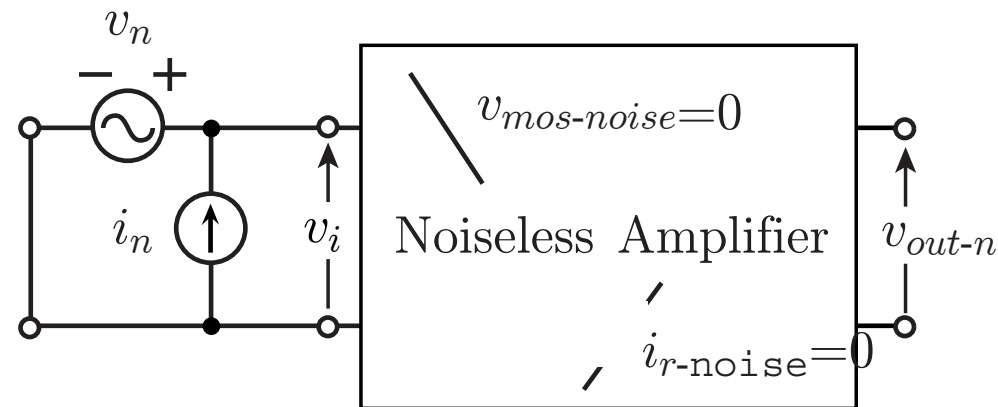
低雑音増幅回路の設計

増幅回路の出力雑音のみ考慮

v_n の求め方

$$v_n = \frac{v_N}{A}$$

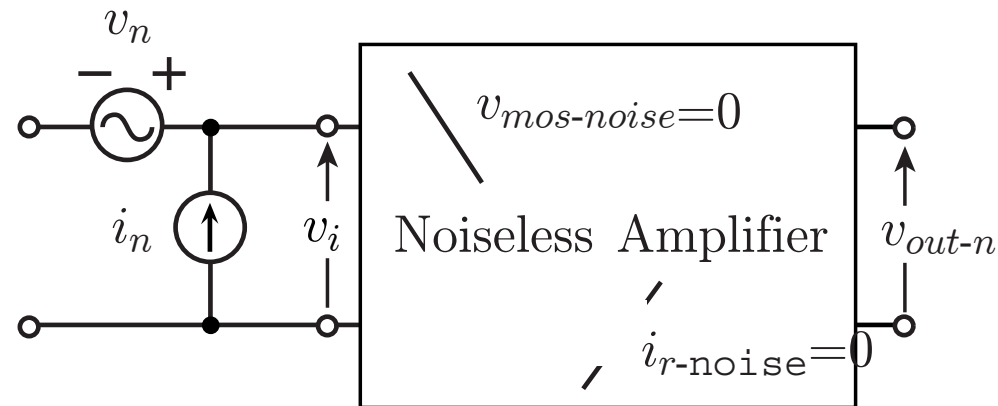
A : 電圧利得



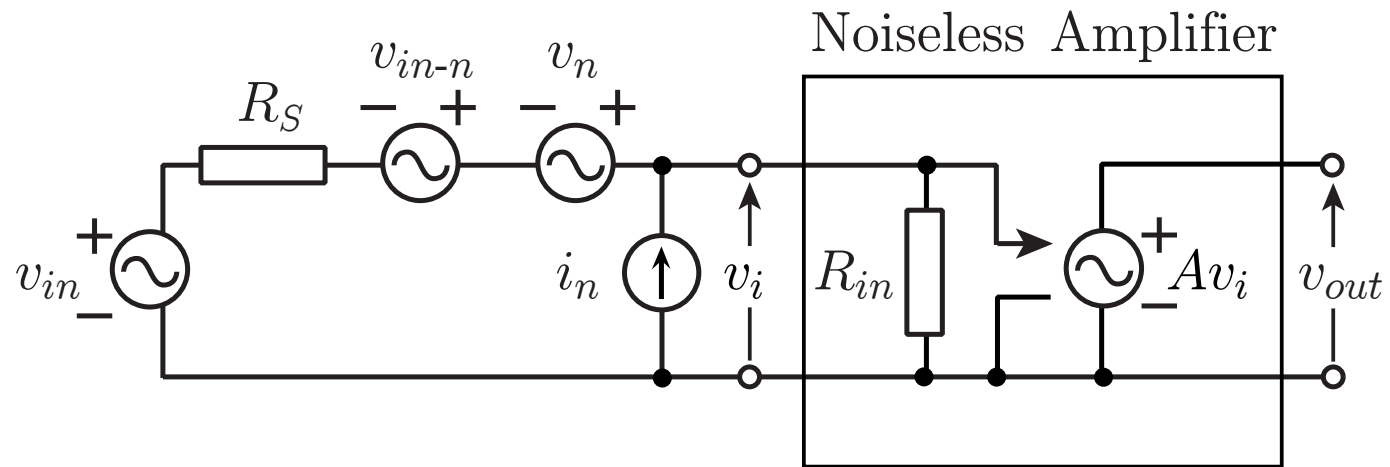
i_n の求め方

$$i_n = \frac{v_N}{Z_T}$$

Z_T : 伝達インピーダンス

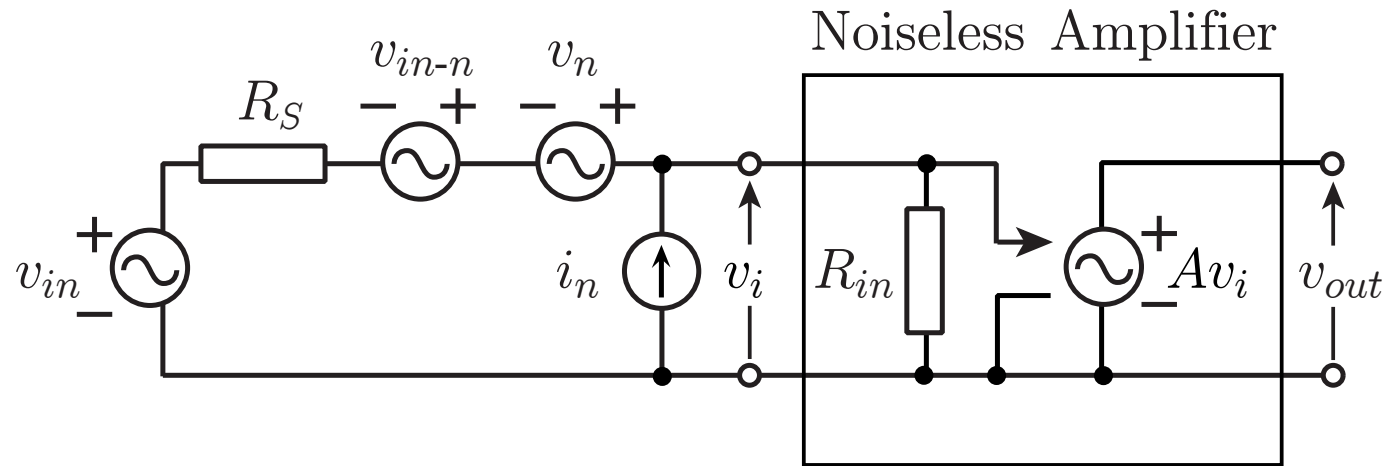


雑音整合



$$\overline{v_{in-n}^2} = 4kTR_S$$

$$v_S = \frac{AR_{in}}{R_S + R_{in}} v_{in}$$



v_{in-n} と v_n , v_{in-n} と i_n は無相関

v_n と i_n は相関有り

$$\overline{v_{out-n}^2} = \frac{A^2 R_{in}^2}{|R_S + R_{in}|^2} \left\{ \overline{v_{in-n}^2} + \overline{(v_n + R_S i_n)^2} \right\}$$

$$\overline{e_n^2} = 4kTR_S \qquad v_S = \frac{AR_{in}}{R_S + R_{in}} v_{in}$$

$$\overline{v_{out-n}^2} = \frac{A^2 R_{in}^2}{|R_S + R_{in}|^2} \left\{ \overline{v_{in-n}^2} + \overline{(v_n + R_S i_n)^2} \right\}$$

$$SNR_{in} = \frac{\overline{v_{in}^2}}{\overline{v_{in-n}^2}}$$

$$SNR_{out} = \frac{\overline{v_{out-s}^2}}{\overline{v_{out-n}^2}} = \frac{\overline{v_{in}^2}}{\overline{v_{in-n}^2} + \overline{(v_n + R_S i_n)^2}}$$

