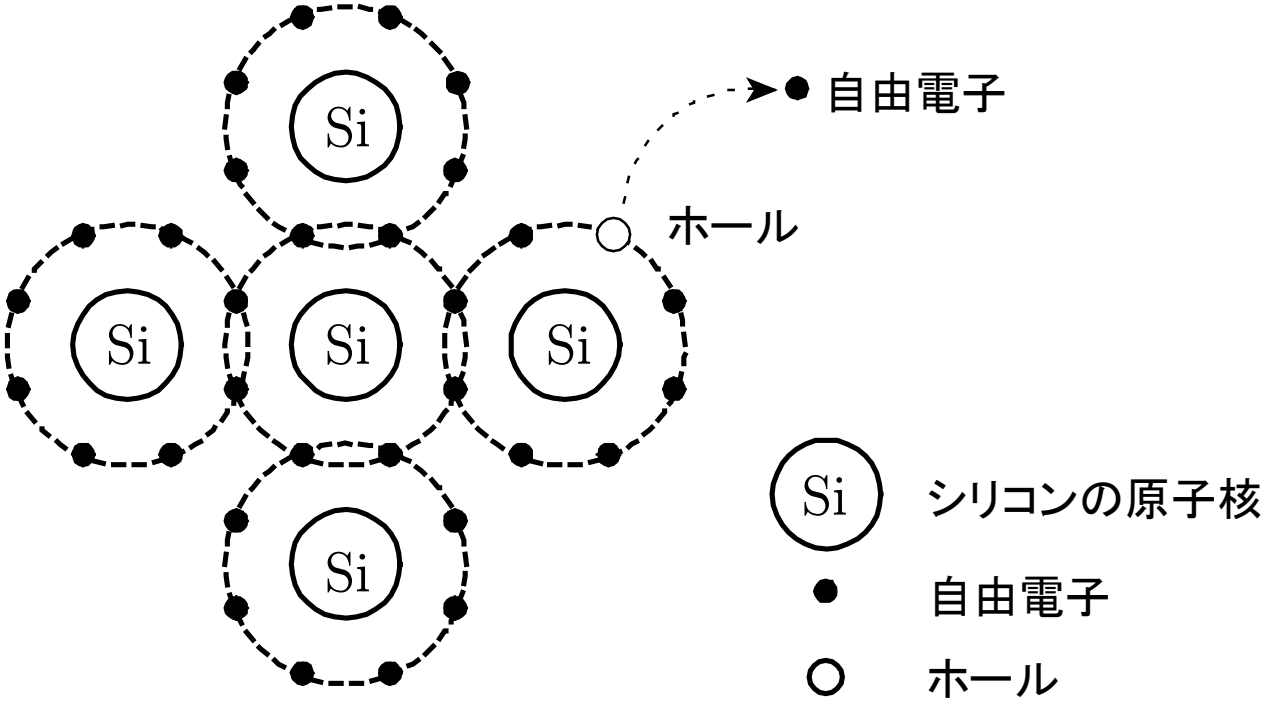
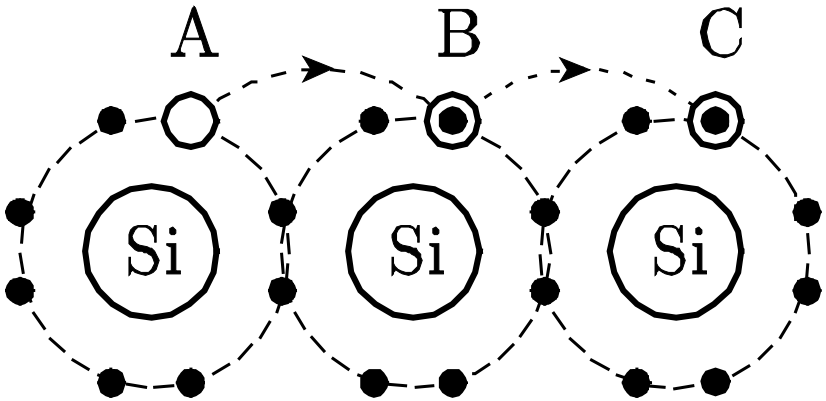
トランジスタのモデリング

