電磁アクチュエータ

12 同期モータの基礎

同期モータの無負荷起電力

a相の電機子巻線に鎖交する磁束は

$$\phi_a = SB_{max}\cos(\omega t)$$

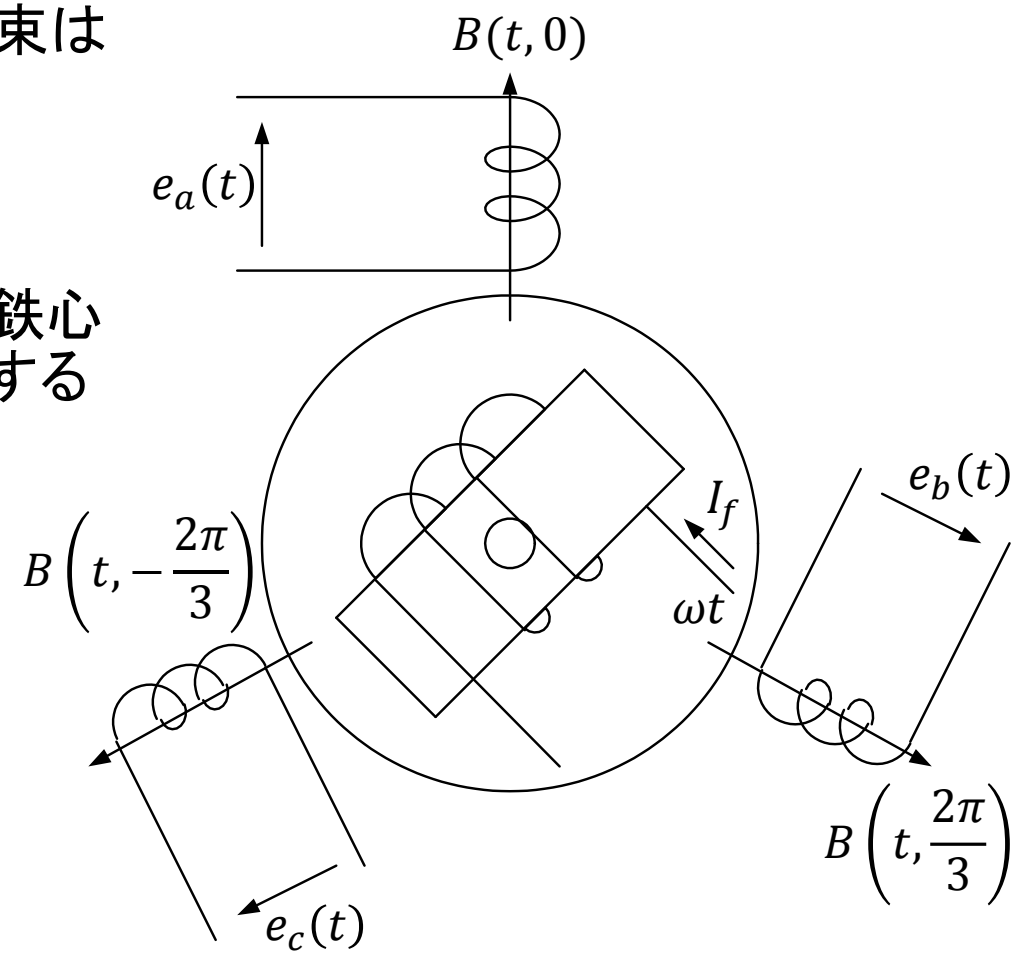
S: 電機子コイルの面積

界磁巻線に直流電流I_fを通じる鉄心の磁気抵抗を無視すると，発生する最大磁束密度は

$$B_{max} = \frac{\mu_0 N_f I_f}{\ell_g}$$

N_f: 界磁巻線の巻数

ℓ_g: ギャップ長



同期モータの無負荷起電力

$$e_a = N_a \frac{d\phi_a}{dt} = N_a S B_{max} \frac{d}{dt}\cos(\omega t)$$
$$= N_a N_f \frac{\mu_0 S}{\ell_g} I_f \omega \sin(\omega t)$$
$$= \frac{N_a N_f}{\mathcal{R}_g} I_f \omega \sin(\omega t)$$