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{\overline{v_{in-n}^2} + \overline{(v_n + R_S i_n)^2}}{\overline{v_{in-n}^2}} = 1 + \frac{\overline{(v_n + R_S i_n)^2}}{4kTR_S}$$

$$F = 1 + \frac{\overline{(v_n + R_S i_n)^2}}{4kTR_S} = 1 + \frac{\overline{\left(\frac{v_n}{\sqrt{R_S}} + \sqrt{R_S} i_n \right)^2}}{4kT}$$

相加・相乗平均の定理

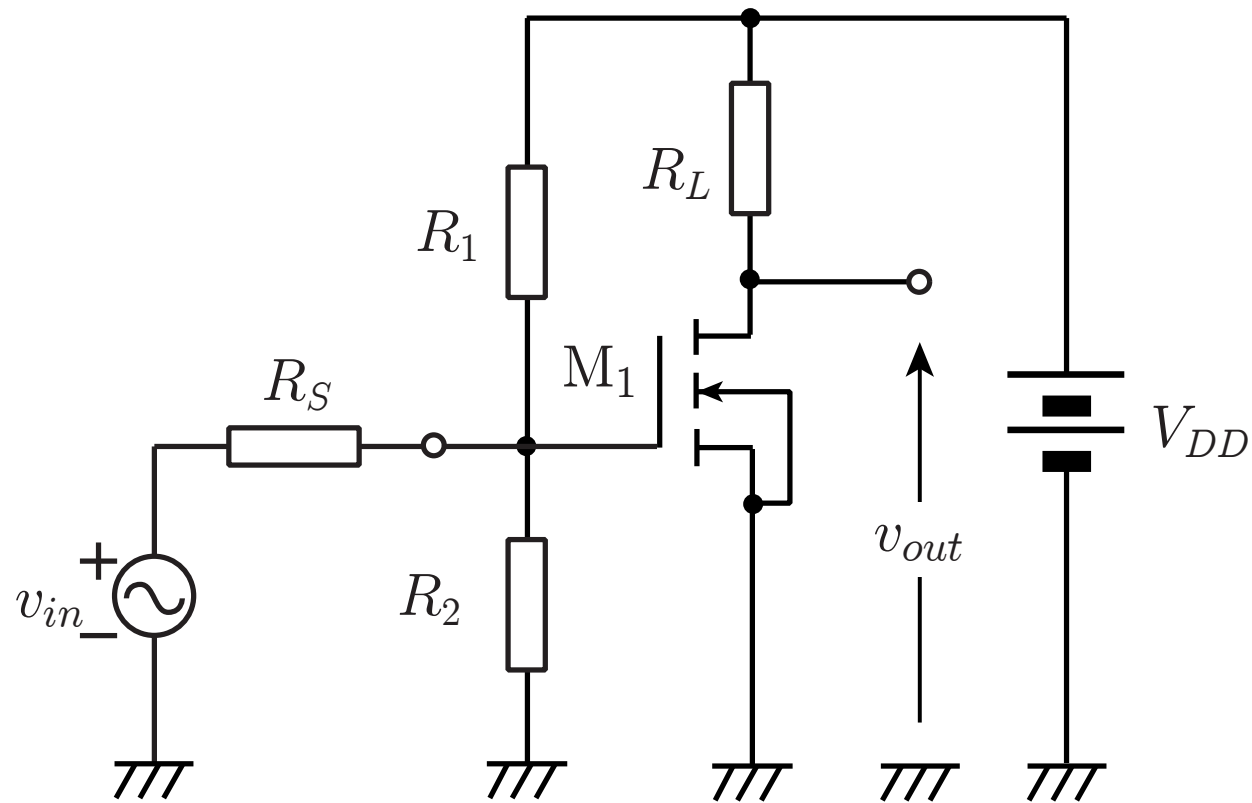
$$a + b \geq 2\sqrt{ab}$$

$\frac{v_n}{\sqrt{R_S}} = \sqrt{R_S} i_n$ のとき F が最小

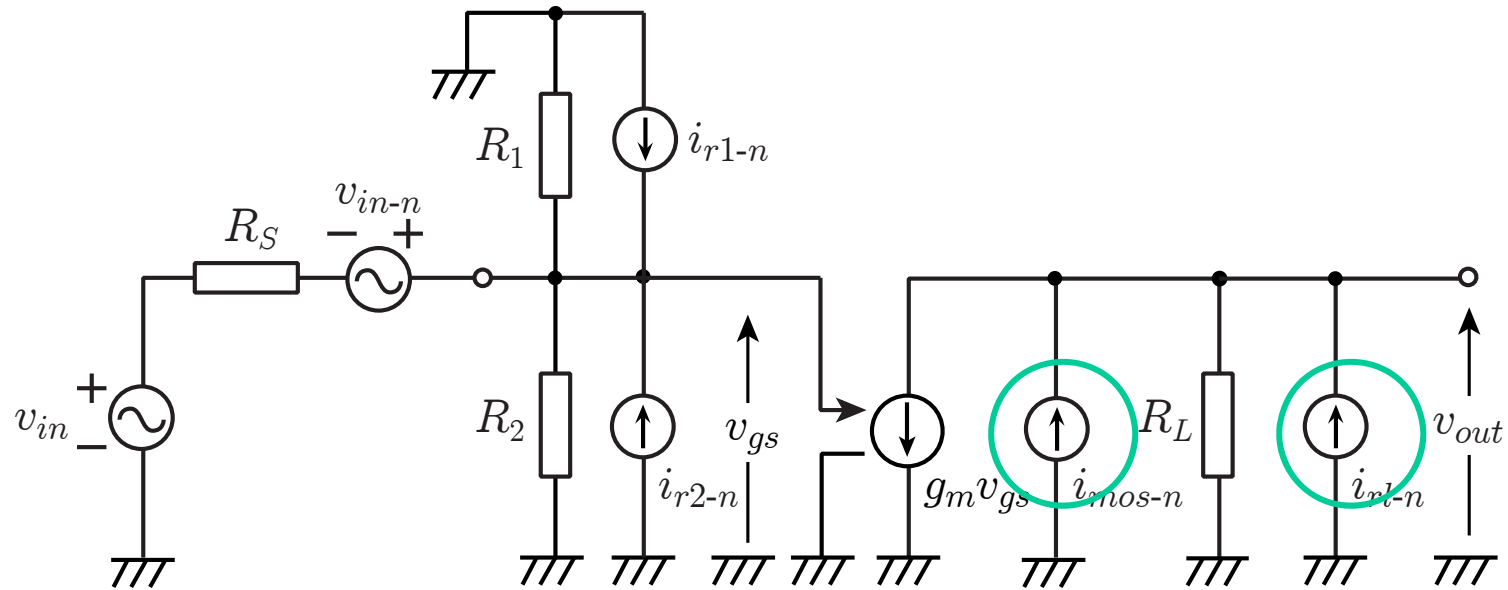
$$R_S = \frac{v_n}{i_n}$$

実際の増幅回路の雑音解析例

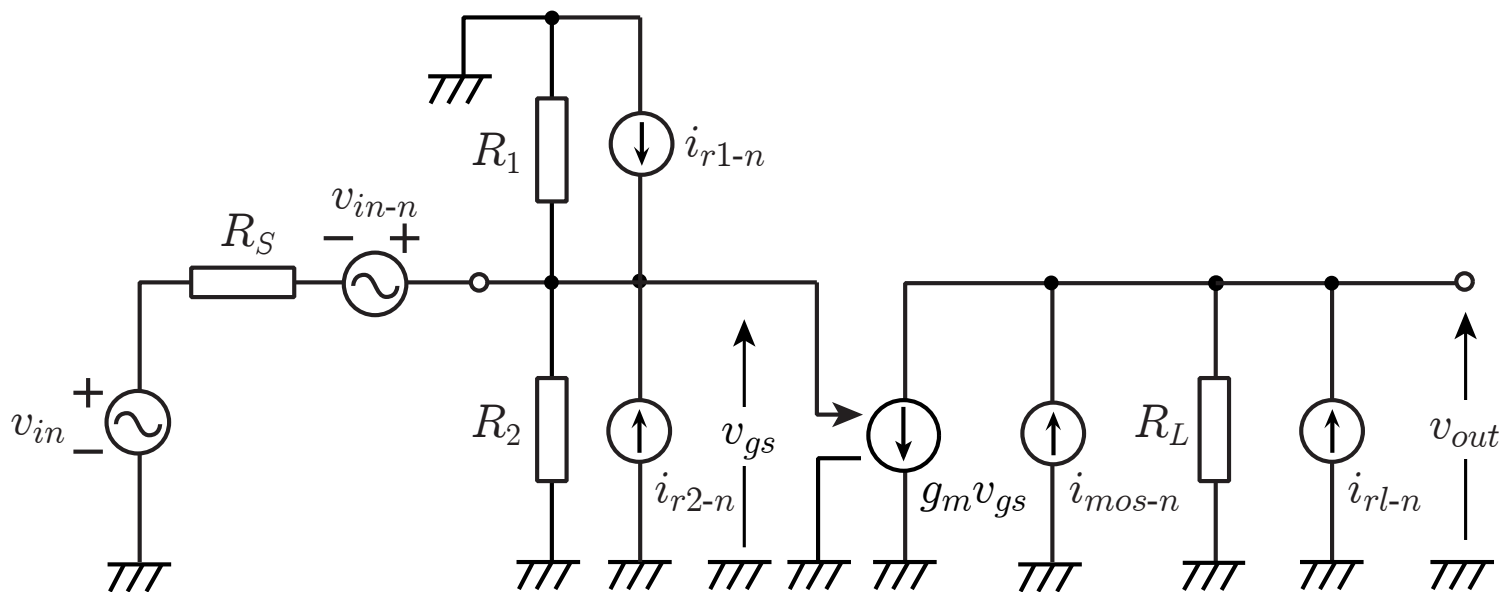
ソース接地増幅回路



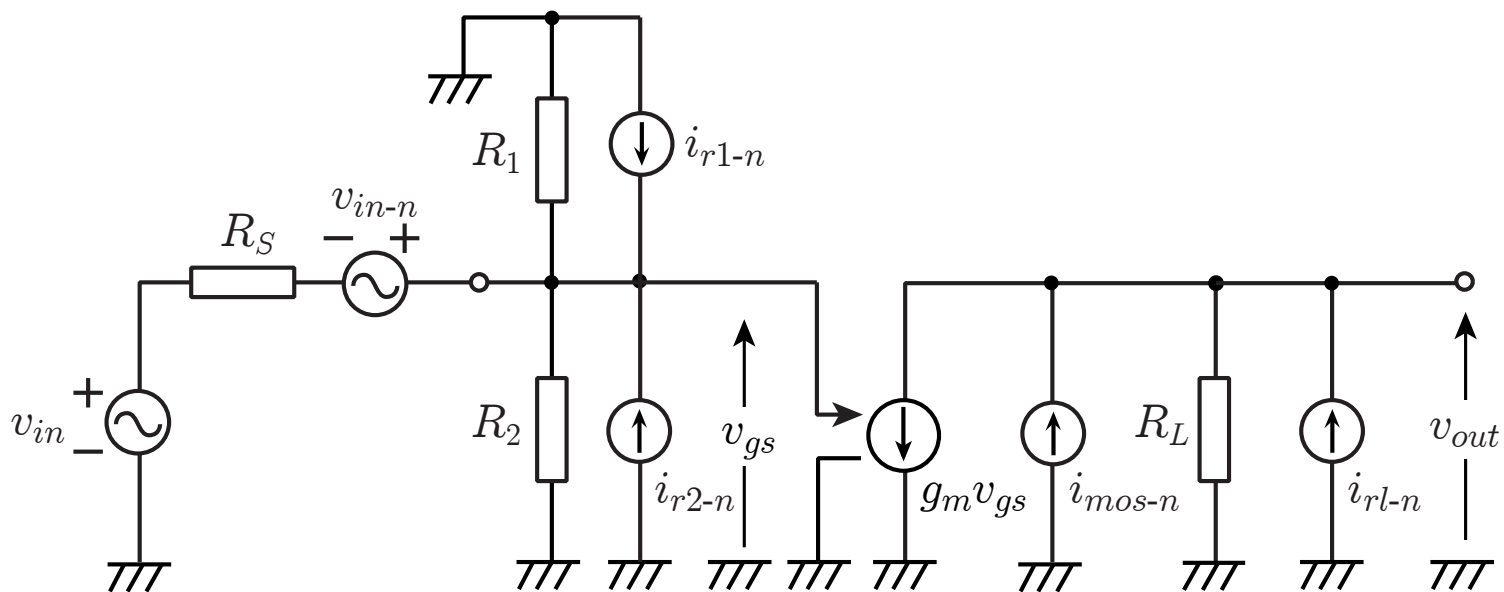
ソース接地増幅回路の小信号モデル



$$\overline{v_{o-n1}^2} = R_L^2 (\overline{i_{rl-n}^2} + \overline{i_{mos-n}^2})$$