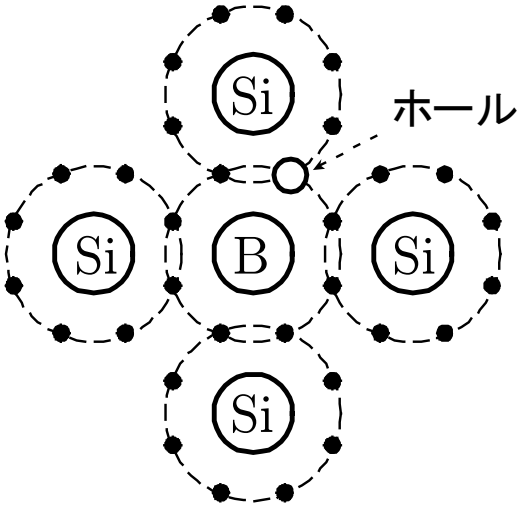
pn接合

真性半導体



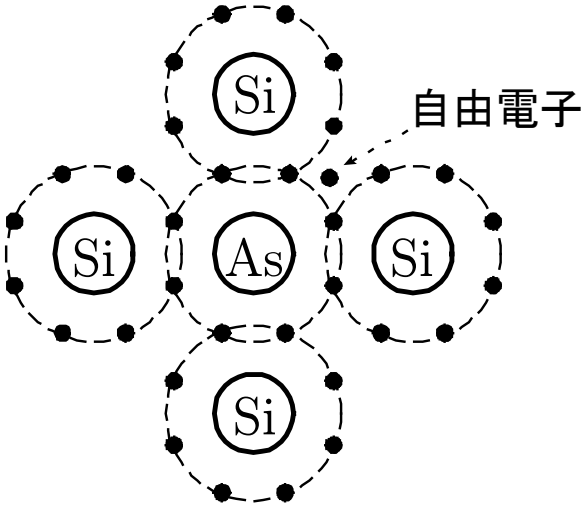


不純物半導体



B ホウ素の原子核

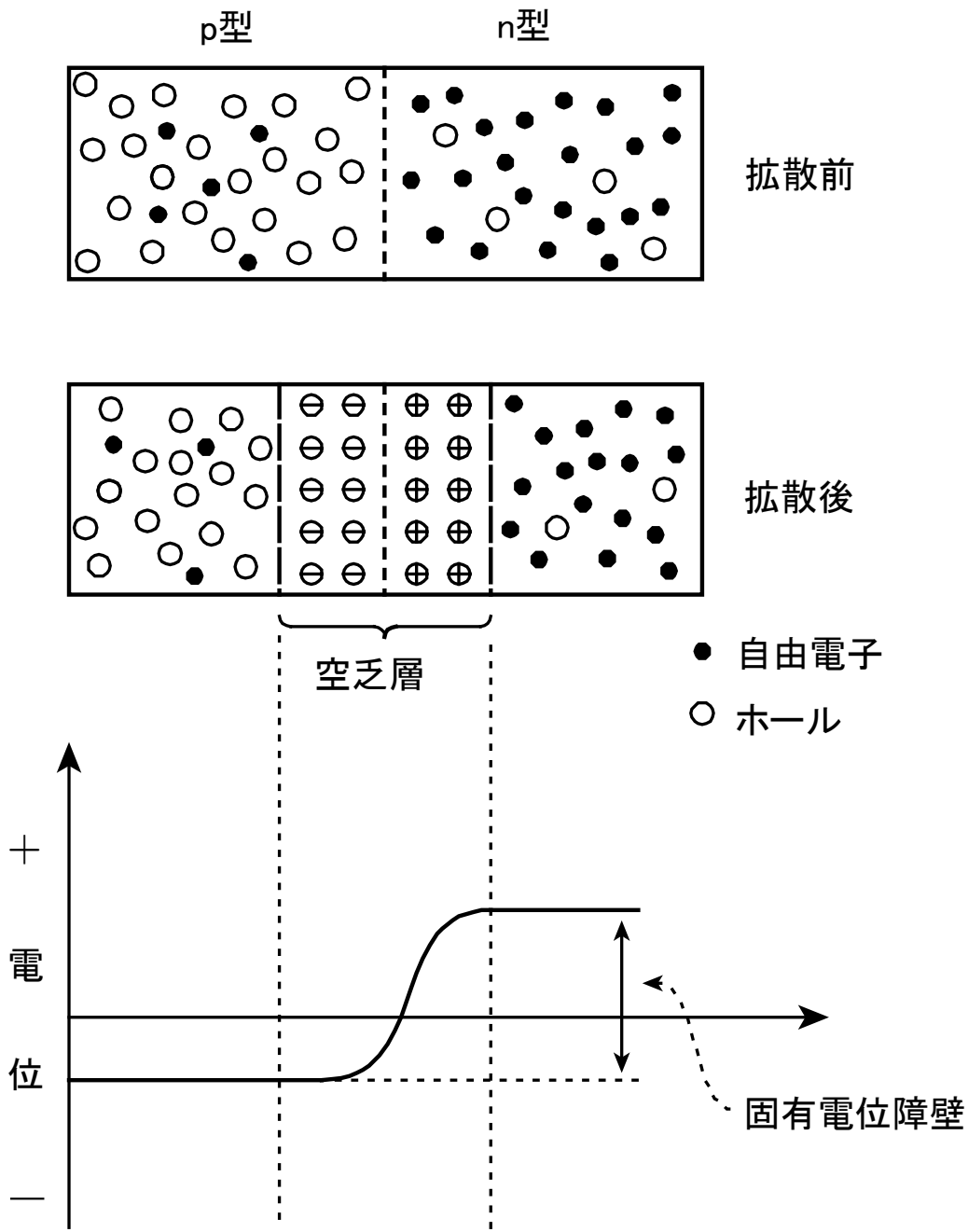
(a) p型半導体



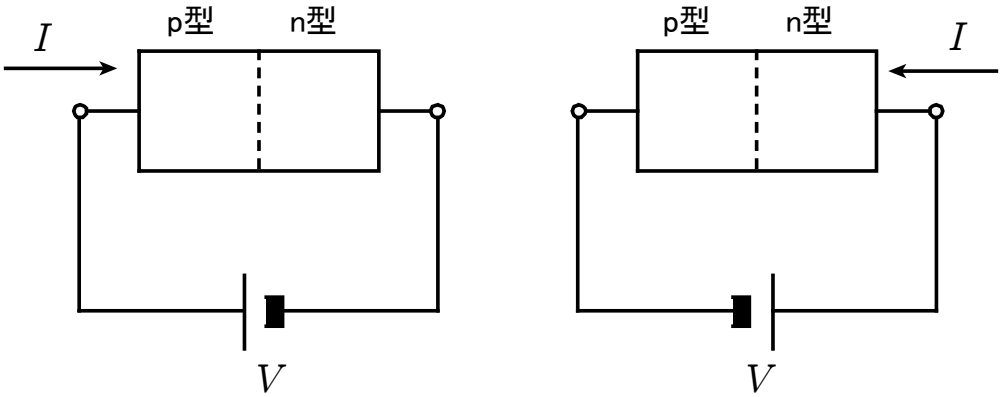
As ヒ素の原子核

(b) n型半導体

拡散と再結合

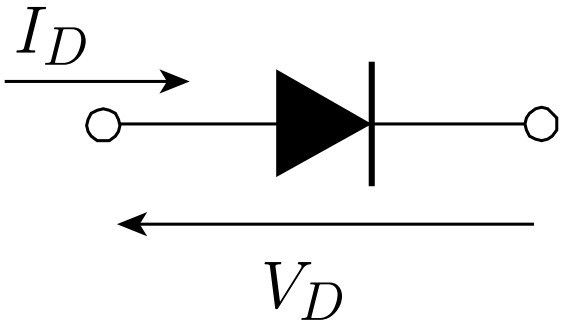


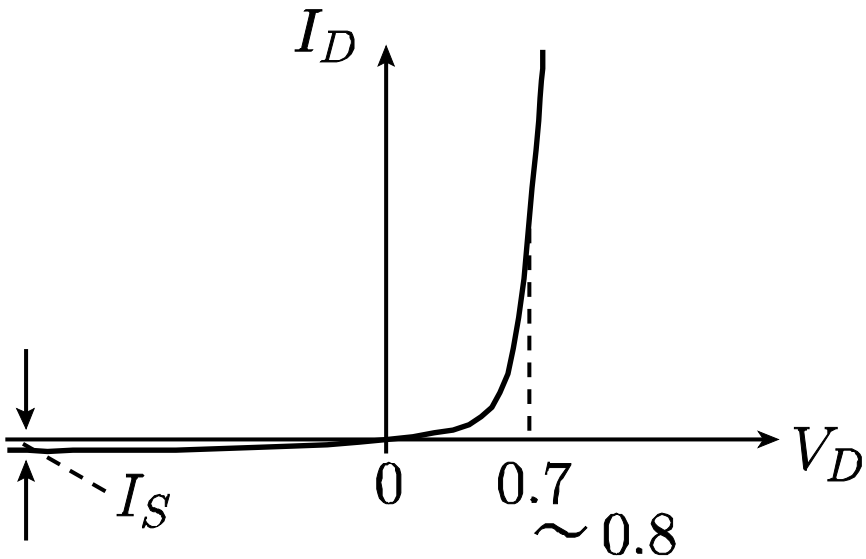
pn接合ダイオードの特性



(a) 順方向バイアス　　(b) 逆方向バイアス

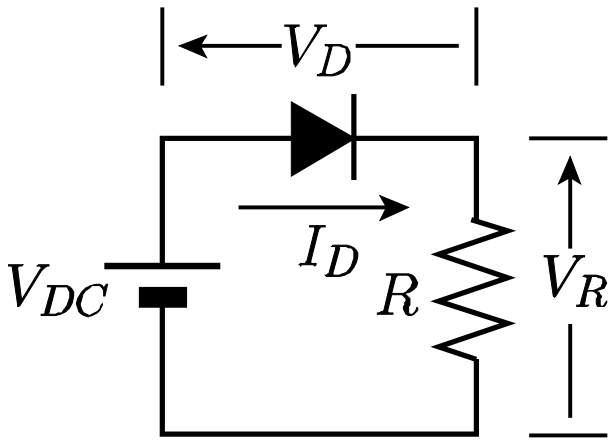
$$I_D = I_S\left\{\exp\left(\frac{qV_D}{kT}\right)-1\right\}$$





問：V_Dが負の時にI_Dが零でないのは何故か.

計算ができない？

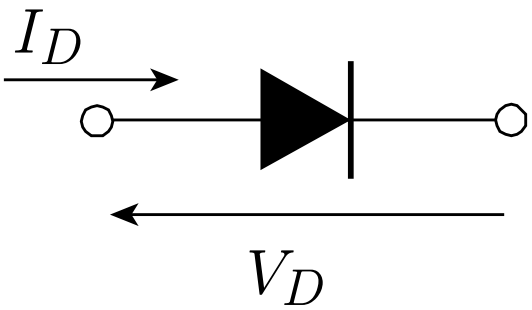


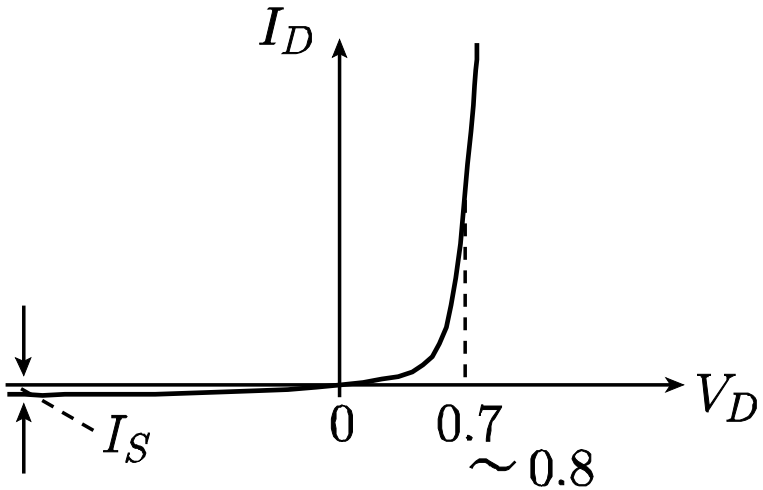
R=800Ω，I_S=10^{-15}A，k=1.38×10^{-23}J·K^{-1}，

q=1.6×10^{-19}C，T=300K

$$I_D=I_S\left\{\exp\left(\frac{qV_D}{kT}\right)-1\right\} \qquad I_D=\frac{V_{DC}-V_D}{R}$$

未知の変数はV_DとI_Dの2個で，方程式も2個だが，
V_DやI_Dについて解くことができない．





実際のpn接合ダイオードの特性

I_S=10^{-15}A，k=1.38×10^{-23}J·K^{-1}，q=1.6×10^{-19}C，T=300K

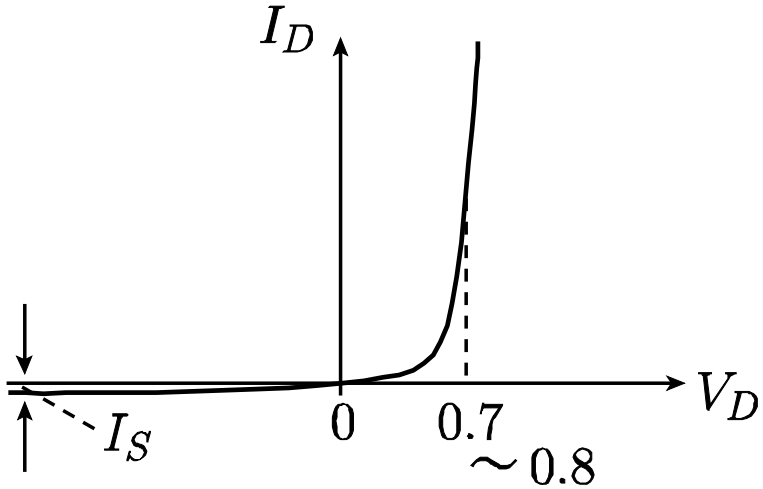
$$V_D=\frac{kT}{q}\ln\left(\frac{I_D}{I_S}+1\right)$$ より

I_D=0.10mAのときV_D=0.655V

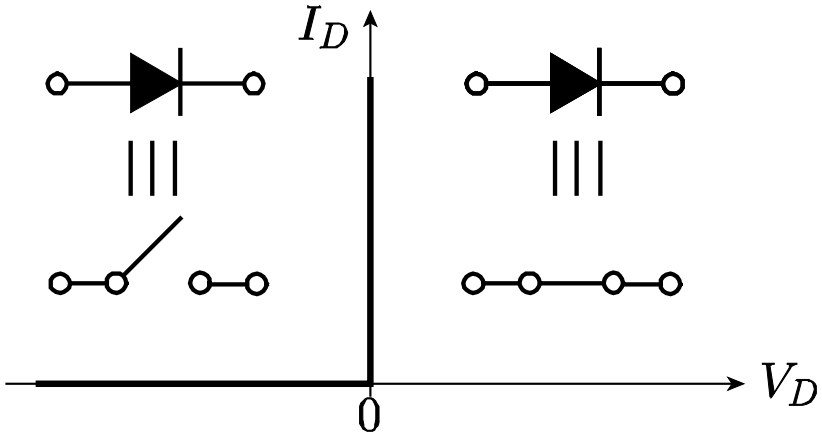
I_D=1.0mAのときV_D=0.715V

I_D=10mAのときV_D=0.775V

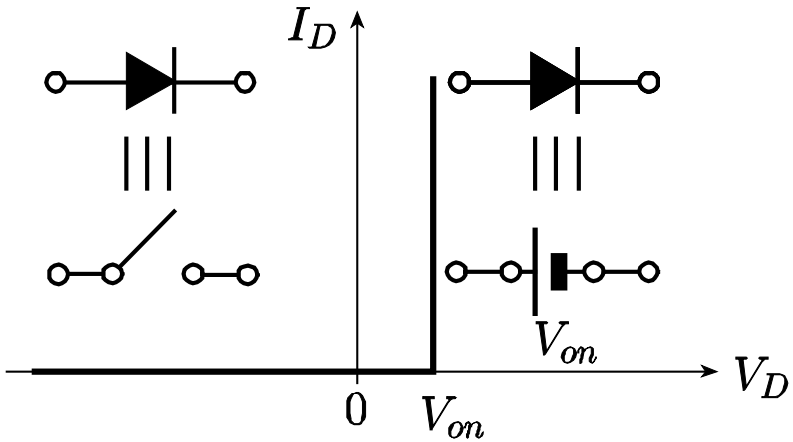
I_Dが大きく変化してもV_Dはほぼ一定.



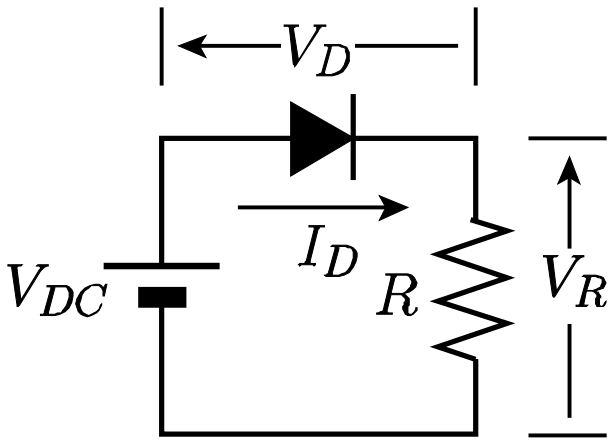
実際のpn接合ダイオードの特性



理想ダイオード特性



近似ダイオード特性



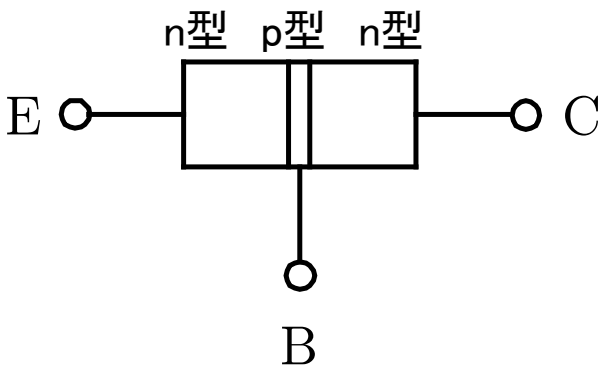
R=800Ω, I_S=10^{-15}A, k=1.38×10^{-23}J·K^{-1},

q=1.6×10^{-19}C, T=300K, V_D=0.70V

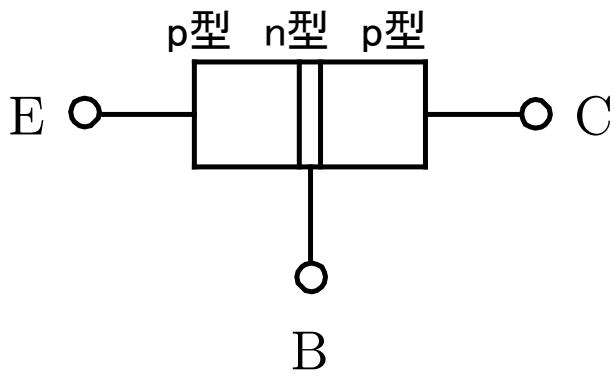
近似計算: V_D=0.70V $I_D=\dfrac{V_{DC}-V_D}{R}$=1.0mA