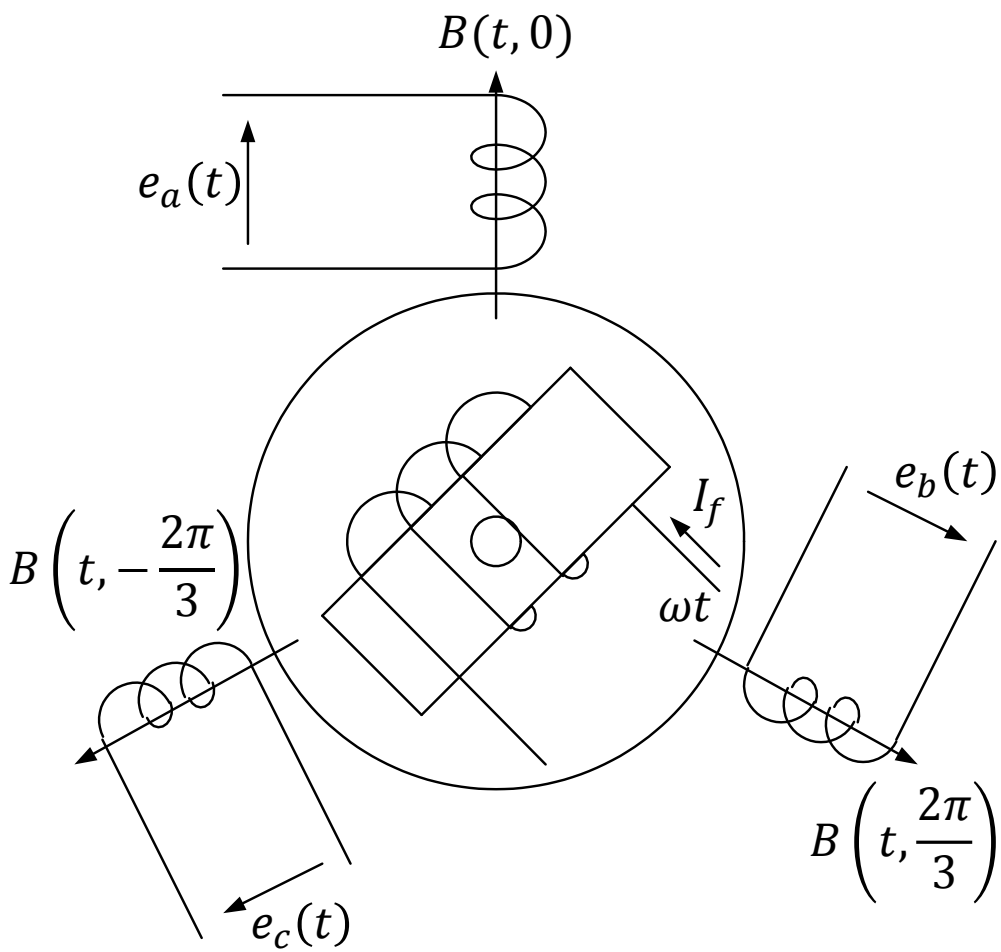
N_a: 電機子巻線の巻数
S: 電機子磁極の面積

ここで，$\frac{N_a N_f}{\mathcal{R}_g}$は界磁巻線と電機子巻線の間の相互インダクタンス$M$であるので，

$$e_a = M I_f \omega \sin(\omega t)$$

であり，極対数$\omega = p\omega_m$であるので，

$$e_a = p M I_f \omega_m \sin(\omega t)$$



三相巻線に誘起する起電力

起電力の実効値は,

$$E_0 = \frac{pMI_f\omega_m}{\sqrt{2}} = \frac{MI_f\omega}{\sqrt{2}}$$

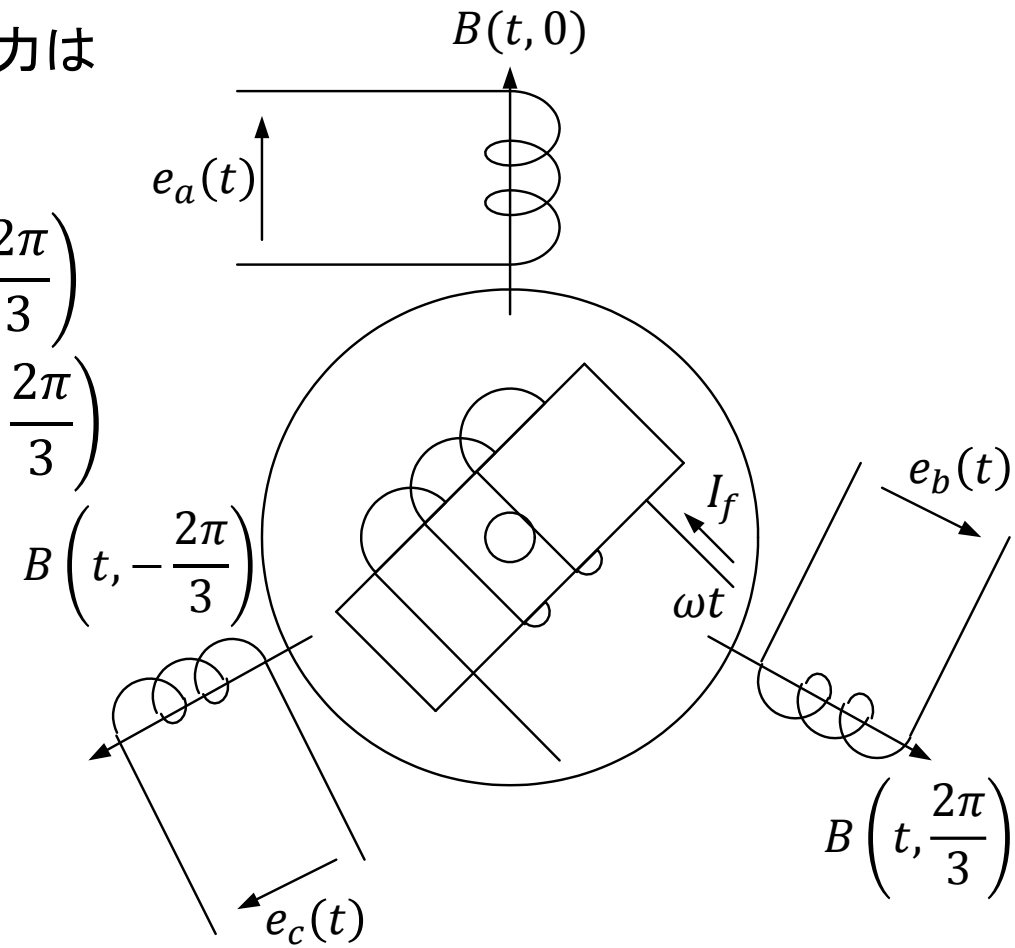
であるので，各コイルに誘導する起電力は

$$e_a = e(t,0) = \sqrt{2}E_0\sin(\omega t)$$

$$e_b = e\left(t,\frac{2\pi}{3}\right) = \sqrt{2}E_0\sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$e_c = e\left(t,-\frac{2\pi}{3}\right) = \sqrt{2}E_0\sin\left(\omega t + \frac{2\pi}{3}\right)$$

三相平衡電圧が誘起する



三相巻線に誘起する起電力

三相巻線の一相に交流電流を流した場合，

$$i_a = \sqrt{2} I \cos \omega t$$

交番磁界が発生する。

$$B_a(t, \theta) = \frac{N_a \mu_0}{\ell_g} \sqrt{2} I \cos \theta \cos \omega t$$

巻線に誘起する起電力は

$$e_a(t) = -N_a \frac{d\phi_a}{dt} = -N_a S \frac{dB_a(t, 0)}{dt}$$

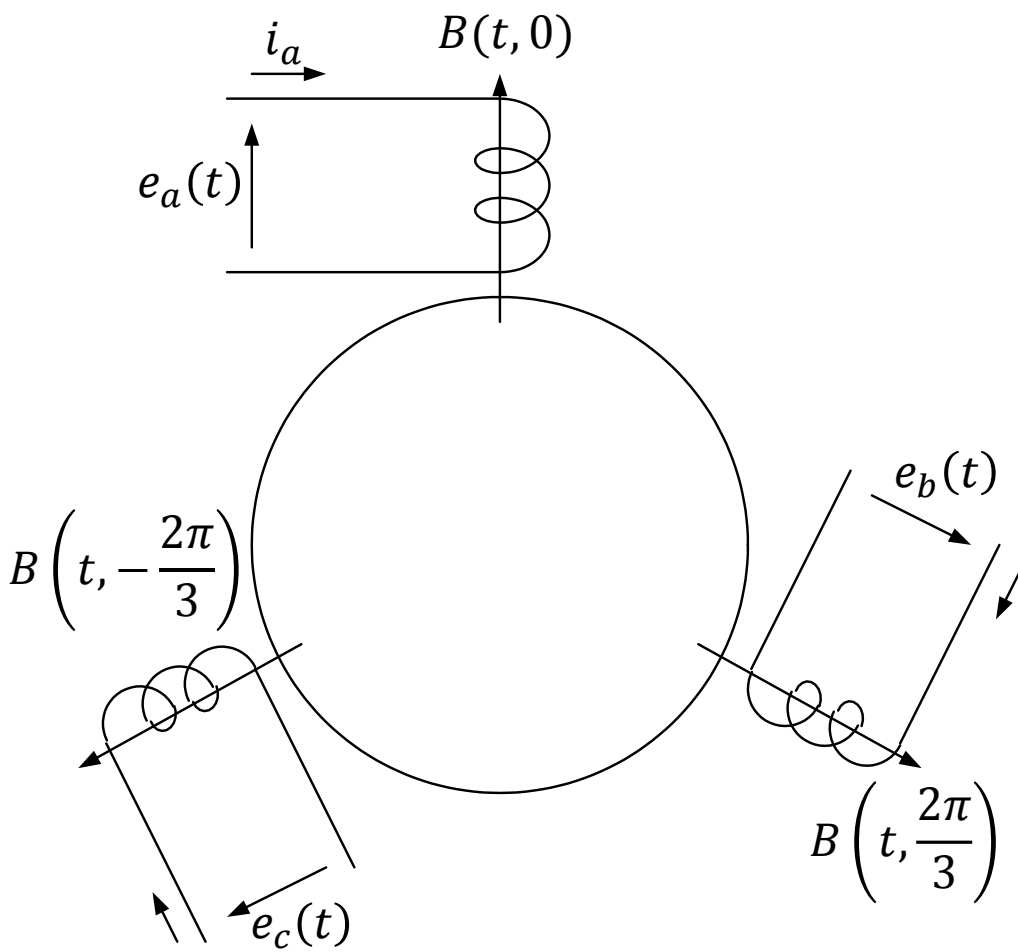
$$= -\frac{{N_a}^2 \mu_0 S}{\ell_g} \sqrt{2} I \omega \sin(\omega t)$$