$$\overline{v_{o-n}^2} = \left(g_m R_L \frac{R_1 R_2 R_S}{R_1 R_2 + R_2 R_S + R_S R_1} \right)^2 (\overline{i_{r1-n}^2} + \overline{i_{r2-n}^2})$$



$$\overline{v_{o-n3}}^2 = \left(g_m R_L \frac{R_1 R_2}{R_1 R_2 + R_2 R_S + R_S R_1} \right)^2 \overline{v_{in-n}}^2$$

$$\begin{aligned}
\overline{v_{out-n}}^2 &= \overline{v_{o-n1}}^2 + \overline{v_{o-n2}}^2 + \overline{v_{o-n3}}^2 \\
&= R_L^2 (\overline{i_{rl-n}}^2 + \overline{i_{mos-n}}^2) \\
&\quad + \left(g_m R_L \frac{R_1 R_2 R_S}{R_1 R_2 + R_2 R_S + R_S R_1} \right)^2 (\overline{i_{r1-n}}^2 + \overline{i_{r2-n}}^2) \\
&\quad + \left(g_m R_L \frac{R_1 R_2 R_S}{R_1 R_2 + R_2 R_S + R_S R_1} \right)^2 \overline{v_{in-n}}^2 \\
\overline{v_{out-s}}^2 &= \left(g_m R_L \frac{R_1 R_2}{R_1 R_2 + R_2 R_S + R_S R_1} \right)^2 \overline{v_{in}}^2 \\
SNR_{in} &= \frac{\overline{v_{in}}^2}{\overline{v_{in-n}}^2}
\end{aligned}$$