真の値: V_D=0.7145V, I_D=0.982mA

バイポーラトランジスタのモデリング



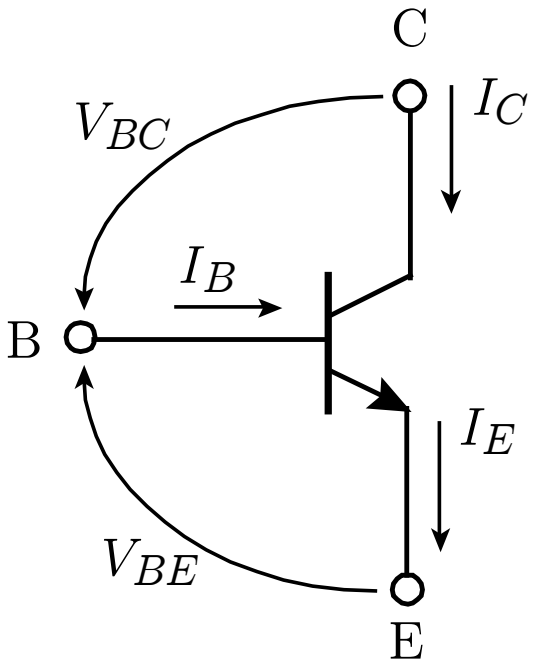
(a) npnトランジスタ



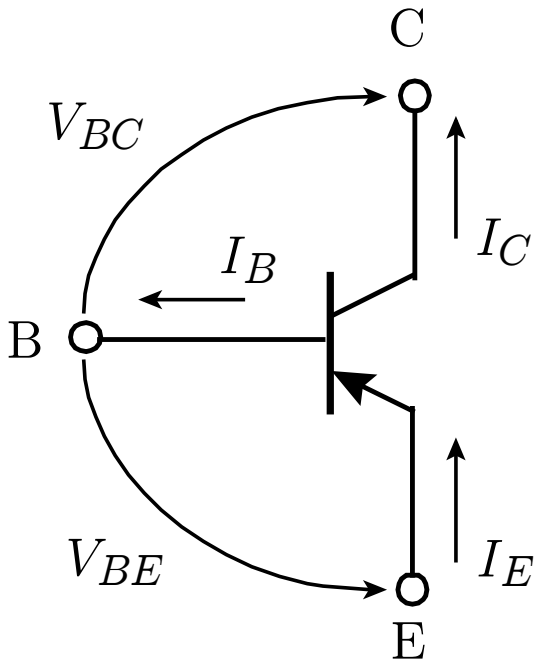
(b) pnpトランジスタ

E: Emitter, エミッタ
B: Base, ベース
C: Collector, コレクタ

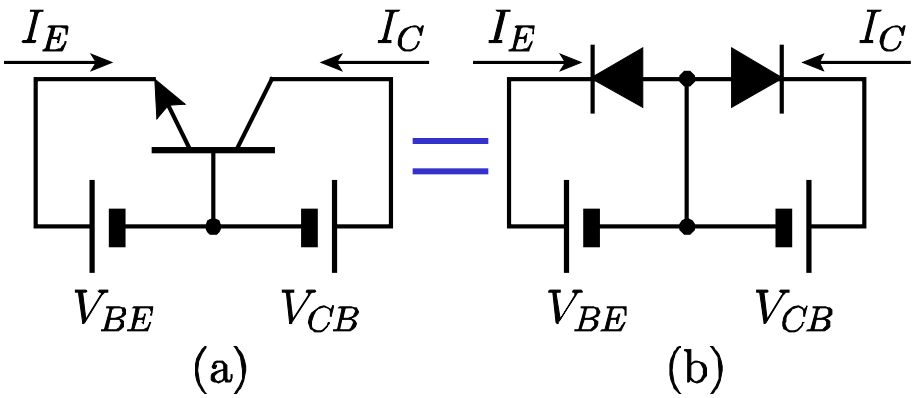
トランジスタの記号



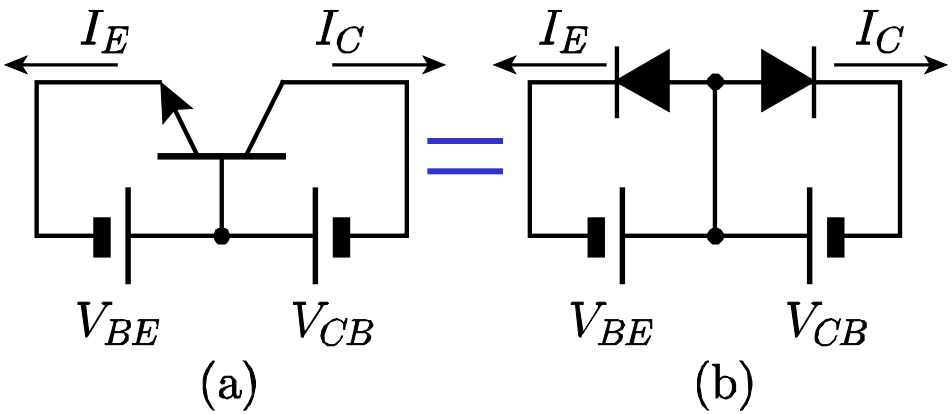
(a) npnトランジスタ



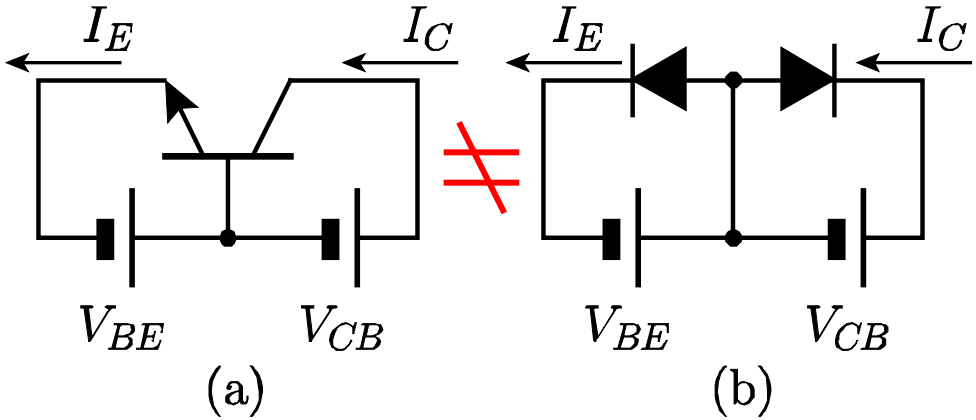
(b) pnpトランジスタ



遮断領域： $V_{BE}<0, V_{BC}<0$

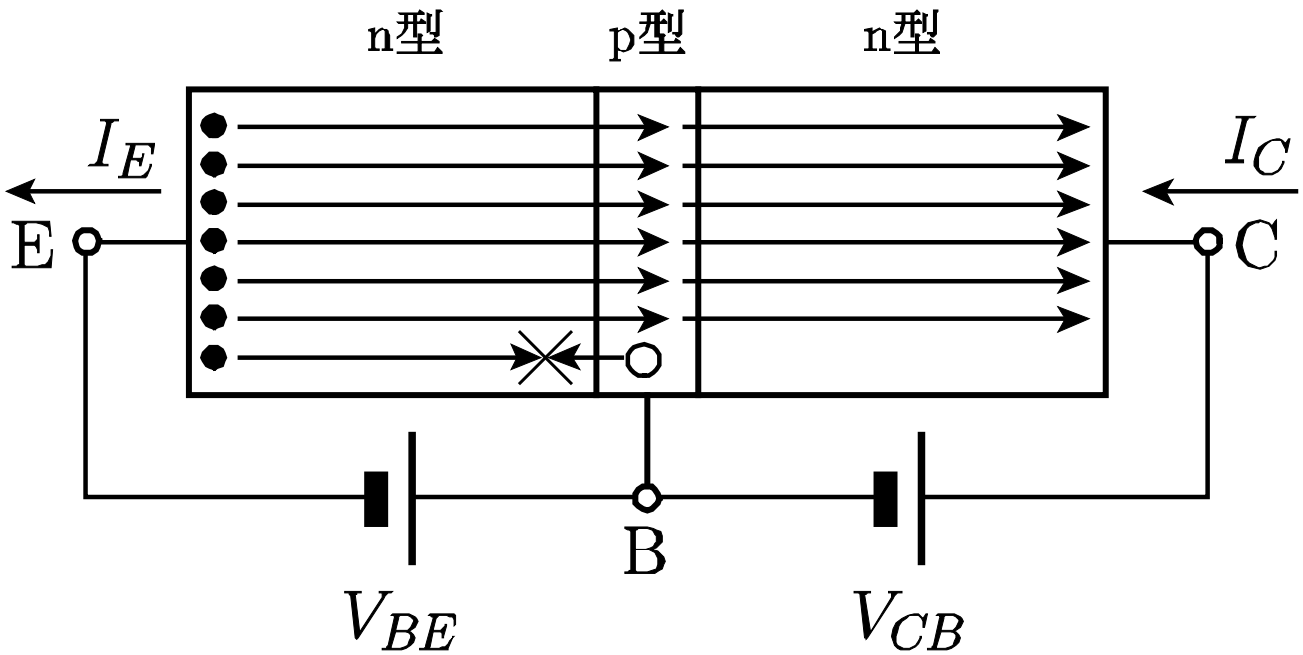


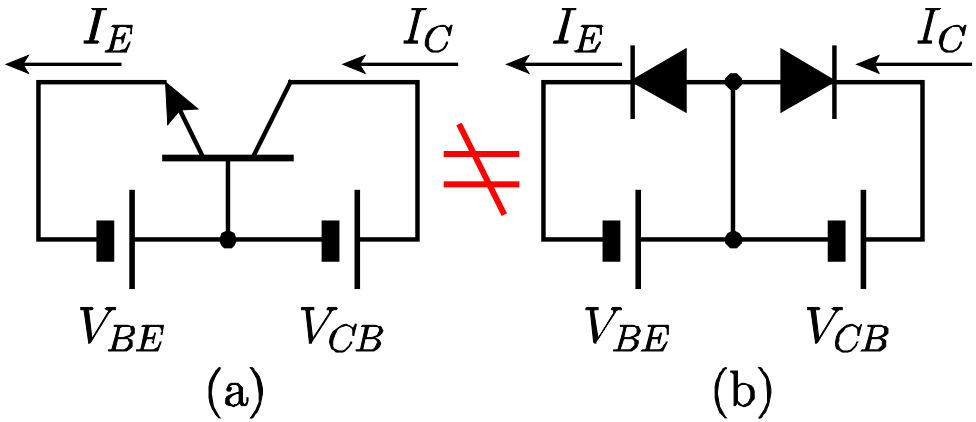
飽和領域：$V_{BE}>0, V_{BC}>0$



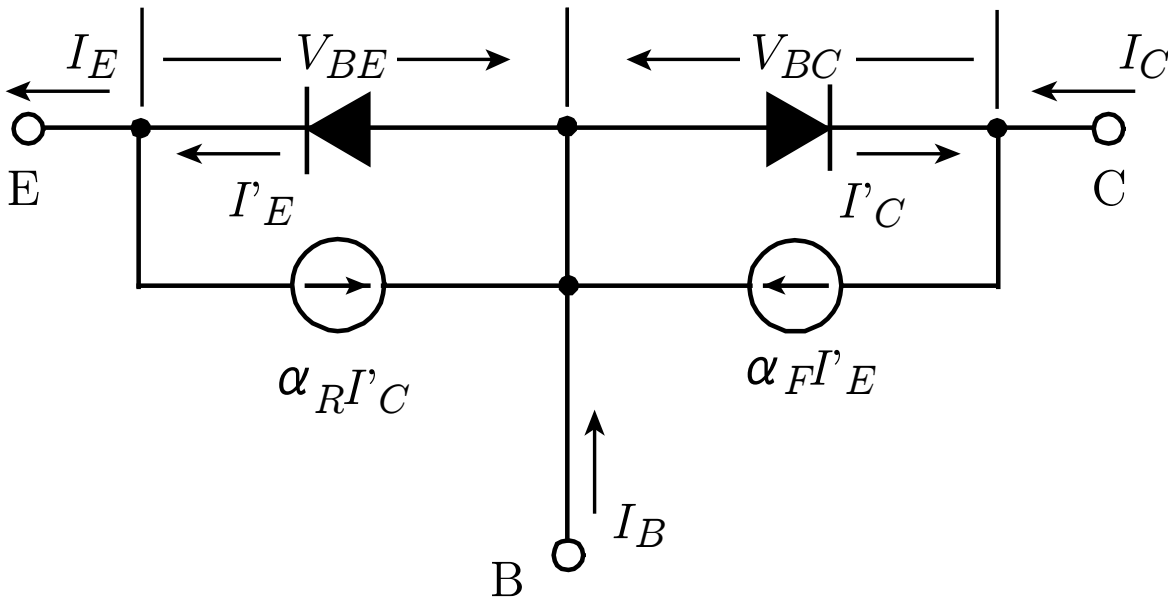
能動活性領域：$V_{BE}>0, V_{BC}<0$

能動活性領域におけるキャリアの移動





Ebers-Mollモデル：制御電源を用いた表現



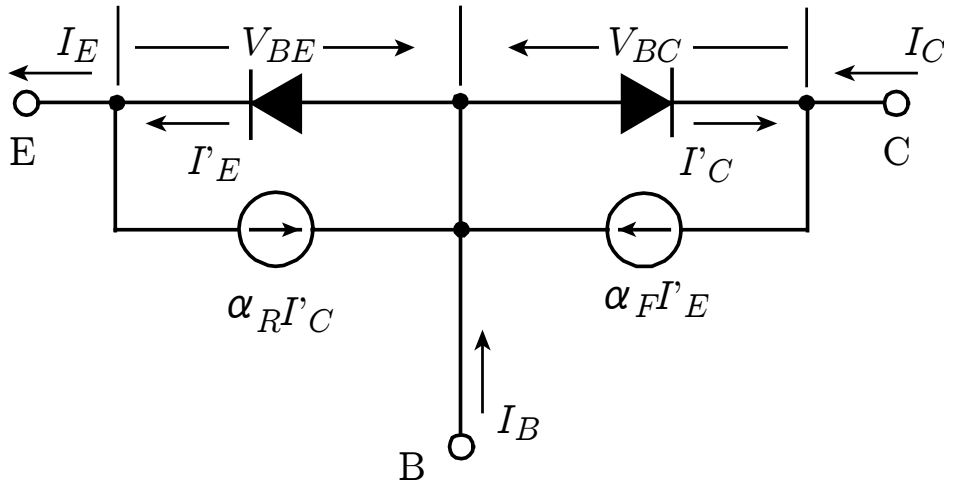