ただし，$L_0 = \frac{{N_a}^2 \mu_0 S}{\ell_g}$ 自己インダクタンスで，

リアクタンス$X_0 = \omega L_0$を用いると，

$$\dot{E}_a = j\omega L_0 \dot{I} = jX_0 \dot{I}$$

ある。



電機子電流により誘起する起電力

三相巻線に平衡三相交流電流を流した場合，

$$\begin{cases} i_a = \sqrt{2}I\cos\omega t \\ i_b = \sqrt{2}I\cos\left(\omega t - \dfrac{2\pi}{3}\right) \\ i_c = \sqrt{2}I\cos\left(\omega t + \dfrac{2\pi}{3}\right) \end{cases}$$

$$\begin{cases} B_a = \dfrac{N_a\mu_0}{\ell_g}\sqrt{2}I\cos\theta\cos\omega t \\ B_b = \dfrac{N_a\mu_0}{\ell_g}\sqrt{2}I\cos\left(\theta - \dfrac{2\pi}{3}\right)\cos\left(\omega t - \dfrac{2\pi}{3}\right) \\ B_c = \dfrac{N_a\mu_0}{\ell_g}\sqrt{2}I\cos\left(\theta + \dfrac{2\pi}{3}\right)\cos\left(\omega t + \dfrac{2\pi}{3}\right) \end{cases}$$

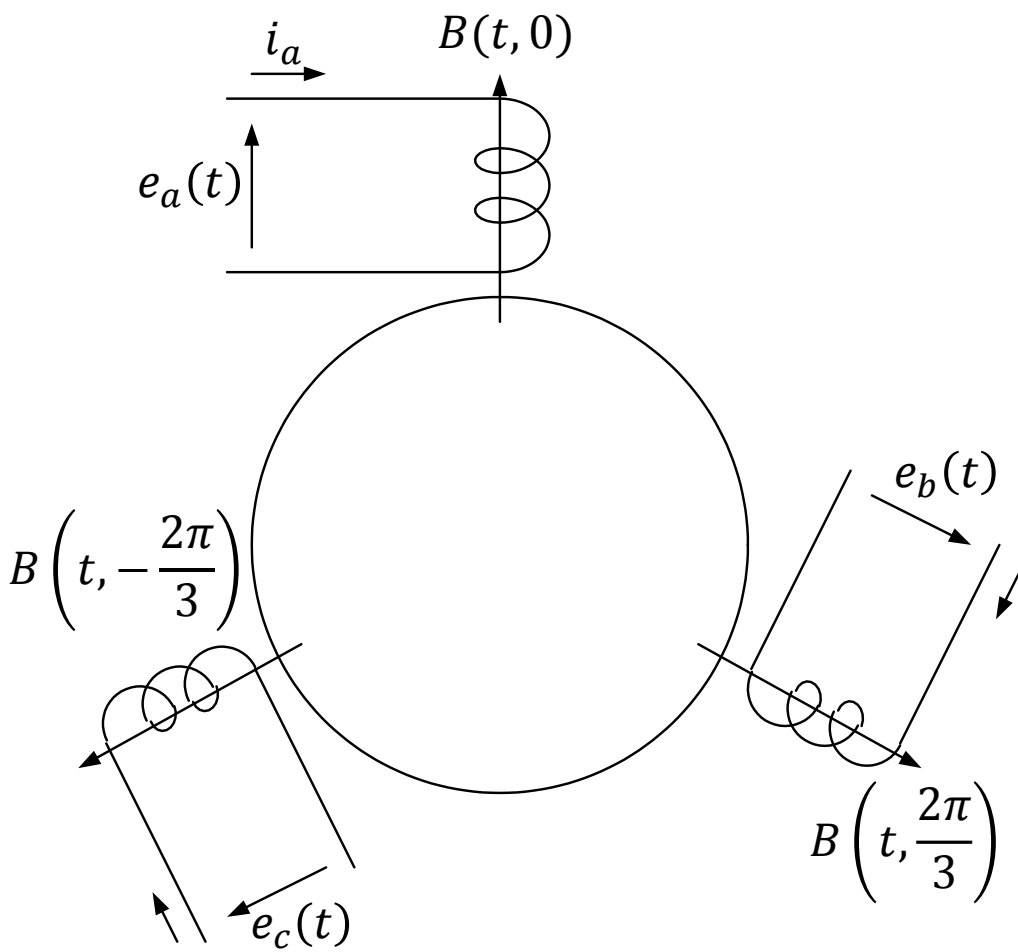
$$B_a + B_b + B_c = \frac{3}{2}\frac{N_a\mu_0}{\ell_g}\sqrt{2}I\cos(\theta - \omega t)$$

巻線に誘起する起電力は

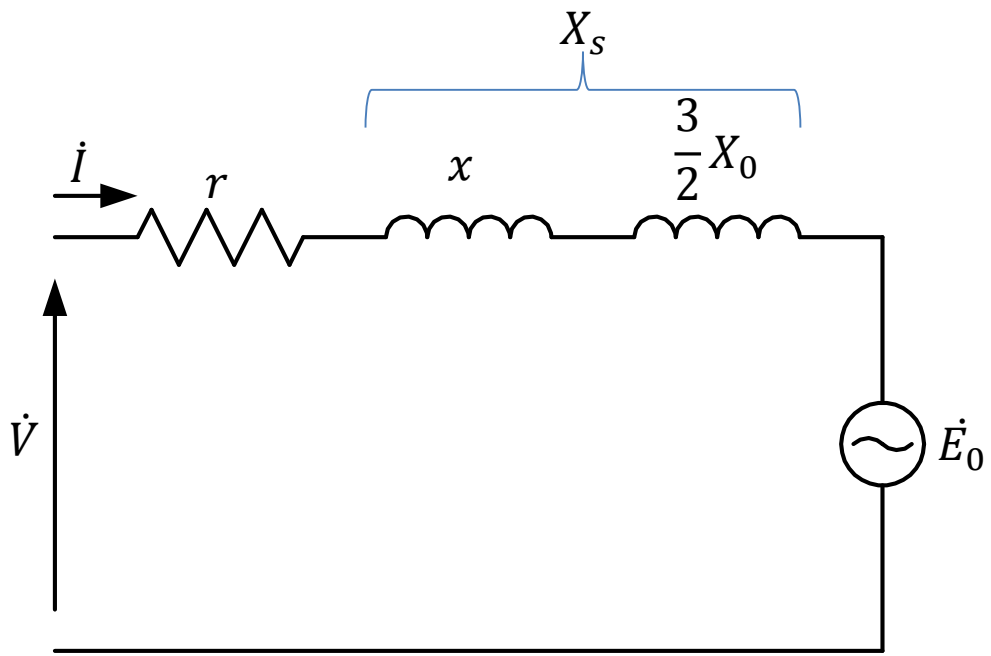
$$e_a(t) = -\frac{3}{2}\frac{{N_a}^2\mu_0 S}{\ell_g}\sqrt{2}I\omega\sin(\omega t)$$