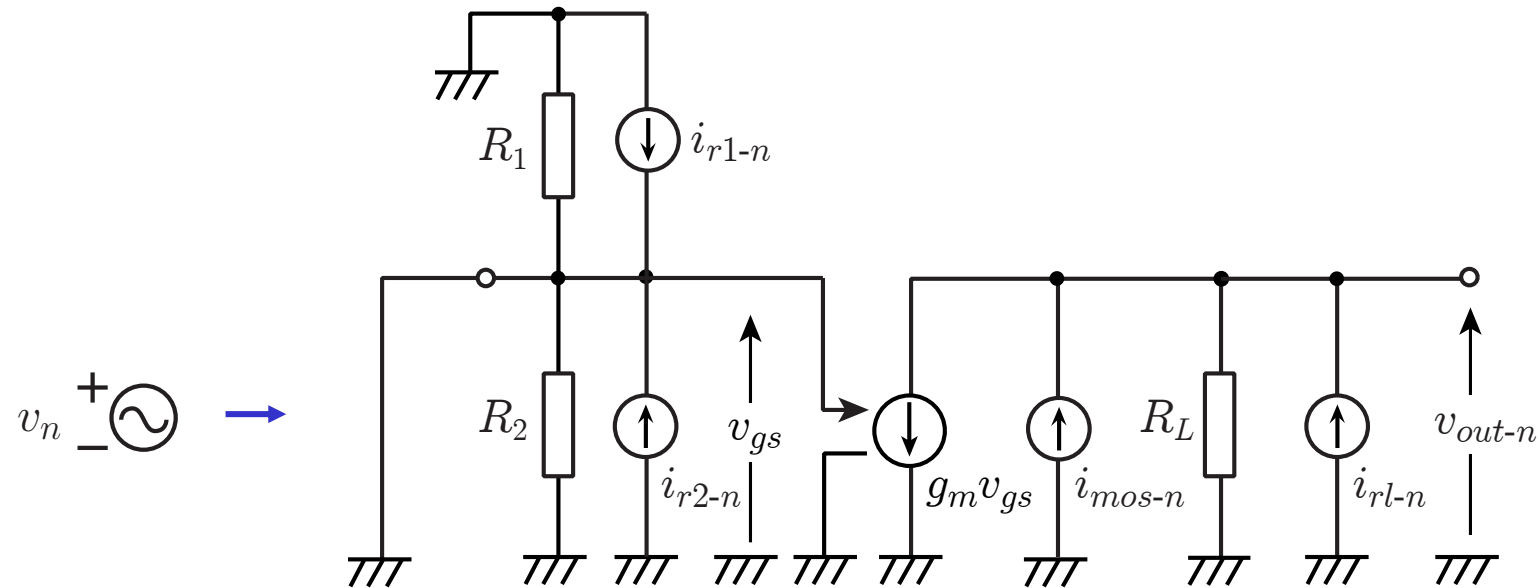
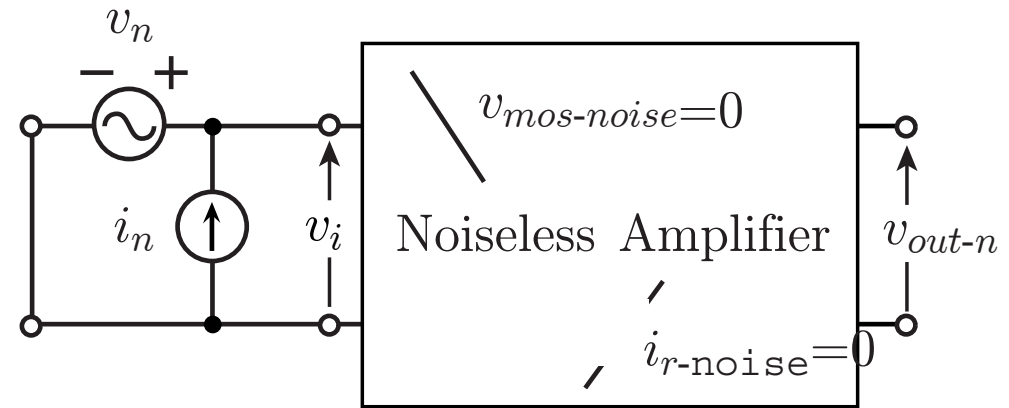
$$\overline{v_{out-n}^2} = \overline{v_{o-n1}^2} + \overline{v_{o-n2}^2} + \overline{v_{o-n3}^2}$$

$$\overline{v_{out-s}^2} = \left(g_m R_L \frac{R_1 R_2}{R_1 R_2 + R_2 R_S + R_S R_1} \right)^2 \overline{v_{in}^2}$$

$$SNR_{in} = \frac{\overline{v_{in}^2}}{\overline{v_{in-n}^2}} \quad \overline{v_{in-n}^2} = 4kTR_S$$

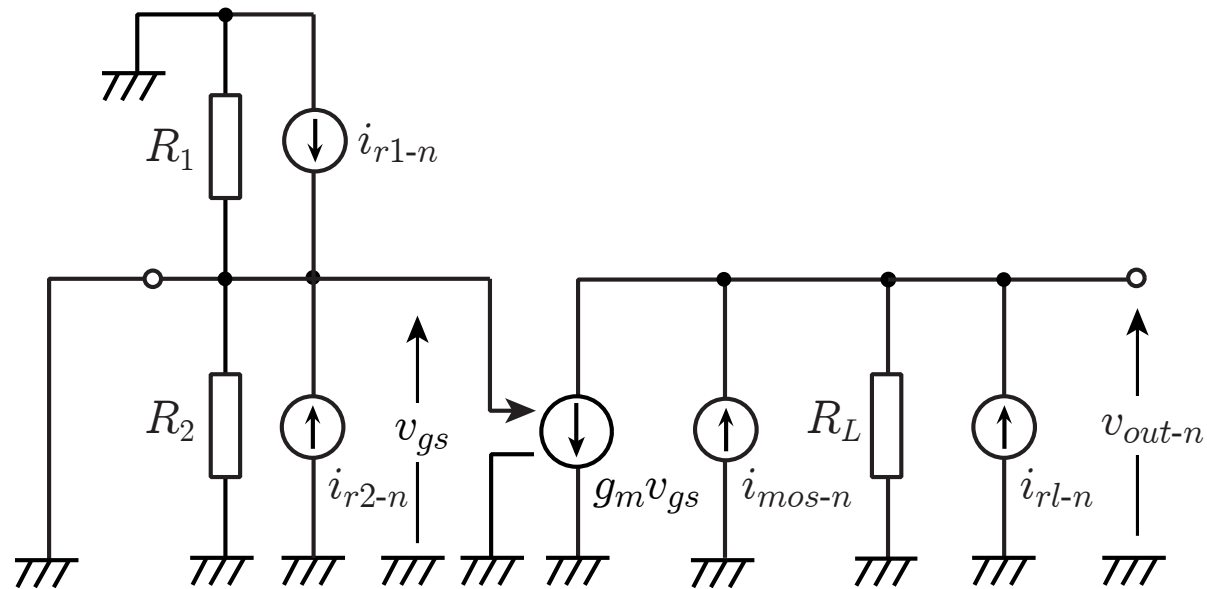
$$\begin{aligned} F &= \frac{SNR_{in}}{SNR_{out}} \\ &= 1 + \frac{1}{4kTR_S} \left\{ \left(\frac{R_1 R_2 + R_2 R_S + R_S R_1}{g_m R_1 R_2} \right)^2 (\overline{i_{rl-n}^2} + \overline{i_{mos-n}^2}) \right. \\ &\quad \left. + R_S^2 (\overline{i_{r1-n}^2} + \overline{i_{r2-n}^2}) \right\} \end{aligned}$$