遮断領域：　$V_{BE}<0, V_{BC}<0$

能動活性領域：　$V_{BE}>0, V_{BC}<0$

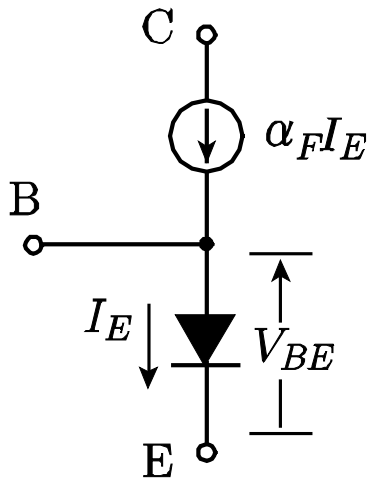
飽和領域：　$V_{BE}>0, V_{BC}>0$

逆方向能動活性領域：$V_{BE}<0, V_{BC}>0$

能動活性領域：　$V_{BE}>0,\ V_{BC}<0$

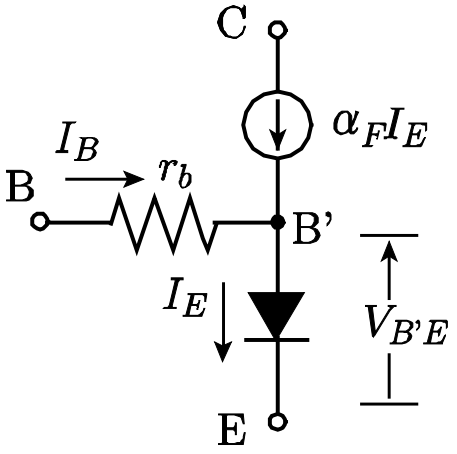


$I_C' \approx 0$



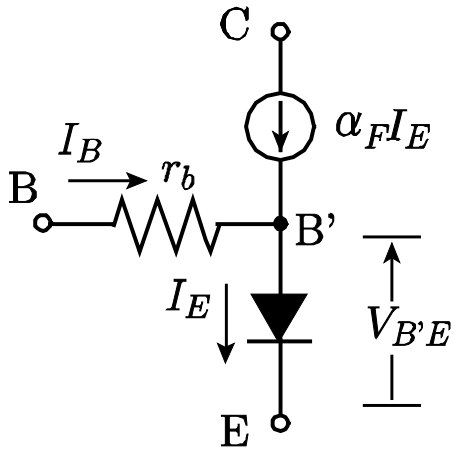
モデルの精度の向上

ベース広がり抵抗を考慮



トランジスタのモデルの簡単化

ベース広がり抵抗を考慮したモデル

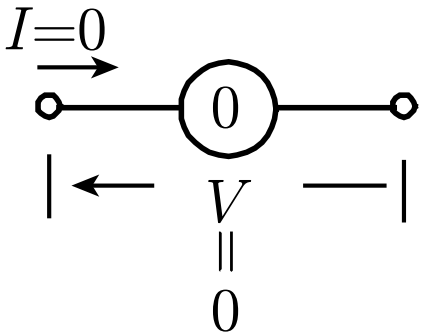


$$\alpha_F \to 1$$

$$I_B = I_E - I_C = (1-\alpha_F) I_E \to 0$$

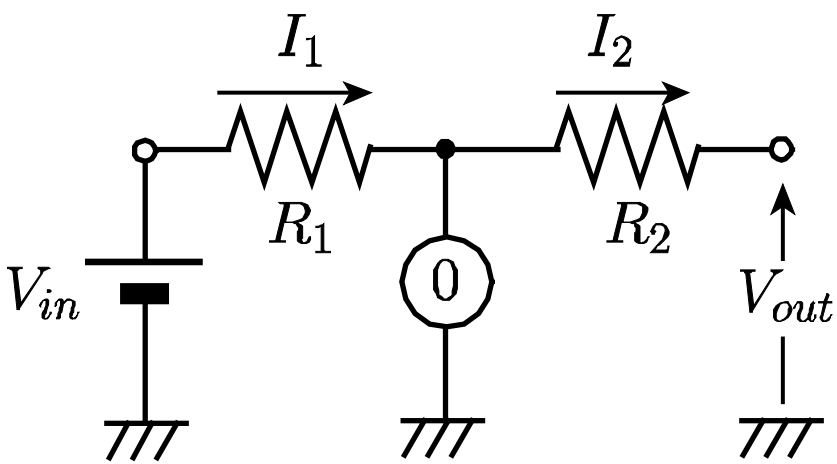
$$V_{BB'} = r_b I_B \to 0$$

B-B'間



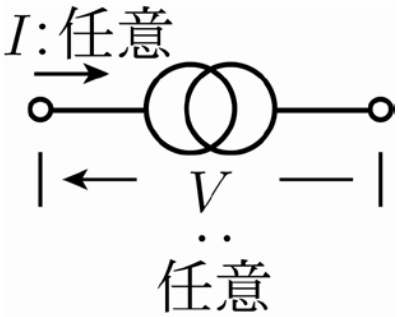
(a) ナレータ

ナレータ単独使用の問題点



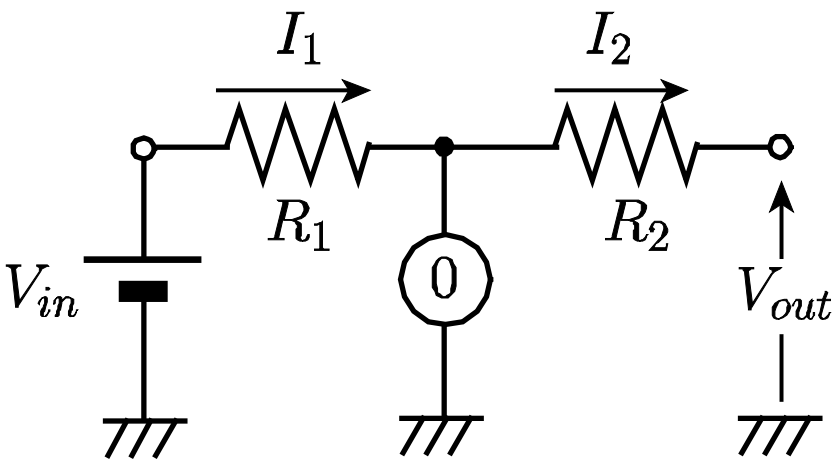
$$I_1 = \frac{V_{in}}{R_1} \quad \rightarrow \quad I_2 = ?$$