つまり，

$$\dot{E}_a = j\frac{3}{2}\omega L_0\dot{I} = j\frac{3}{2}X_0\dot{I}$$

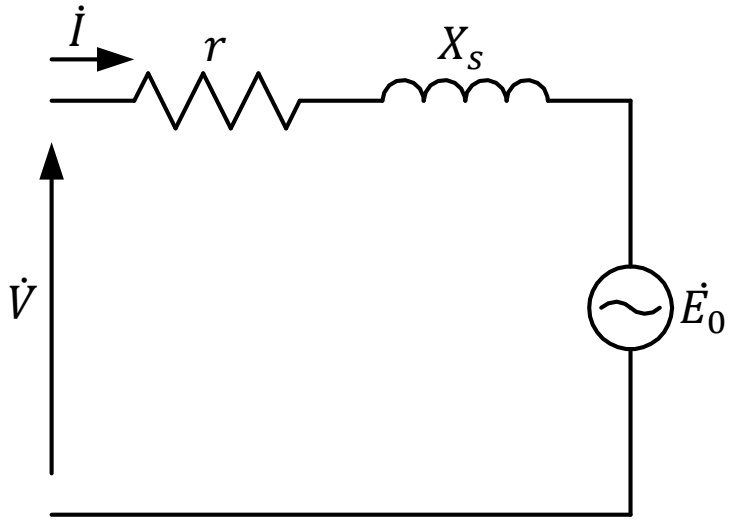


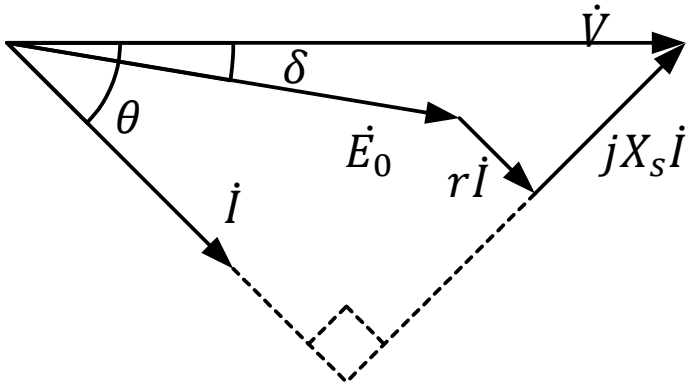
同期モータの等価回路



E_0 : 誘導起電力
X_0 : 自己リアクタンス
x : 漏れリアクタンス
r : 電機子巻線抵抗
X_s : 同期リアクタンス

同期モータのフェーザ図





θ : 力率角

端子電圧$\dot{V}$と電機子電流$\dot{I}$の間の角度

δ : トルク角

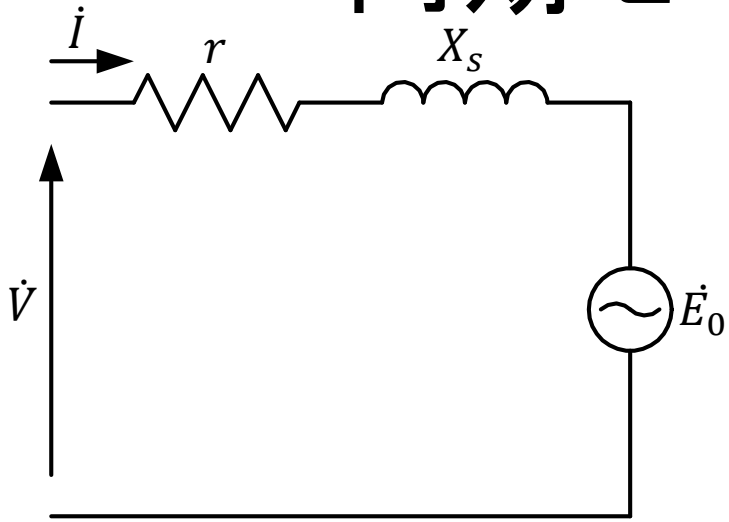
端子電圧$\dot{V}$と誘導起電力$\dot{E}_0$の間の角度

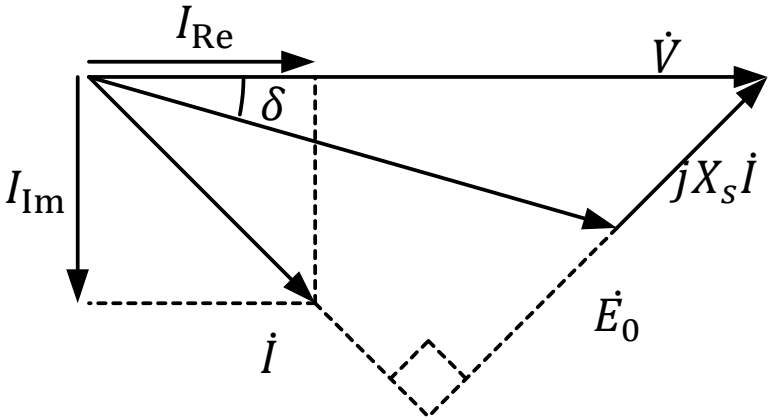
$\dot{V}$は回転磁界より90度進み

$\dot{E}_0$は界磁磁界より90度進み

δ は回転磁界と界磁磁界の間の角でもある。

同期モータのトルク





巻線抵抗rを無視すると電機子電流は

$$\dot{I} = I_{\mathrm{Re}} + jI_{\mathrm{Im}} = \frac{\dot{V} - \dot{E}_0}{jX_s} = \frac{E_0 \sin\delta}{X_s} - j\frac{V - E_0\cos\delta}{X_s}$$

三相分の電力は

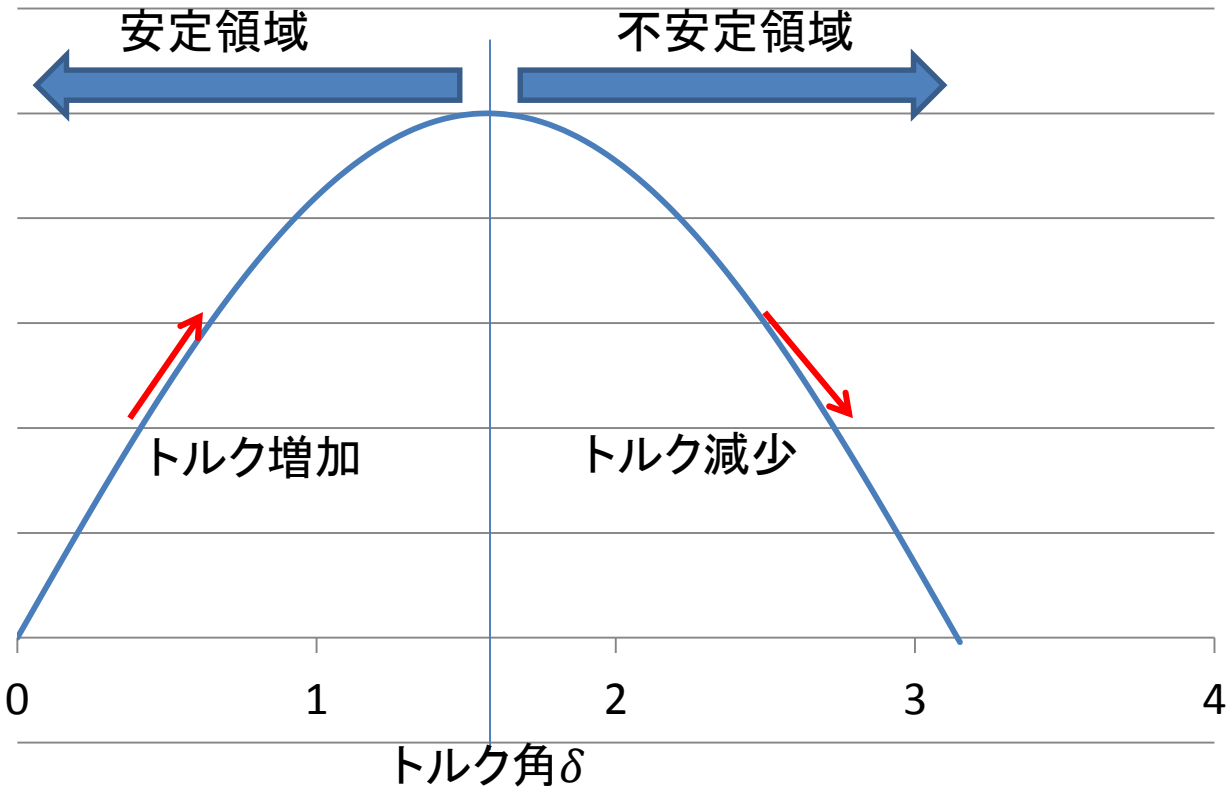
$$P = 3VI_{\mathrm{Re}} = 3\frac{VE_0 \sin\delta}{X_s}$$