v_n の求め方



$$A = \frac{v_{out}}{v_n} = -g_m R_L$$

$$v_n = \frac{v_{out-n}}{A}$$

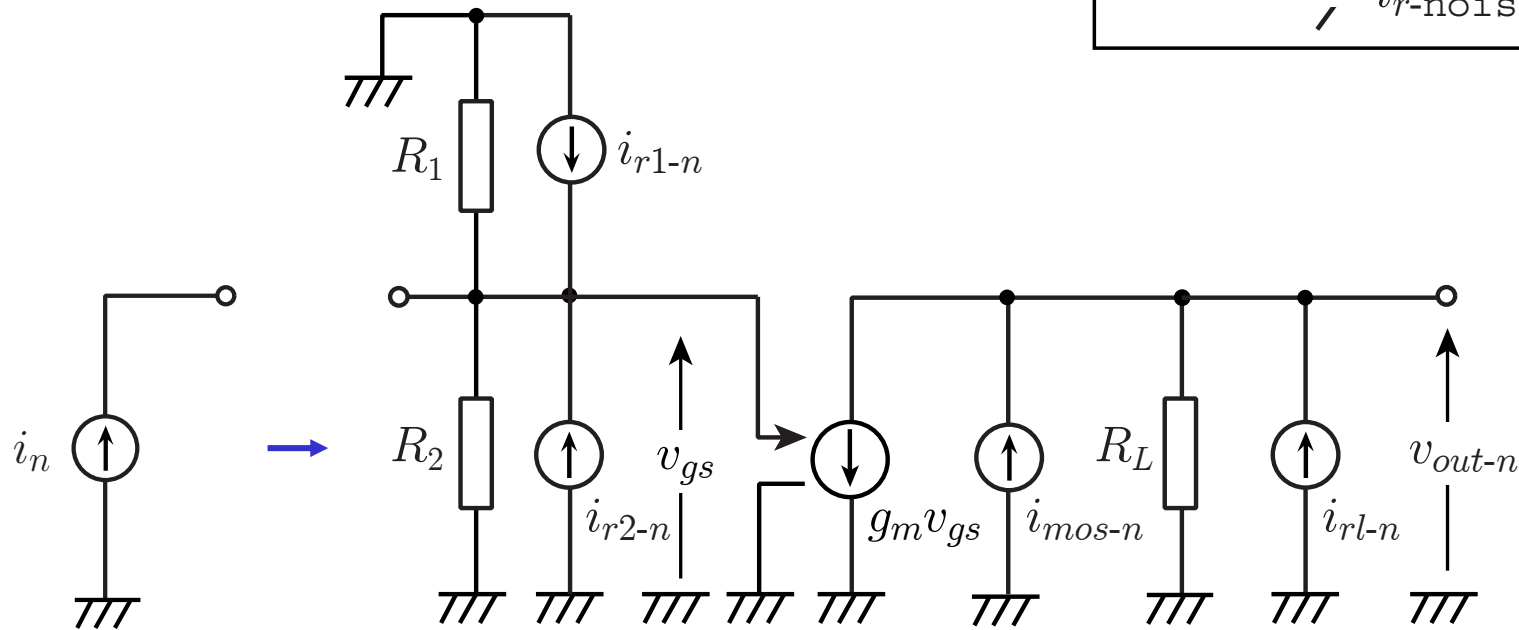
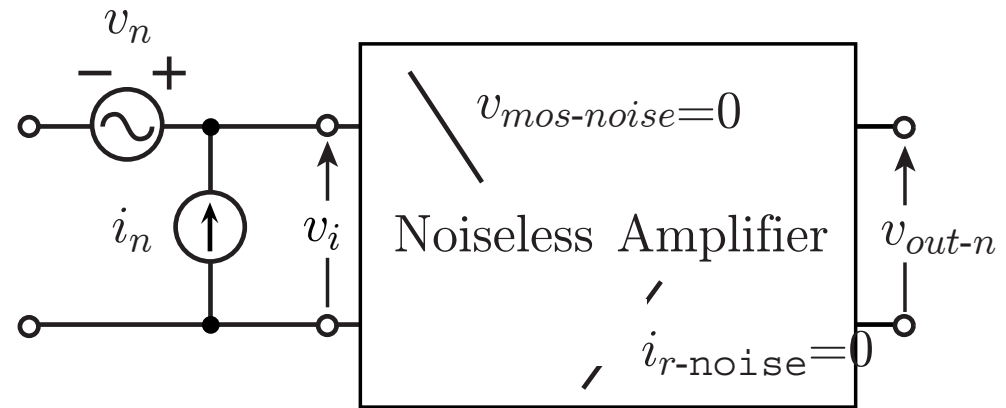


$$v_{out-n} = R_L (i_{rl-n} + i_{mos-n})$$

$$A = \frac{v_{out}}{v_n} = -g_m R_L$$

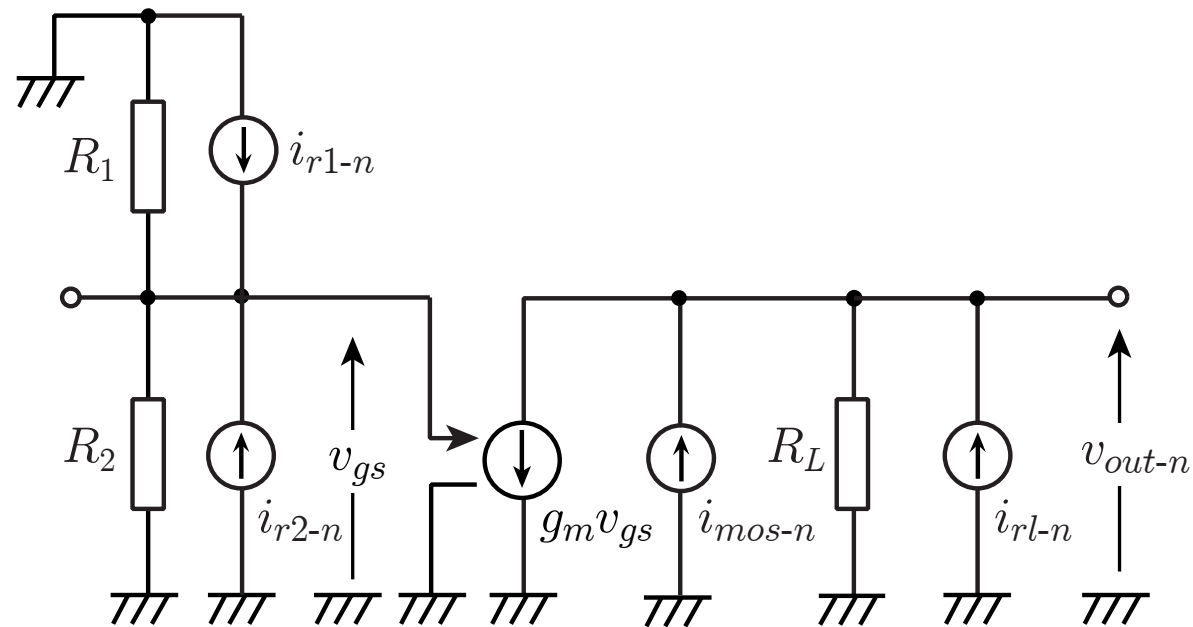
$$v_n = -\frac{i_{rl-n} + i_{mos-n}}{g_m}$$