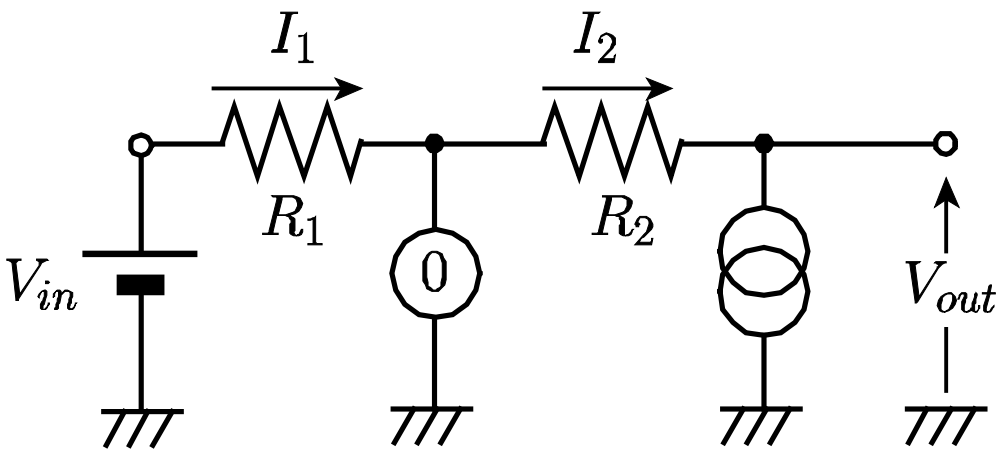
ノレータの導入



(b) ノレータ

矛盾の解消



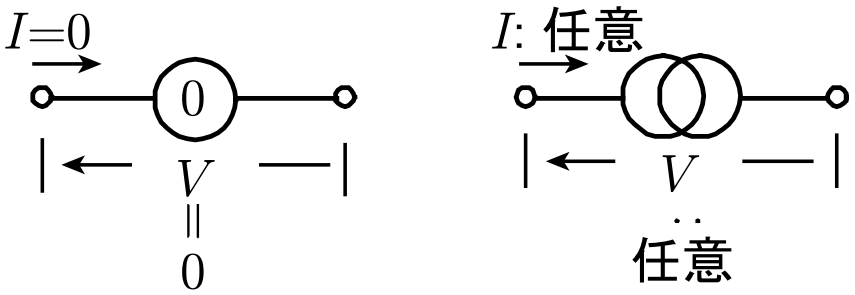


$$I_1 = \frac{V_{in}}{R_1}$$

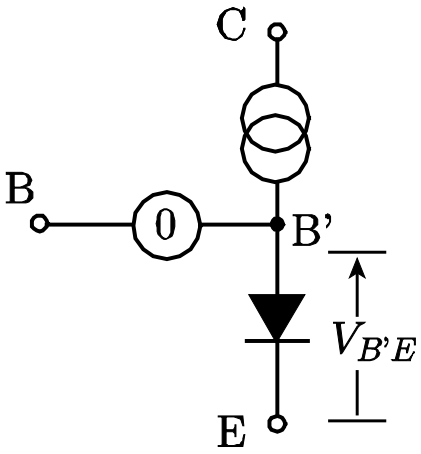
$$I_2 = I_1$$

$$V_{out} = 0 - R_2 I_2 = \frac{-R_2}{R_1} V_{in}$$

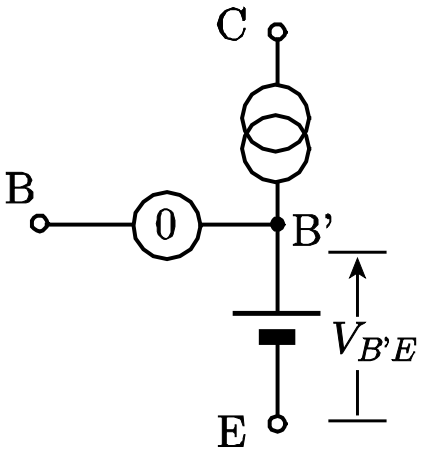
ナレータノレータによるトランジスタのモデリング



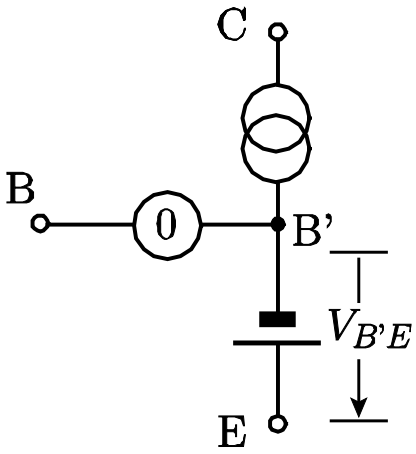
(a) ナレータ　　(b) ノレータ



npnトランジスタ
モデル（1）

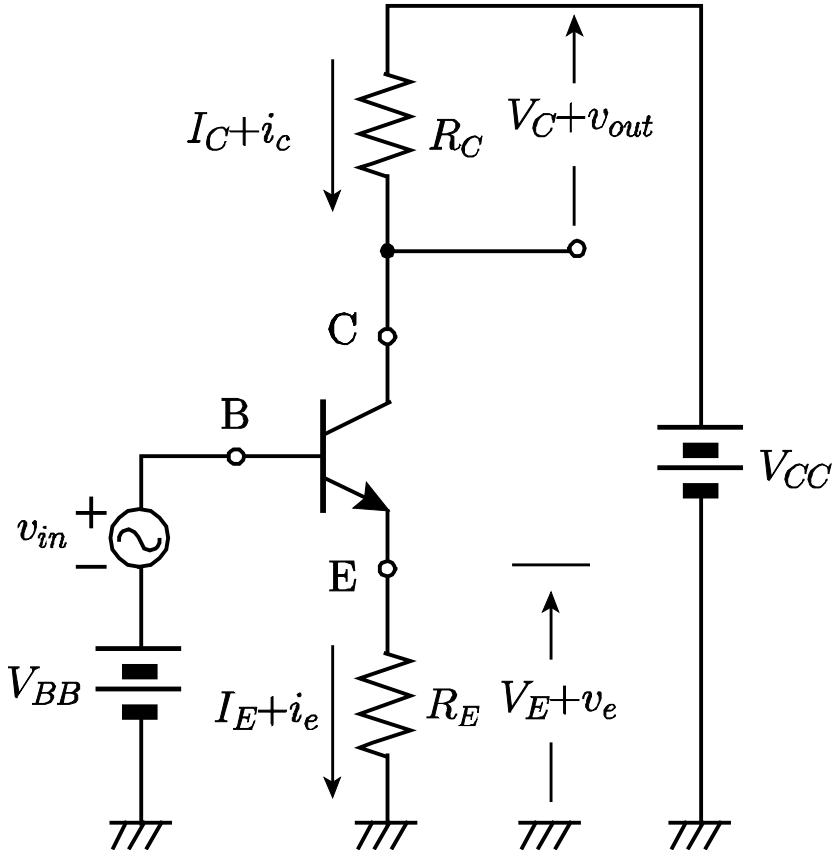


npnトランジスタ
モデル（2）

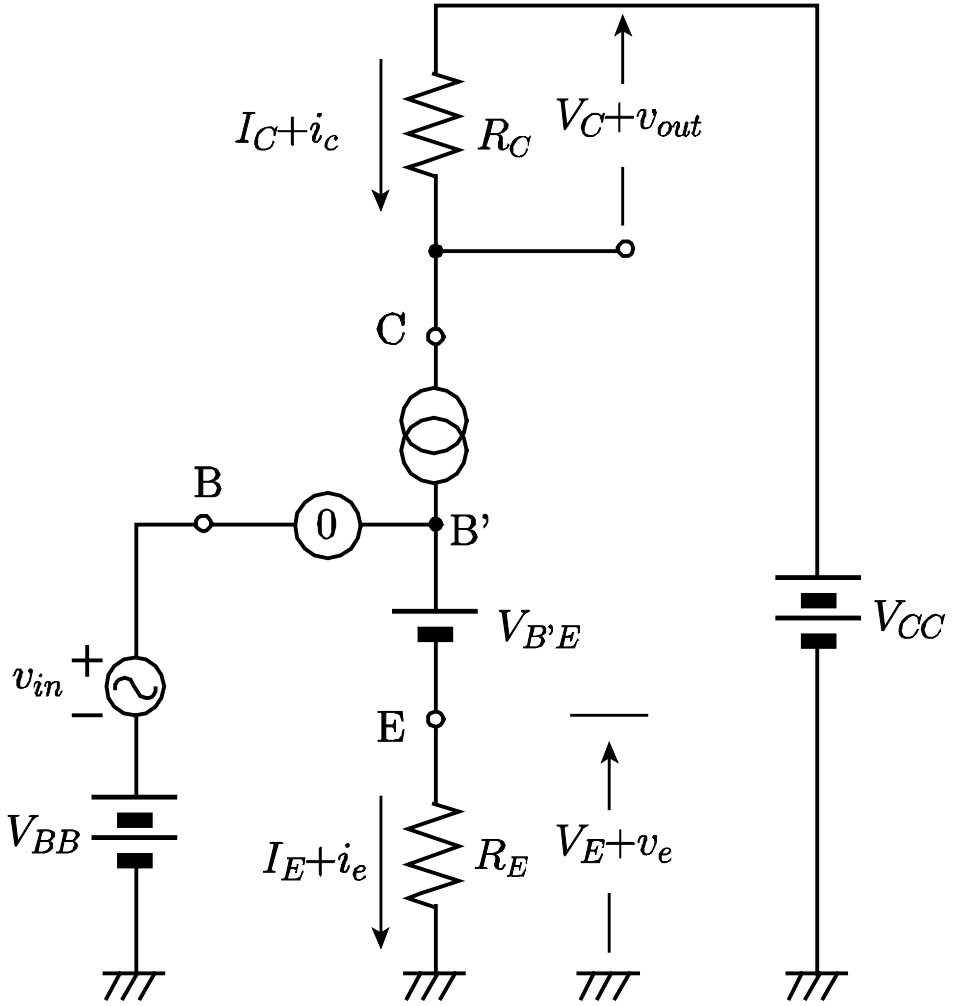


pnpトランジスタ
モデル

トランジスタの増幅作用

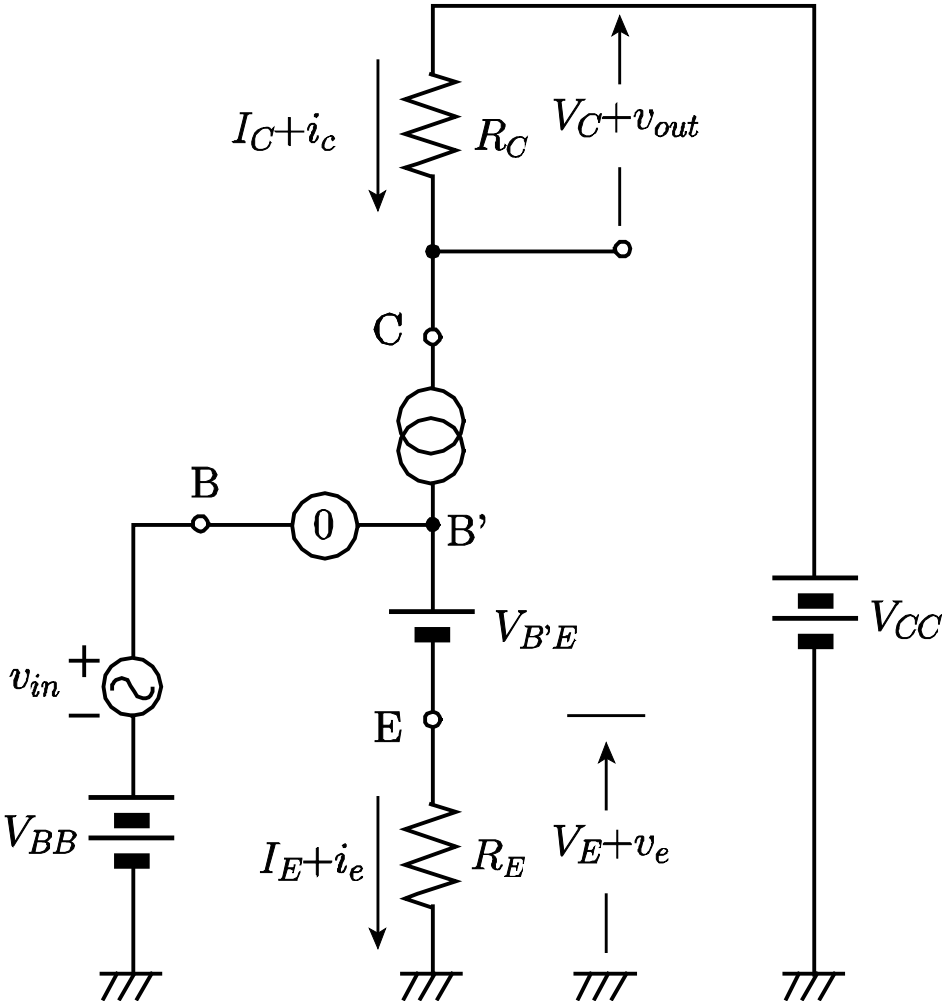


トランジスタ回路



トランジスタ回路のナレータ・ノレータ モデルによる表現

ただし，v_{in}=0のときi_e=0，v_e=0，i_c=0，v_{out}=0



$$V_E=V_{BB}-V_{B'E}$$

$$I_E=\frac{V_E}{R_E}$$

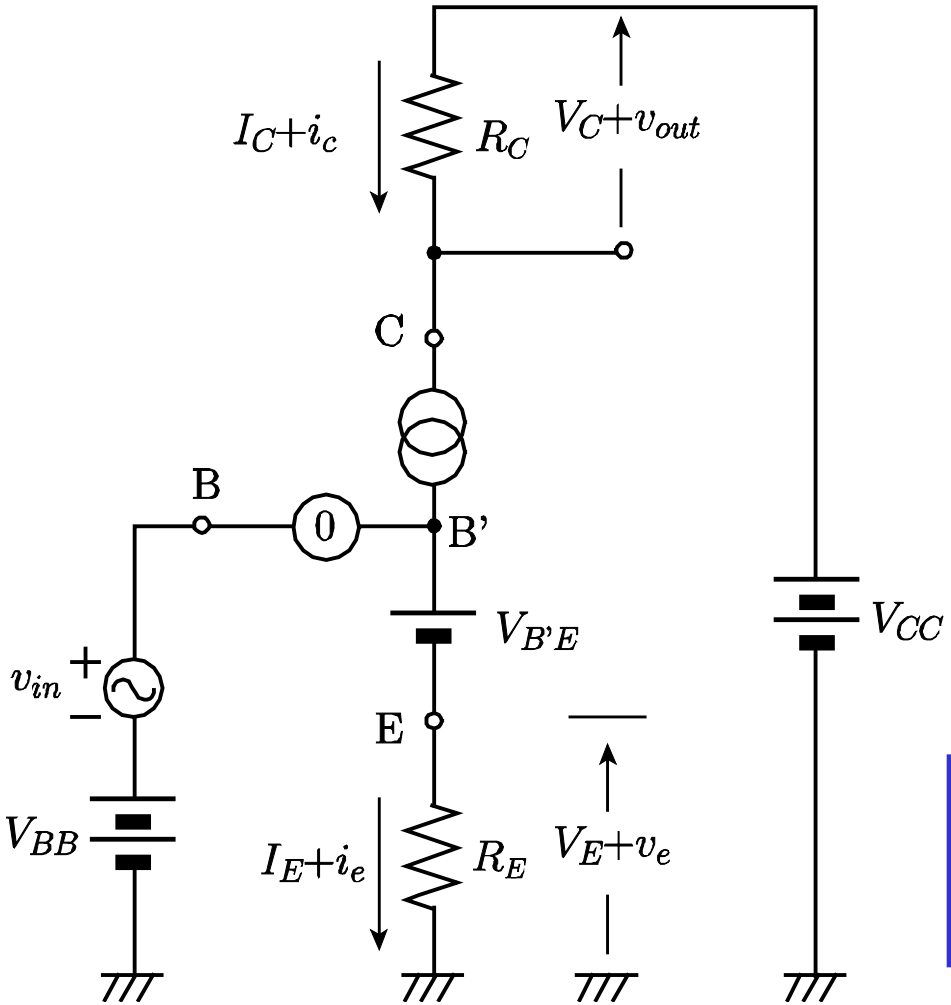
$$I_C=I_E$$

$$V_C=R_C I_C=\frac{R_C(V_{BB}-V_{B'E})}{R_E}$$

小文字：小信号，大文字：大信号

$$V_E+v_e=V_{BB}+v_{in}-V_{B'E},\quad I_E+i_e=\frac{V_E+v_e}{R_E},\quad I_C+i_c=I_E+i_e$$