トルクは

$$T = \frac{P}{\omega_m} = 3p\frac{VE_0 \sin\delta}{\omega X_s}$$

同期モータのトルク角と安定性

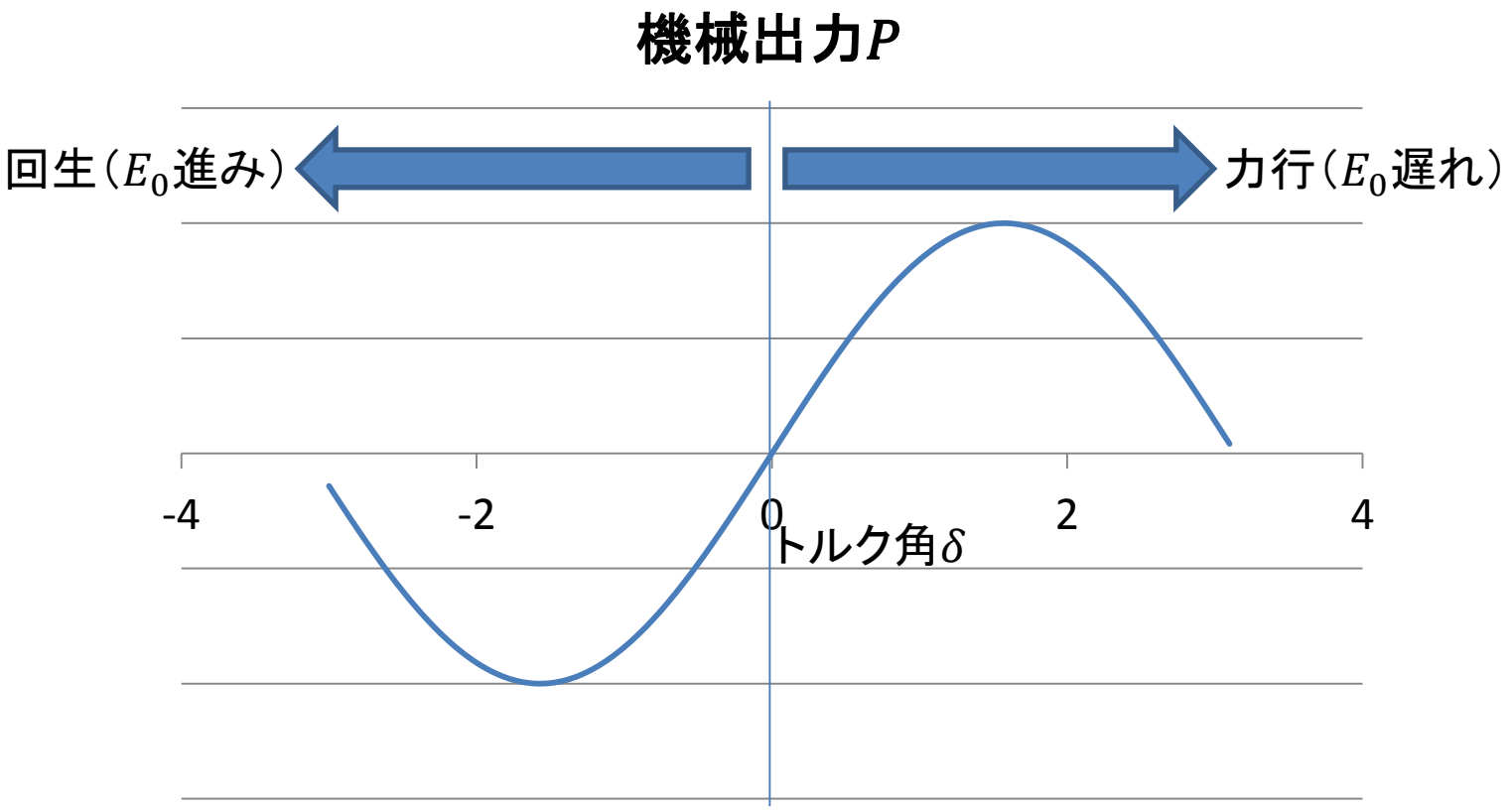
- 機械出力は$P = 3\dfrac{VE_0 \sin\delta}{X_s}$
- 負荷が増加すると回転子が遅れ，δが増加する