i_n の求め方



$$Z_T = -g_m R_L \frac{R_1 R_2}{R_1 + R_2}$$

$$i_n = \frac{v_{out-n}}{Z_T}$$

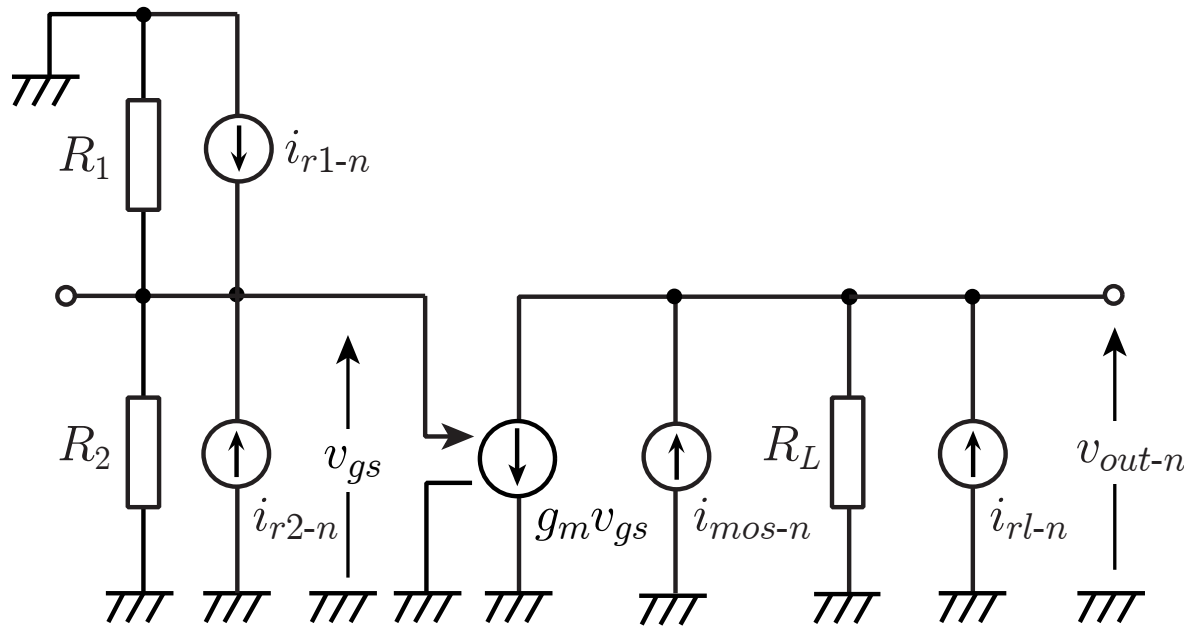


$$v_{o-n1} = R_L (i_{rl-n} + i_{mos-n})$$

$$v_{o-n2} = -g_m R_L \frac{R_1 R_2}{R_2 + R_1} (i_{r1-n} + i_{r2-n})$$

$$v_{out-n} = v_{o-n1} + v_{o-n2}$$

$$v_{out-n} = R_L (i_{rl-n} + i_{mos-n}) - g_m R_L \frac{R_1 R_2}{R_2 + R_1} (i_{r1-n} + i_{r2-n})$$



$$i_n = \frac{v_{out-n}}{Z_T}$$

$$Z_T = -g_m R_L \frac{R_1 R_2}{R_2 + R_1}$$

$$i_n = -\frac{R_1 + R_2}{g_m R_1 R_2} (i_{rl-n} + i_{mos-n}) + i_{r1-n} + i_{r2-n}$$

$$v_n = -\frac{i_{rl-n} + i_{mos-n}}{g_m}$$

$$i_n = -\frac{R_1 + R_2}{g_m R_1 R_2} (i_{rl-n} + i_{mos-n}) + i_{r1-n} + i_{r2-n}$$

$$F = 1 + \frac{\overline{(v_n + R_S i_n)^2}}{4kTR_S} \text{ であるので, まず } v_n + R_S i_n \text{ を求める}$$

$$\begin{aligned} v_n + R_S i_n = & -\frac{i_{rl-n} + i_{mos-n}}{g_m} - \frac{R_S (R_1 + R_2)}{g_m R_1 R_2} (i_{rl-n} + i_{mos-n}) \\ & + R_S (i_{r1-n} + i_{r2-n}) \end{aligned}$$

$$v_n + R_S i_n = -\frac{i_{rl-n} + i_{mos-n}}{g_m} - \frac{R_S (R_1 + R_2)}{g_m R_1 R_2} (i_{rl-n} + i_{mos-n}) \\ + R_S (i_{r1-n} + i_{r2-n})$$

これより $F = 1 + \frac{\overline{(v_n + R_S i_n)^2}}{4kTR_S}$ となる

$$F = 1 + \frac{1}{4kTR_S} \left\{ \left(\frac{R_1 R_2 + R_2 R_S + R_S R_1}{g_m R_1 R_2} \right)^2 \overline{(i_{rl-n}^2 + i_{mos-n}^2)} \right. \\ \left. + R_S^2 \overline{(i_{r1-n}^2 + i_{r2-n}^2)} \right\}$$