$$V_C+v_{out}=R_C(I_C+i_c)=\frac{R_C(V_{BB}+v_{in}-V_{B'E})}{R_E}$$



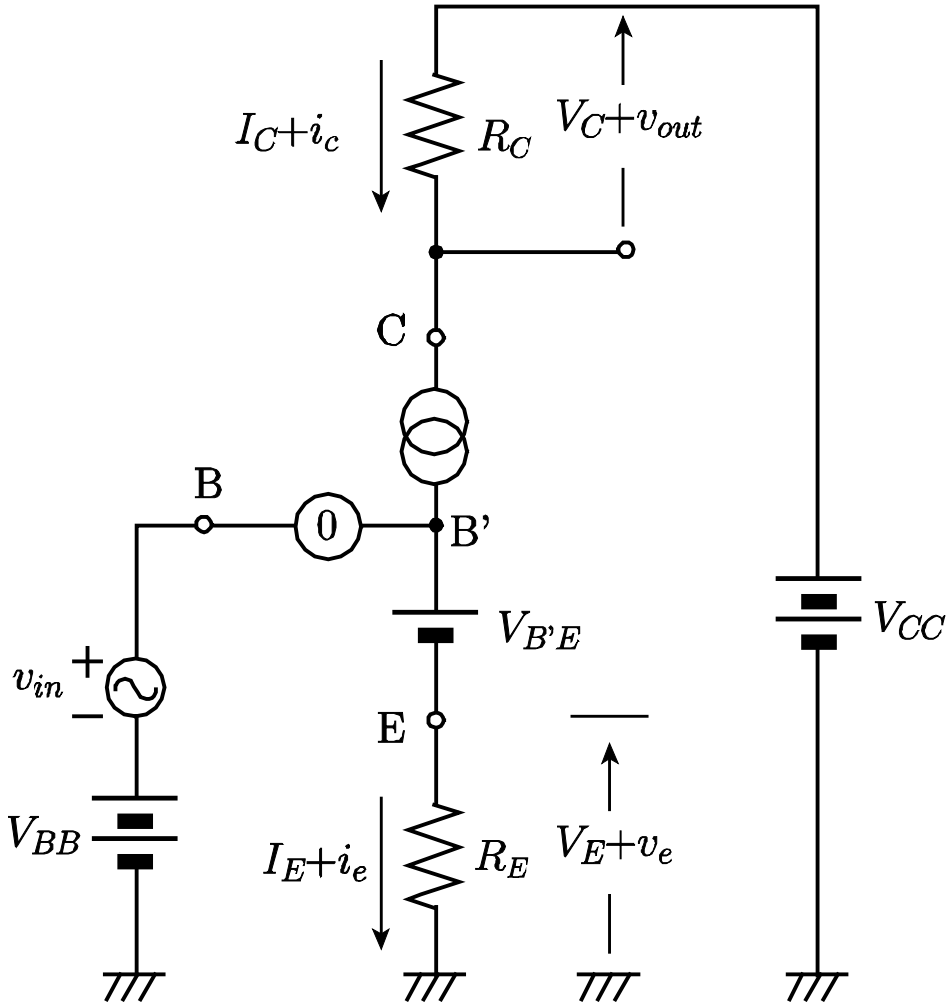
$$v_e=v_{in}$$

$$i_e=\frac{v_e}{R_E}$$

$$i_c=i_e$$

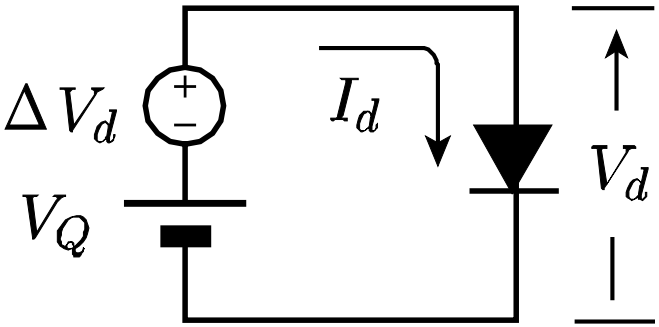
$$v_{out}=\frac{R_C}{R_E}v_{in}$$

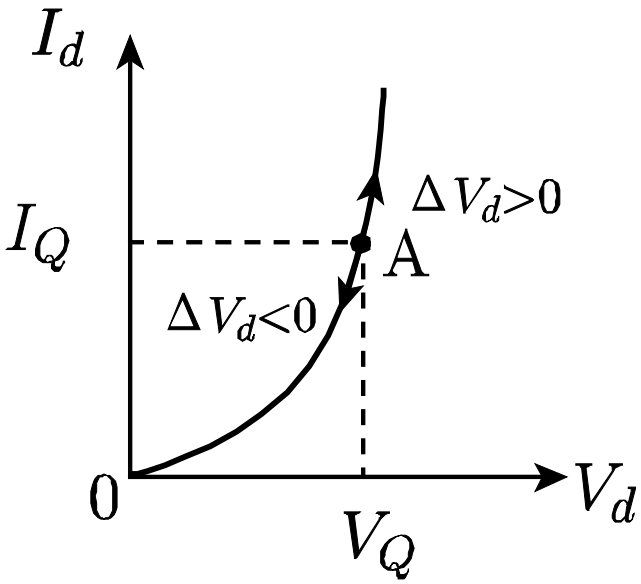
$A_v=\dfrac{R_C}{R_E}$: 電圧増幅利得



問：v_{in}=0のときV_E及びV_Cを求めよ．
ただし，V_{BB}=1.0V，$V_{B'E}$=0.70V，
R_E=3.0kΩ，R_C=15kΩとする．

直流成分(大信号)と信号成分(小信号)の分離



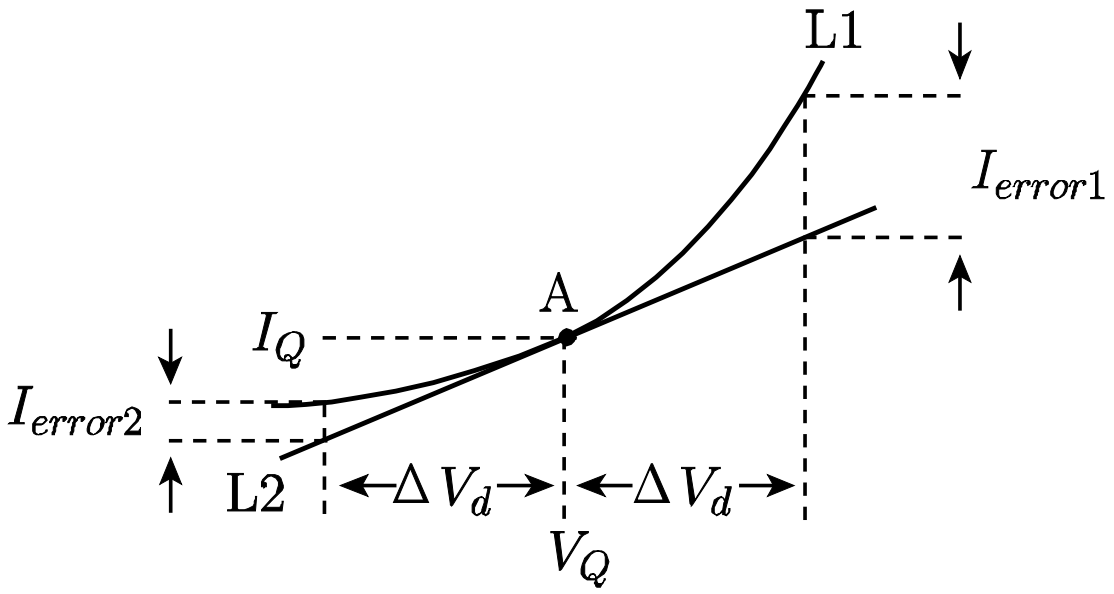


$$I_d \approx I_Q + \left.\frac{\partial I_d}{\partial V_d}\right|_{V_d=V_Q} \cdot \Delta V_d$$

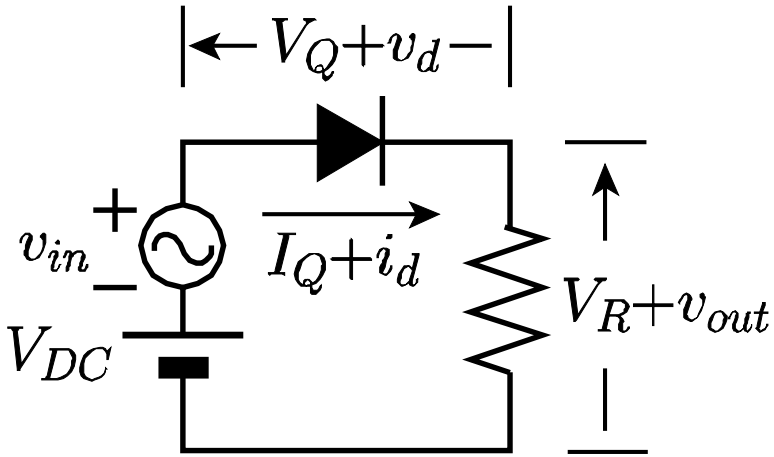
信号成分＝直流成分からの変化分

直線近似の導入

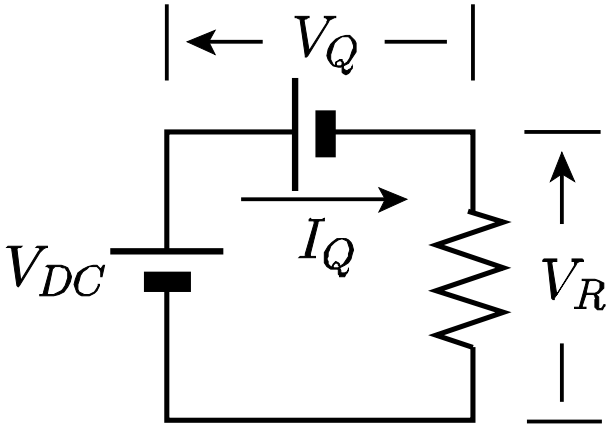
$$\Delta I_d = \left.\frac{\partial I_d}{\partial V_d}\right|_{V_d = V_Q} \cdot \Delta V_d$$