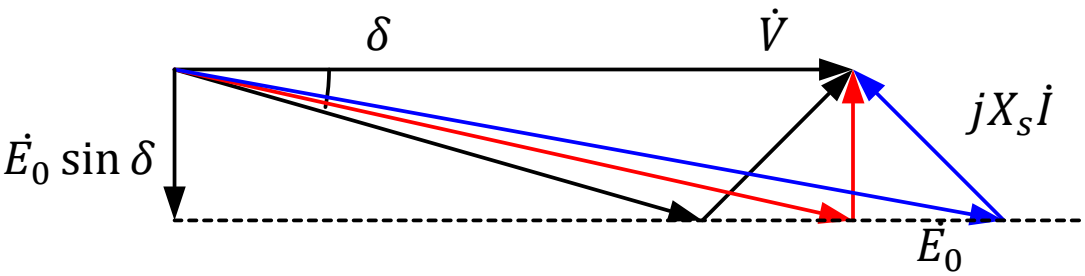
同期モータの機械出力とトルク角

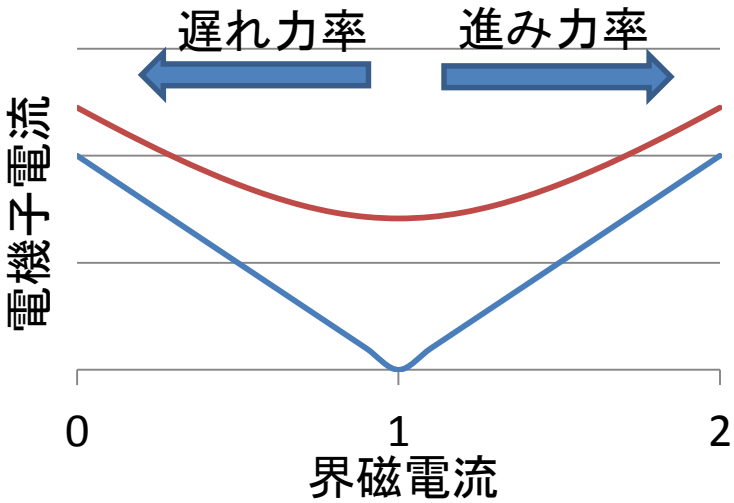
機械出力は

$$P = 3\frac{VE_0 \sin\delta}{X_s}$$



同期モータの入力力率





界磁電流を変えた場合E_0の大きさが変化する

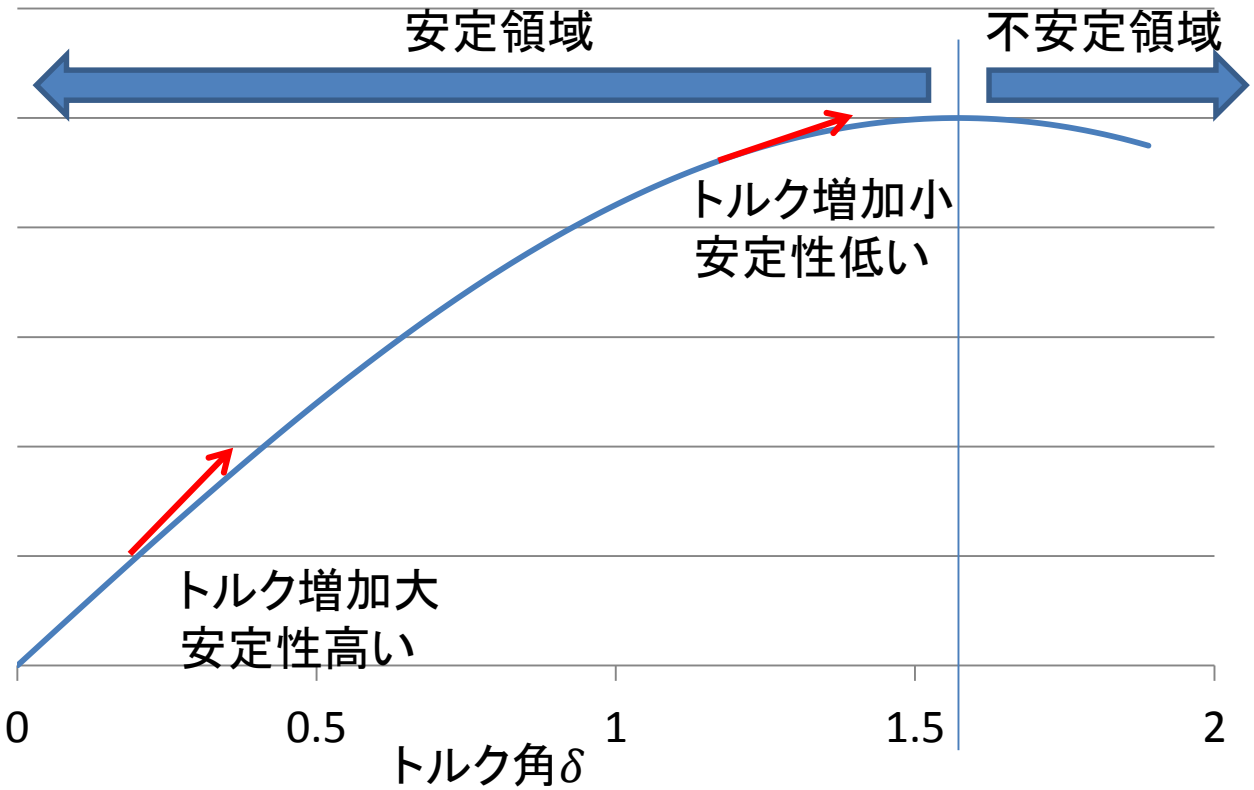
機械出力は$P = 3\dfrac{VE_0 \sin\delta}{X_s}$であるので，

負荷Pを一定とすると

- $E_0 \sin\delta = \dfrac{PX_s}{3V}$は一定
- 電機子電流の振幅位相が変化
- $E_0 \cos\delta = V$のとき，電機子電流最小

同期モータの同期化力

- 同期化力: $\dfrac{\partial P}{\partial \delta} = 3\dfrac{VE_0 \cos\delta}{X_s}$
- 同期モータの安定性を示す指標



Mechanical Equations

Kinetic Energy:

$$W_m = \frac{1}{2} J {\omega_m}^2$$

J : Inertia of the rotor

Power Relationship:

$$P_m - P = \frac{dW_m}{dt} = \omega_m J \frac{d\omega_m}{dt} = p\omega_m J \frac{d^2}{dt^2}\delta$$

T_m: Input Mechanical Torque

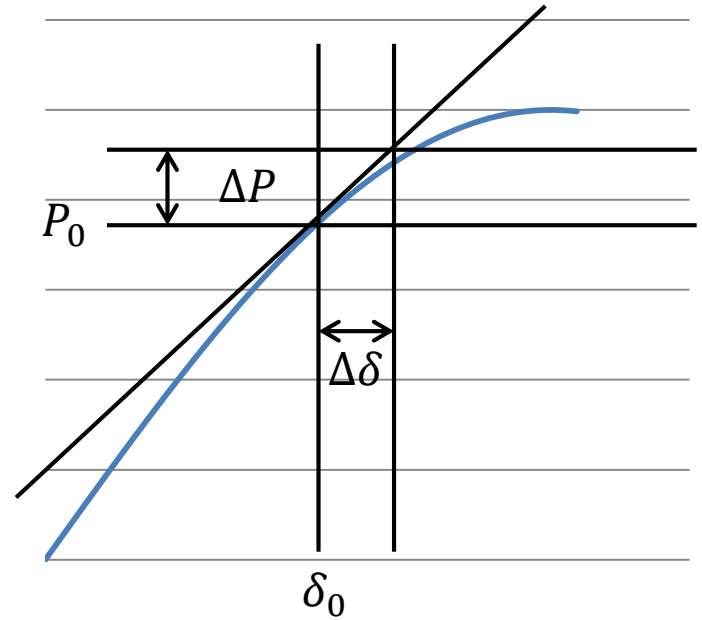
Electric Power:

$$P = \frac{3VE_0}{X_s} \sin\delta$$

Small-Signal Analysis

Approximation:

$$\delta = \delta_0 + \Delta\delta$$



Electric Power:

$$P = \frac{3VE_0}{X_s}\sin\delta$$

$$= P_0 + \Delta P \approx \frac{3VE_0}{X_s}(\sin\delta_0 + \cos\delta_0\,\Delta\delta\,)$$

Dynamic Response

- Assumption:

$$P_m = \frac{3VE_0}{X_s}\sin\delta_0$$

- Power Relationship:

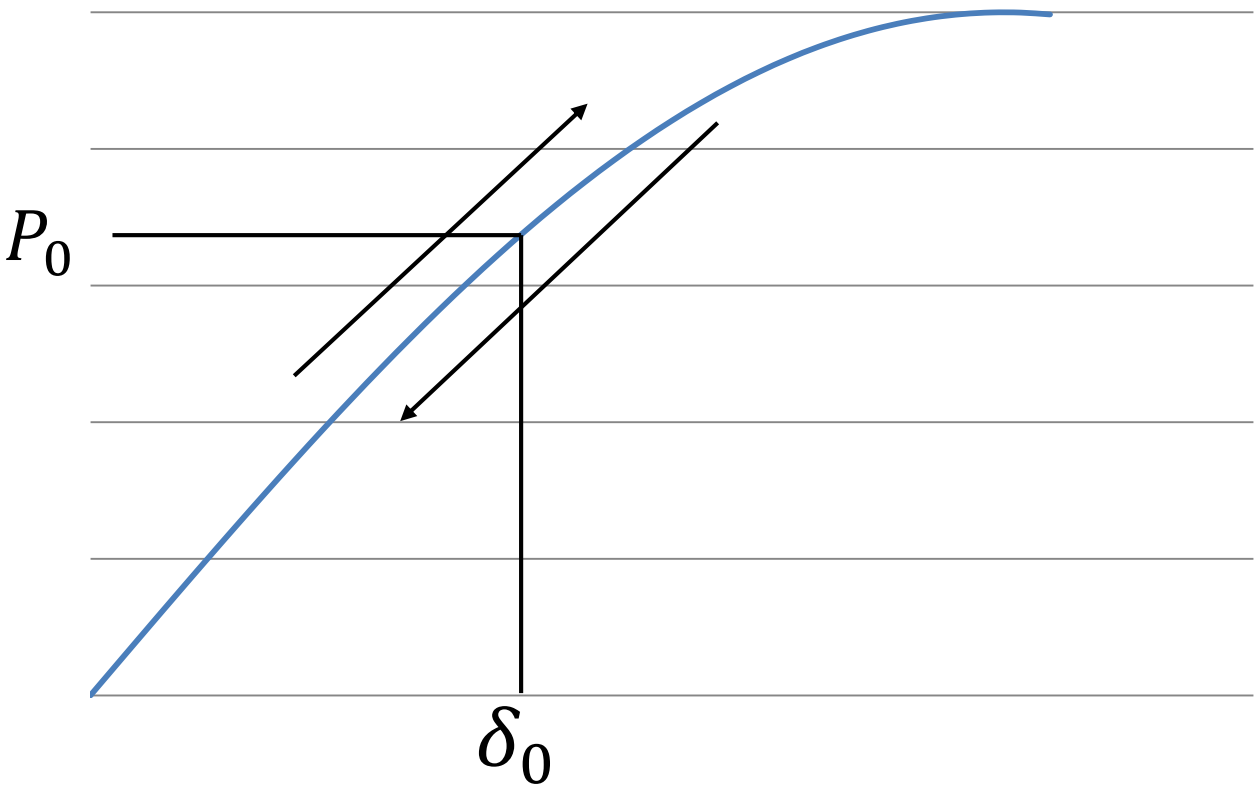
$$P_m - P = -\frac{3VE_0\cos\delta_0}{X_s}\Delta\delta = p\omega_m J\frac{d^2}{dt^2}\Delta\delta$$

$$\frac{d^2}{dt^2}\Delta\delta = -\frac{3VE_0\cos\delta_0}{p\omega_m J X_s}\Delta\delta$$

$$\Delta\delta = C_1\sin\left(\sqrt{\frac{3VE_0\cos\delta_0}{p\omega_m J X_s}}\,t + C_2\right)$$

Power Swings

$$\Delta\delta = C_1 \sin\left(\sqrt{\frac{3VE_0 \cos\delta_0}{p\omega_m J X_s}}\, t + C_2\right)$$



Kinetic Energy Stored in the Rotor

- Initial Position: δ_1

  $\delta < \delta_1$

  $T_m > T$ Decelaration

- Kinetic Energy at δ_0:

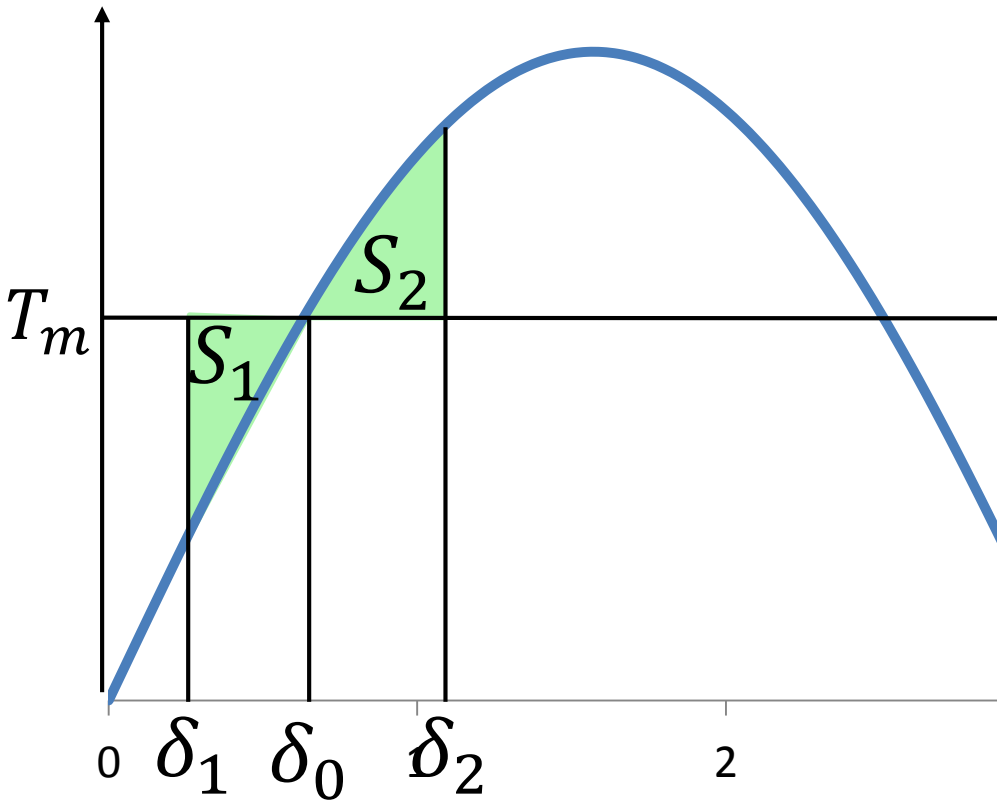
$$S_1 = \int_{\delta_1}^{\delta_0} (T_m - T) d\delta$$

- Released Energy:

  $\delta < \delta_1$

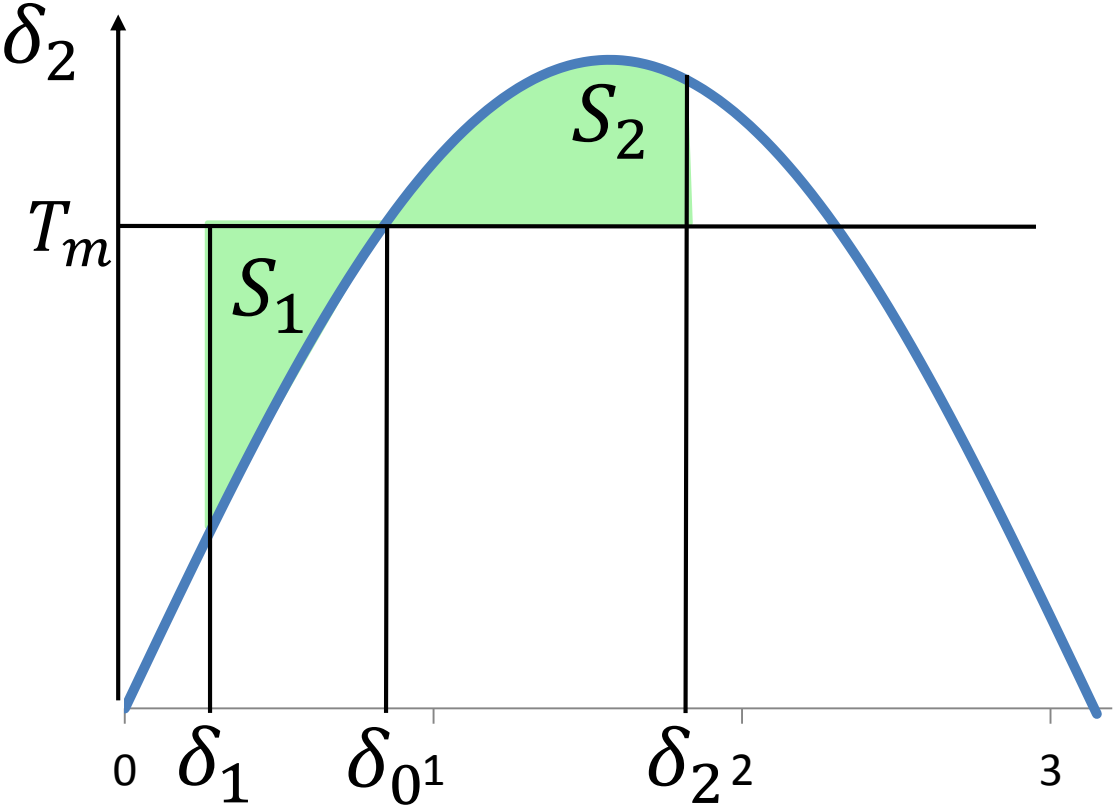
$$S_2 = \int_{\delta_0}^{\delta_2} (T - T_m) d\delta$$