低雑音増幅回路の設計

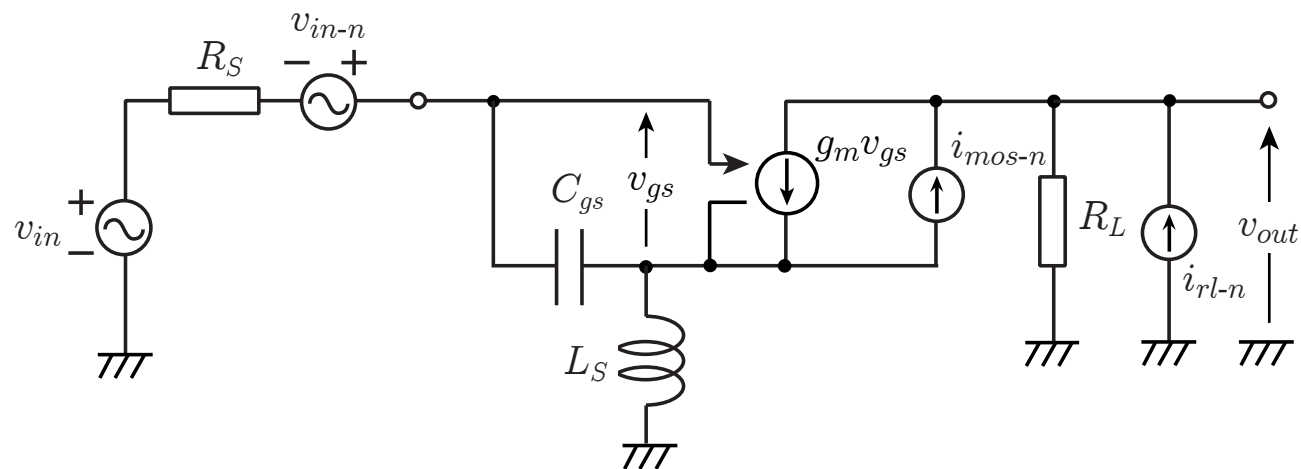
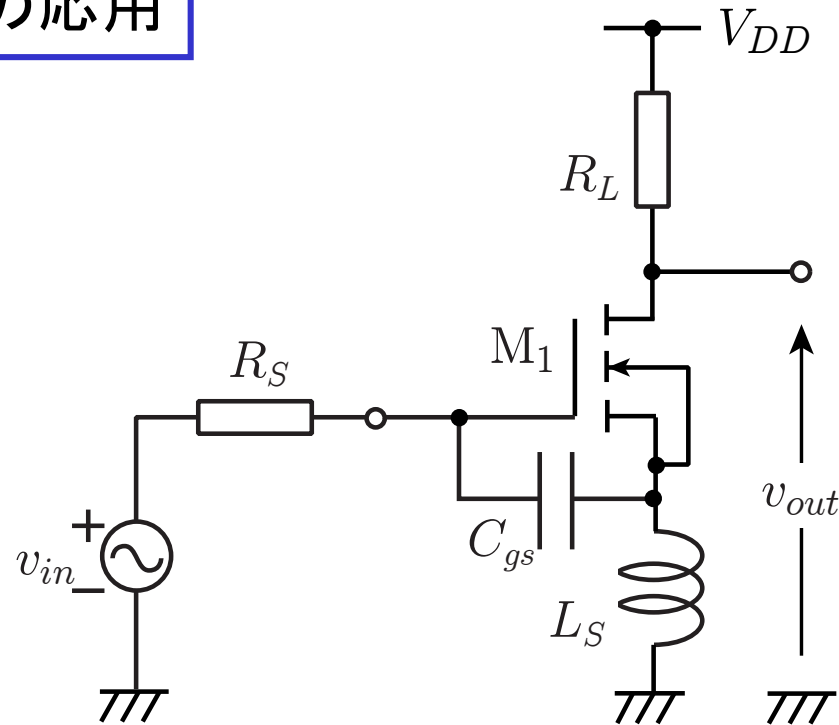
通信機器用低雑音増幅回路

高周波(数GHz)

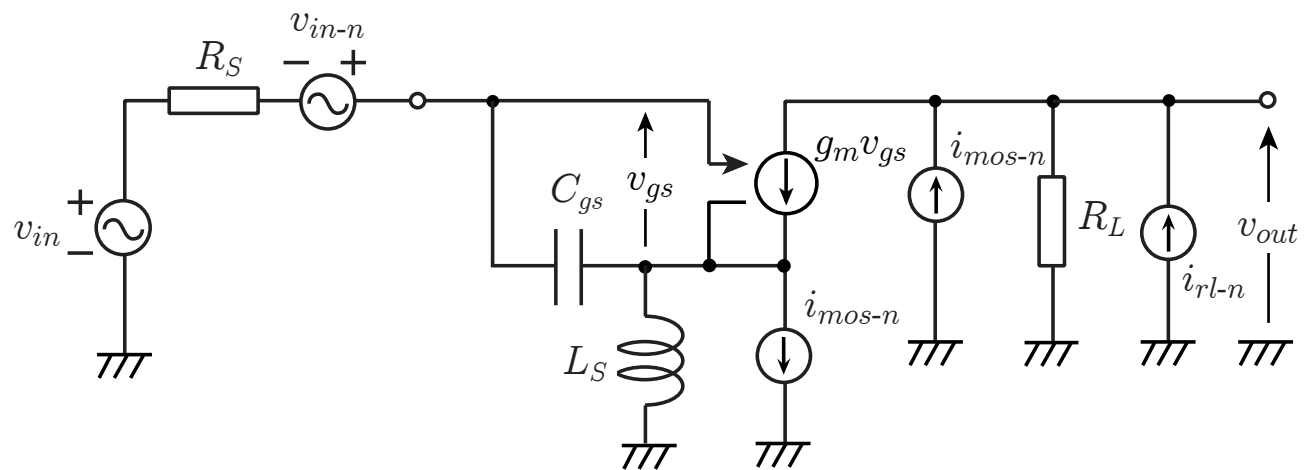
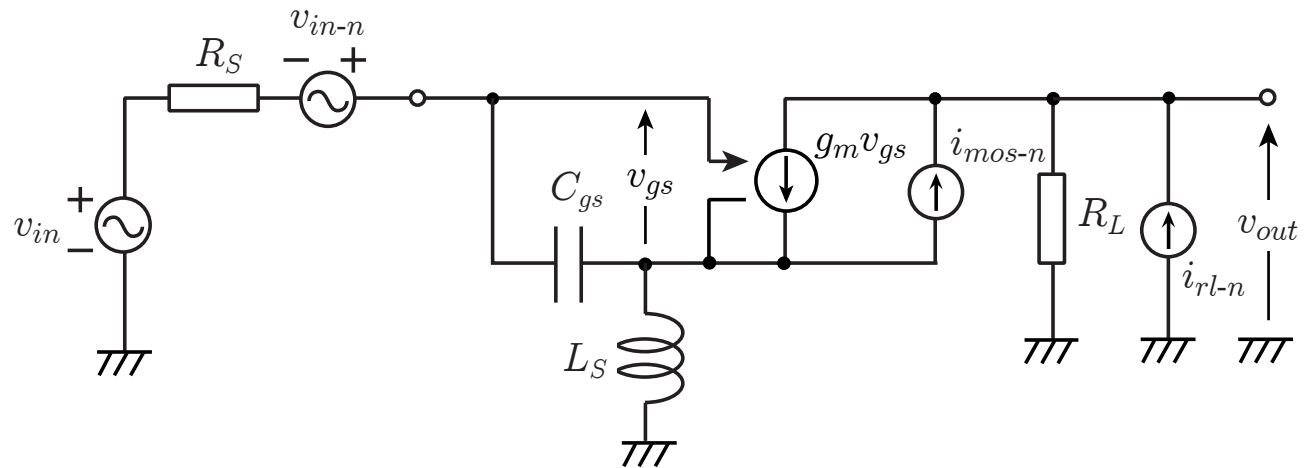
狭帯域