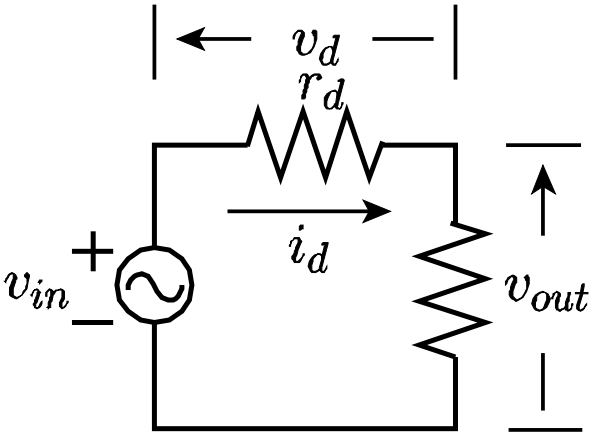
A点を原点とする電圧と電流は比例関係＝抵抗



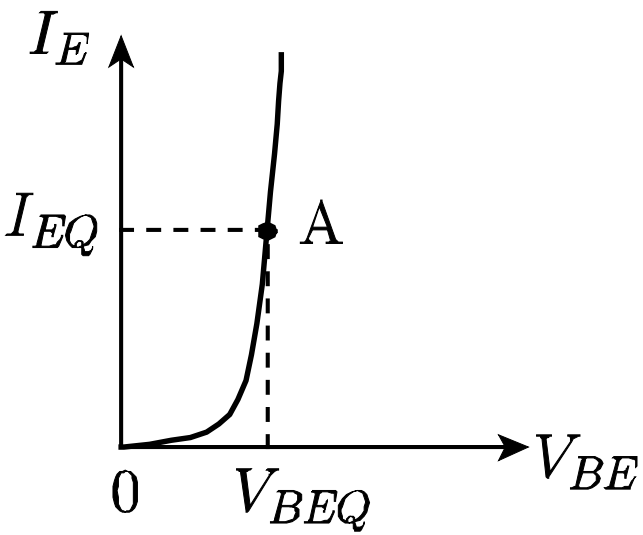
直流成分



信号成分



バイポーラトランジスタの小信号モデル

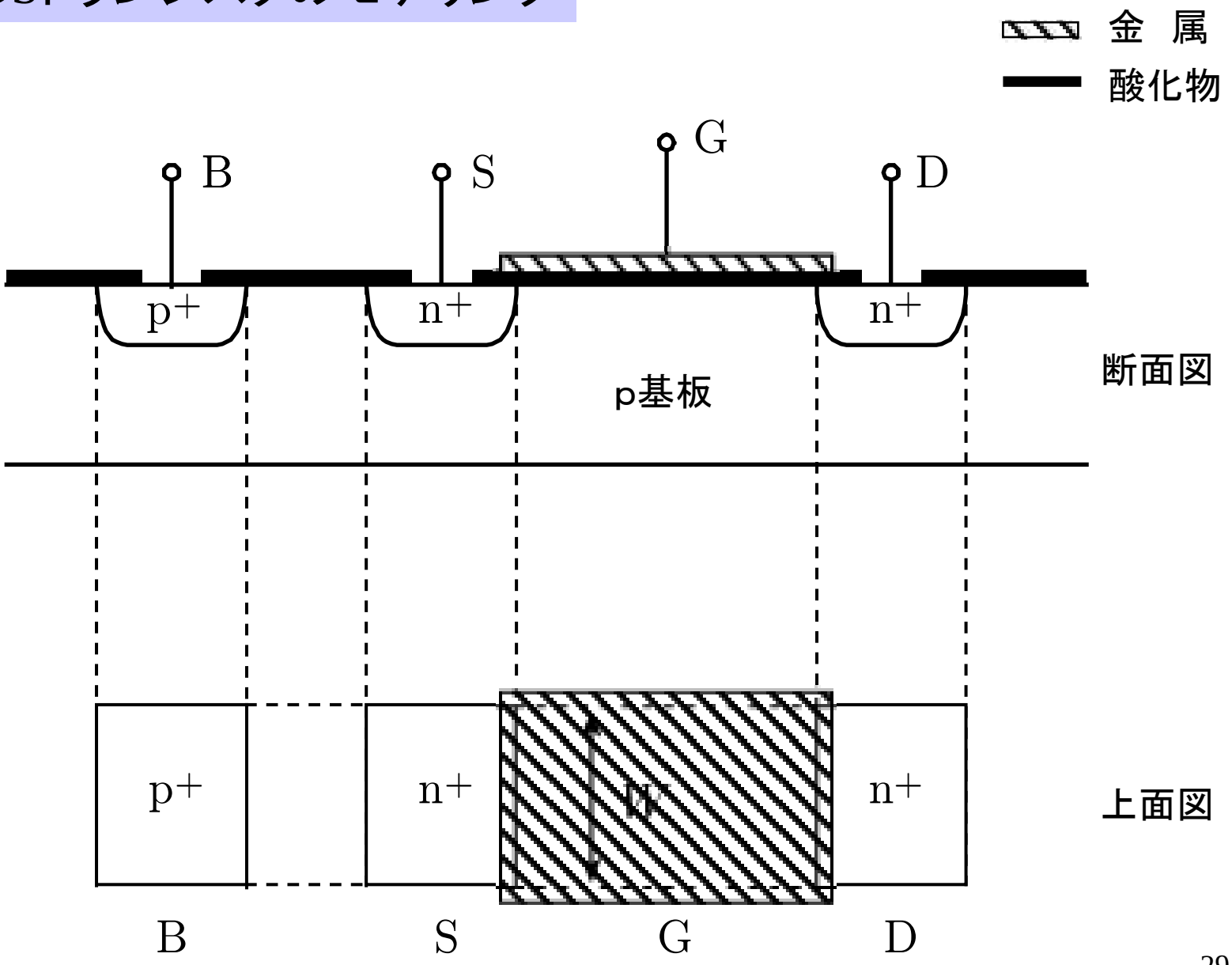


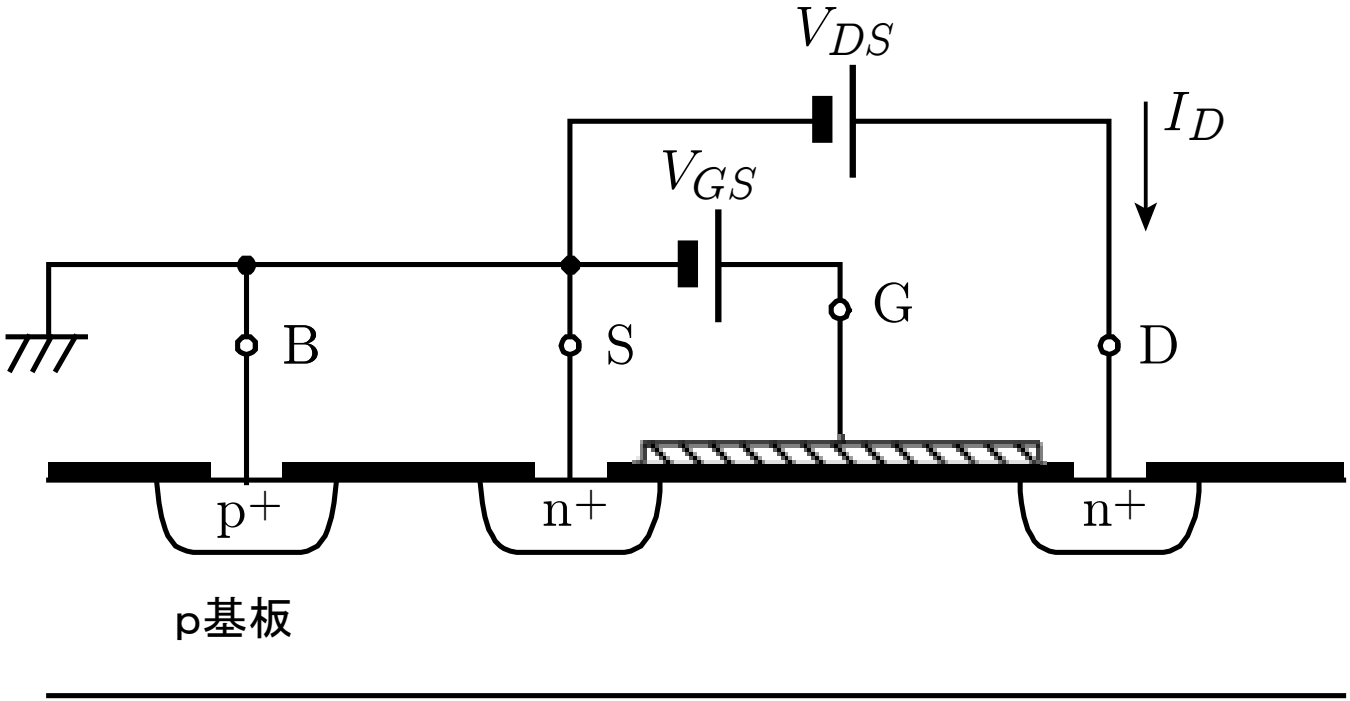
$$I_E = I_{ES}\left\{\exp\left(\frac{qV_{BE}}{kT}\right) - 1\right\}$$

$$I_E = \left.\frac{\partial I_E}{\partial V_{BE}}\right|_{V_{BE}=V_{BEQ}} \cdot \Delta V_{BE} + I_{EQ}$$

$$\boxed{r_e = \left[\left.\frac{\partial I_E}{\partial V_{BE}}\right|_{V_{BE}=V_{BEQ}}\right]^{-1} \approx \frac{kT}{qI_{EQ}} = \frac{0.026[\mathrm{V}]}{I_{EQ}[\mathrm{A}]}}$$

MOSトランジスタのモデリング

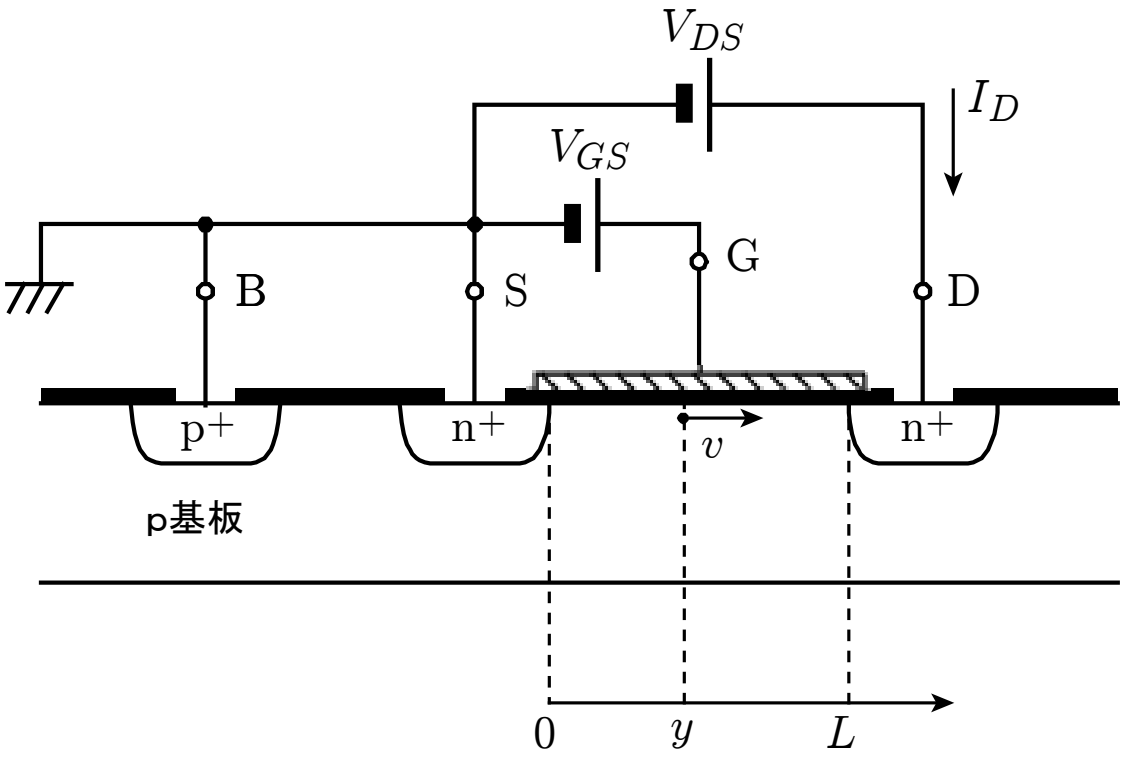




V_{SB}=0と仮定

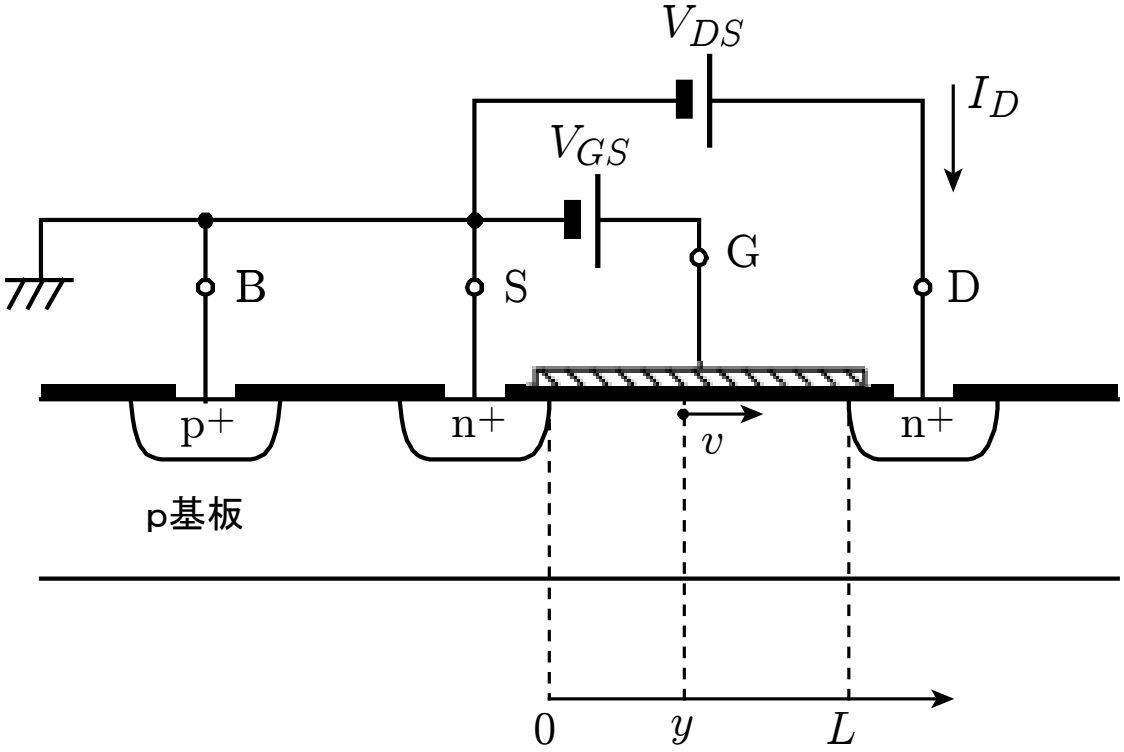
遮断領域：	$V_T > V_{GS}$
非飽和領域：	$V_{GS} - V_T > V_{DS} \geq 0$
飽和領域：	$V_{DS} \geq V_{GS} - V_T > 0$

非飽和領域：$V_{GS}-V_T>V_{DS}\geq 0$



地点yにおけるチャネルの電位：$V(y)$

地点yにおける単位面積当たりの電荷：$Q(y)$



μ : 移動度

C_{OX}: 単位面積当たりのゲート酸化膜容量

$$Q(y) = -C_{OX}\{V_{GS} - V_T - V(y)\}$$

$$I_D = Q(y) \times W \times (-v)$$

$$Q(y) = -C_{OX}\{V_{GS} - V_T - V(y)\}$$

$$v = -\mu E$$

$$I_D = Q(y) \times W \times (-v)$$

$$E = -\frac{dV}{dy}$$

$$\boxed{I_D = \mu C_{OX} W \{V_{GS} - V_T - V(y)\}\frac{dV}{dy}}$$

$$\int_0^L I_D dy = \int_0^L \mu C_{OX} W \{V_{GS} - V_T - V(y)\}\frac{dV}{dy} dy$$

$$= \int_0^{V_{DS}} \mu C_{OX} W \{V_{GS} - V_T - V(y)\} dV$$

$$= \left[\frac{-1}{2}\mu C_{OX} W \{V_{GS} - V_T - V(y)\}^2\right]_0^{V_{DS}}$$

$$\boxed{I_D L = \mu C_{OX} W (V_{GS} - V_T - \frac{V_{DS}}{2}) V_{DS}}$$

$$I_D=\mu C_{OX}\frac{W}{L}(V_{GS}-V_T-\frac{V_{DS}}{2})V_{DS}$$

飽和領域：　　$V_{DS}\geq V_{GS}-V_T>0$

$V_{DS}=V_{GS}-V_T$となったときのI_Dから変化しない.

2乗則

$$I_D=\frac{1}{2}\mu C_{OX}\frac{W}{L}(V_{GS}-V_T)^2$$

遮断領域：　$V_T > V_{GS}$

非飽和領域：　$V_{GS} - V_T > V_{DS} \geq 0$

飽和領域：　$V_{DS} \geq V_{GS} - V_T > 0$

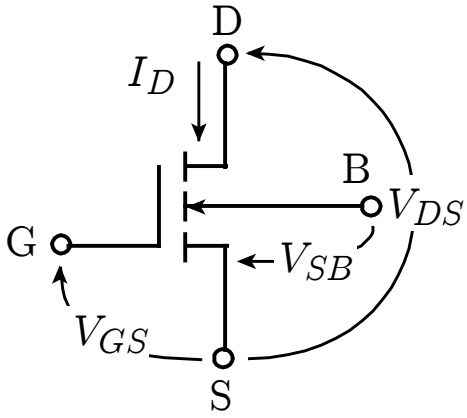
遮断領域：$I_D = 0$

非飽和領域：$I_D = 2K\left(V_{GS} - V_T - \frac{V_{DS}}{2}\right)V_{DS}$

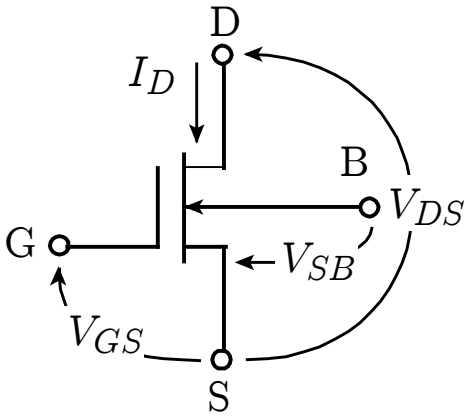
飽和領域：$I_D = K(V_{GS} - V_T)^2$

$K = \frac{1}{2}\mu C_{OX}\frac{W}{L}$: トランスコンダクタンスパラメータ

MOSトランジスタの記号

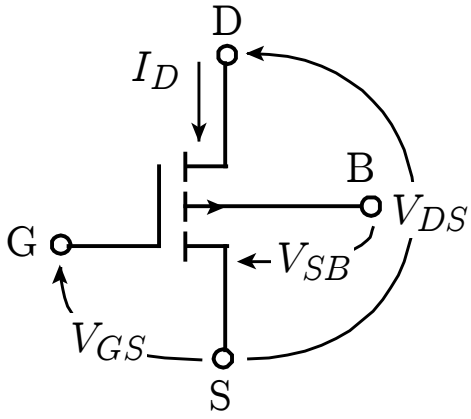


エンハンスメント型

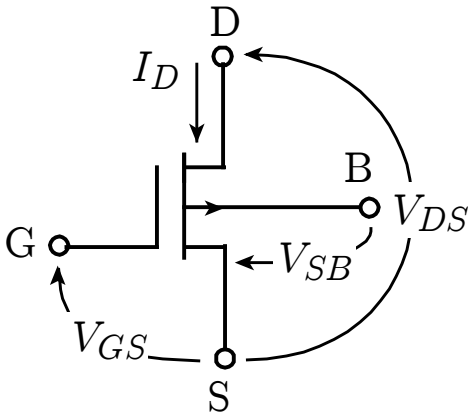


ディプリーション型

nチャネル MOSトランジスタ



エンハンスメント型



ディプリーション型

pチャネル MOSトランジスタ

MOSトランジスタにおける2次的効果

基板効果

$$V_T = V_{T0} + \gamma\left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right)$$

チャネル長変調効果

$$I_D = K(V_{GS} - V_T)^2(1 + \lambda V_{DS})$$

MOSトランジスタの小信号等価回路

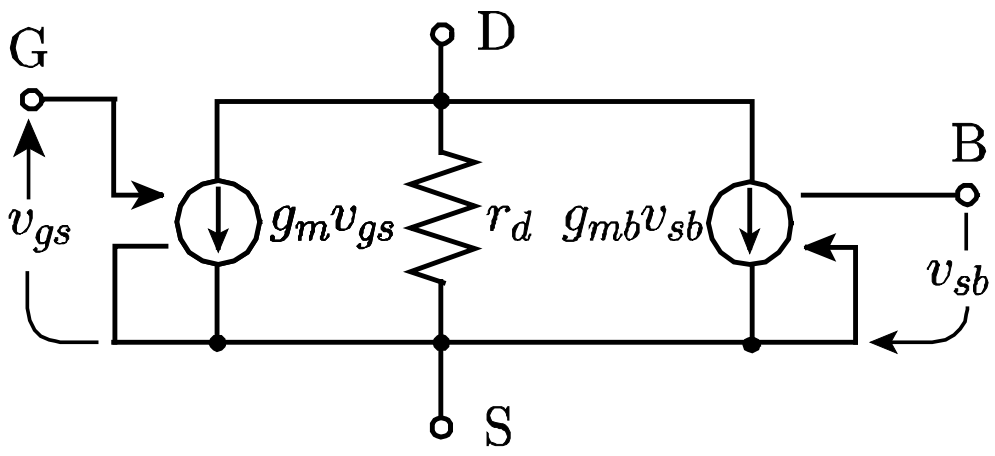
$$I_D = f(V_{GS}, V_{DS}, V_{SB})$$

$$\Delta I_D = \frac{\partial f}{\partial V_{GS}} \Delta V_{GS} + \frac{\partial f}{\partial V_{DS}} \Delta V_{DS} + \frac{\partial f}{\partial V_{SB}} \Delta V_{SB}$$

$$i_d = \frac{\partial f}{\partial V_{GS}} v_{gs} + \frac{\partial f}{\partial V_{DS}} v_{ds} + \frac{\partial f}{\partial V_{SB}} v_{sb}$$

$$\frac{\partial f}{\partial V_{GS}} = g_m, \quad \frac{\partial f}{\partial V_{DS}} = \frac{1}{r_d}, \quad \frac{\partial f}{\partial V_{SB}} = g_{mb}$$

一般的なMOSトランジスタの小信号等価回路



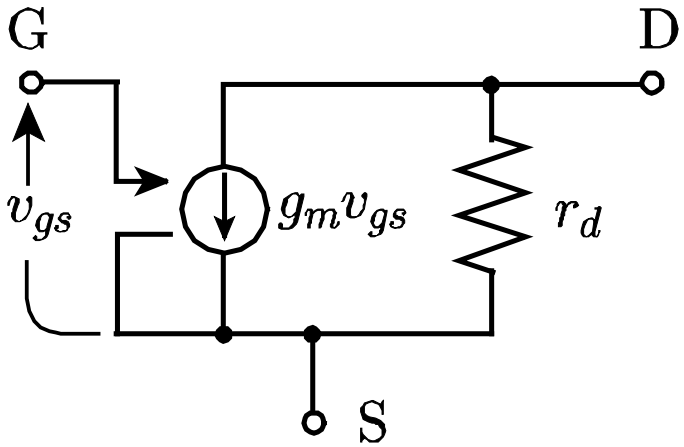
例題

$$I_D = K(V_{GS} - V_T)^2(1+\lambda V_{DS})$$

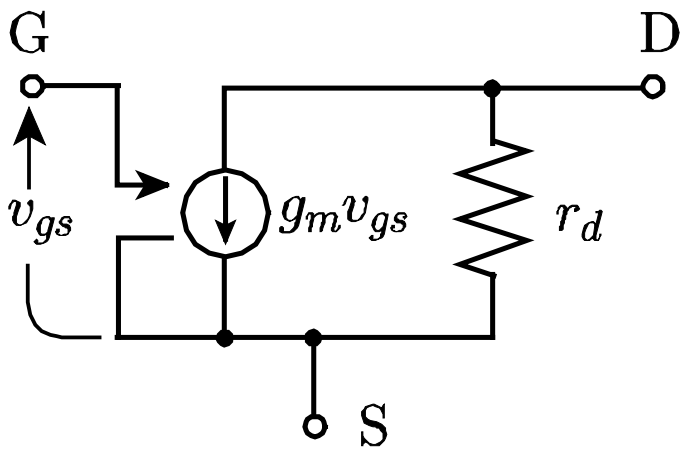
$$g_m = \frac{\partial f}{\partial V_{GS}} = 2K(V_{GS} - V_T)(1+\lambda V_{DS}) = 2\sqrt{KI_D(1+\lambda V_{DS})}$$

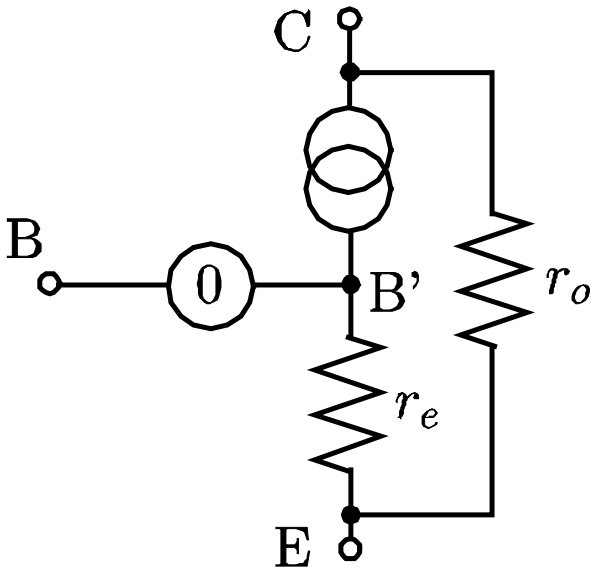
$$r_d = \frac{1}{\dfrac{\partial f}{\partial V_{DS}}} = \frac{1+\lambda V_{DS}}{\lambda I_D}$$

$$g_{mb} = \frac{\partial f}{\partial V_{SB}} = 0$$



問：下記の2個の回路が等価となる条件を示せ.





$$r_d = r_o, \quad g_m = \frac{1}{r_e}$$