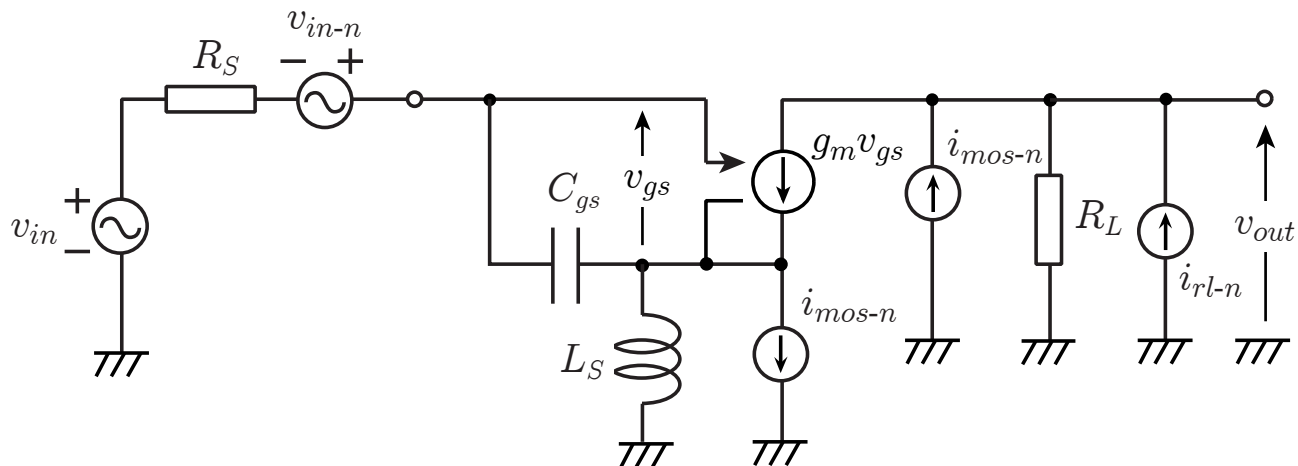
LC共振特性の応用

LC共振特性の応用



電流源の等価変換





$$v_{out-n} = R_L i_{rl-n} + R_L \left(1 - \frac{s L_S g_m}{1 + s L_S g_m + s^2 L_S C_{gs}} \right) i_{mos-n}$$

$$A = \frac{-g_m R_L}{1 + s L_S g_m + s^2 L_S C_{gs}}$$

$$Z_T = -\frac{g_m}{s C_{gs}} R_L$$

$$v_{out-n} = R_L i_{rl-n} + R_L \left(1 - \frac{s L_S g_m}{1 + s L_S g_m + s^2 L_S C_{gs}} \right) i_{mos-n}$$

$$A = \frac{-g_m R_L}{1 + s L_S g_m + s^2 L_S C_{gs}}$$

$$Z_T = -\frac{g_m}{s C_{gs}} R_L$$

$$v_n =$$

$$\frac{1 + s L_S g_m + s^2 L_S C_{gs}}{-g_m} \left\{ i_{rl-n} + \left(1 - \frac{s L_S g_m}{1 + s L_S g_m + s^2 L_S C_{gs}} \right) i_{mos-n} \right\}$$

$$i_n = \frac{s C_{gs}}{-g_m} \left\{ i_{rl-n} + \left(1 - \frac{s L_S g_m}{1 + s L_S g_m + s^2 L_S C_{gs}} \right) i_{mos-n} \right\}$$

$$v_n =$$

$$\frac{1+sL_Sg_m+s^2L_SC_{gs}}{-g_m} \left\{ i_{rl-n} + \left(1 - \frac{sL_Sg_m}{1+sL_Sg_m+s^2L_SC_{gs}} \right) i_{mos-n} \right\}$$

$$i_n = \frac{sC_{gs}}{-g_m} \left\{ i_{rl-n} + \left(1 - \frac{sL_Sg_m}{1+sL_Sg_m+s^2L_SC_{gs}} \right) i_{mos-n} \right\}$$

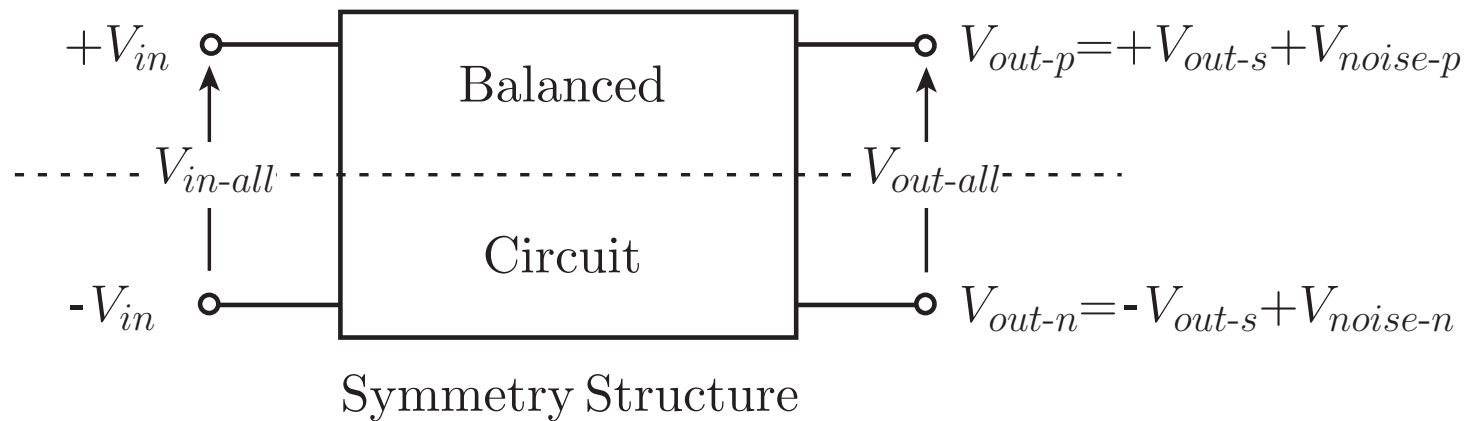
$$\frac{v_n}{i_n} = \frac{1+sL_Sg_m+s^2L_SC_{gs}}{sC_{gs}}$$

$$\frac{1+sL_Sg_m+s^2L_SC_{gs}}{sC_{gs}} = R_S$$

$$f = \frac{1}{2\pi} \sqrt{\frac{1}{L_SC_{gs}}} \text{ のとき } \frac{L_Sg_m}{C_{gs}} \text{ の純抵抗}$$



平衡型構成による雑音の低減？



$$V_{noise-d} = \frac{V_{noise-p} - V_{noise-n}}{2}$$

$$V_{noise-c} = \frac{V_{noise-p} + V_{noise-n}}{2}$$

$$V_{noise-p} = +V_{noise-d} + V_{noise-c}$$

$$V_{noise-n} = -V_{noise-d} + V_{noise-c}$$

$$V_{out-all} = V_{out-p} - V_{out-n} = 2V_{out-s} + 2V_{noise-d}$$

不平衡型回路のSNR

$$V_{out} = V_{out-s} + V_{noise}$$

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T V_{out-s}^2 dt = \overline{V_{out-s}}^2 \quad \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T V_{noise}^2 dt = \overline{V_{noise}}^2$$

$$SNR_{imbal} = \frac{\overline{V_{out-s}}^2}{\overline{V_{noise}}^2}$$

平衡型回路のSNR

$$V_{out-all} = V_{out-p} - V_{out-n} = 2(V_{out-s} + V_{noise-d})$$

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T V_{out-s}^2 dt = \overline{V_{out-s}}^2$$

$$\overline{V_{noise-d}}^2 = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T V_{noise-d}^2 dt = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T V_{noise-c}^2 dt = \frac{1}{2} \overline{V_{noise}}^2$$

$$\text{SNRの改善 : } SNR_{bal} = \frac{\overline{V_{out-s}}^2}{\overline{V_{noise-d}}^2} = 2 \frac{\overline{V_{out-s}}^2}{\overline{V_{noise}}^2}$$

ただし、回路規模は2倍