$$S_1 = S_2$$



Large Power Swing

- Load Angle Exceeds $\frac{\pi}{2}$
- Deceleration at $\delta = \delta_2$



- Stable Requirement:
  $T > T_m$ at $\delta = \delta_2$

Fault Condition

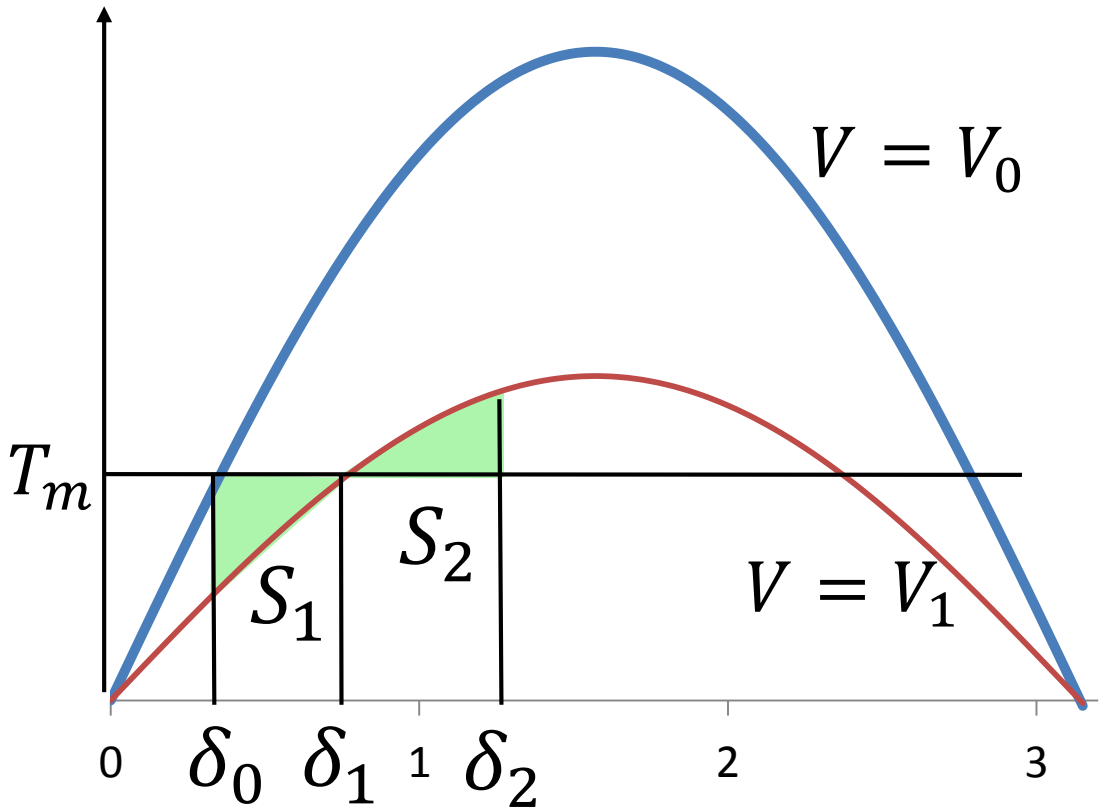
- Initial Load Angle

  δ_0

- Line Voltage

  $V_0 \rightarrow V_1$

- Stable



Short Line Fault

- Initial Load Angle

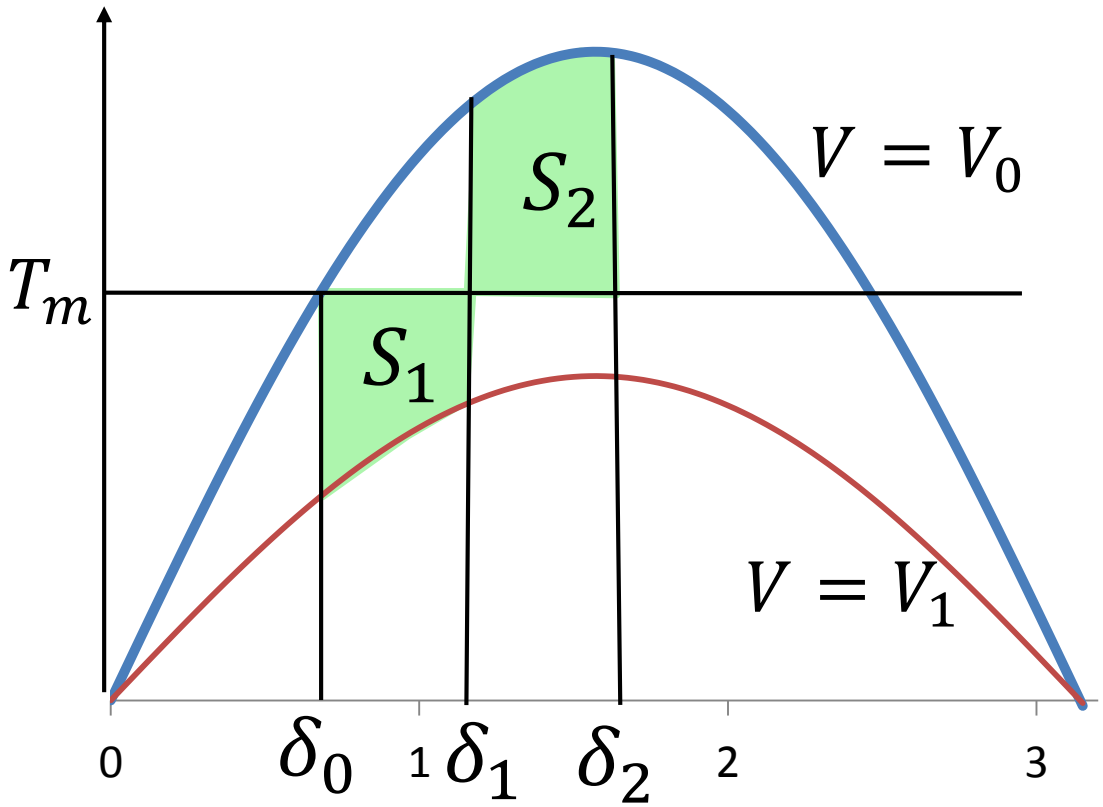
  δ_0

- Line Voltage

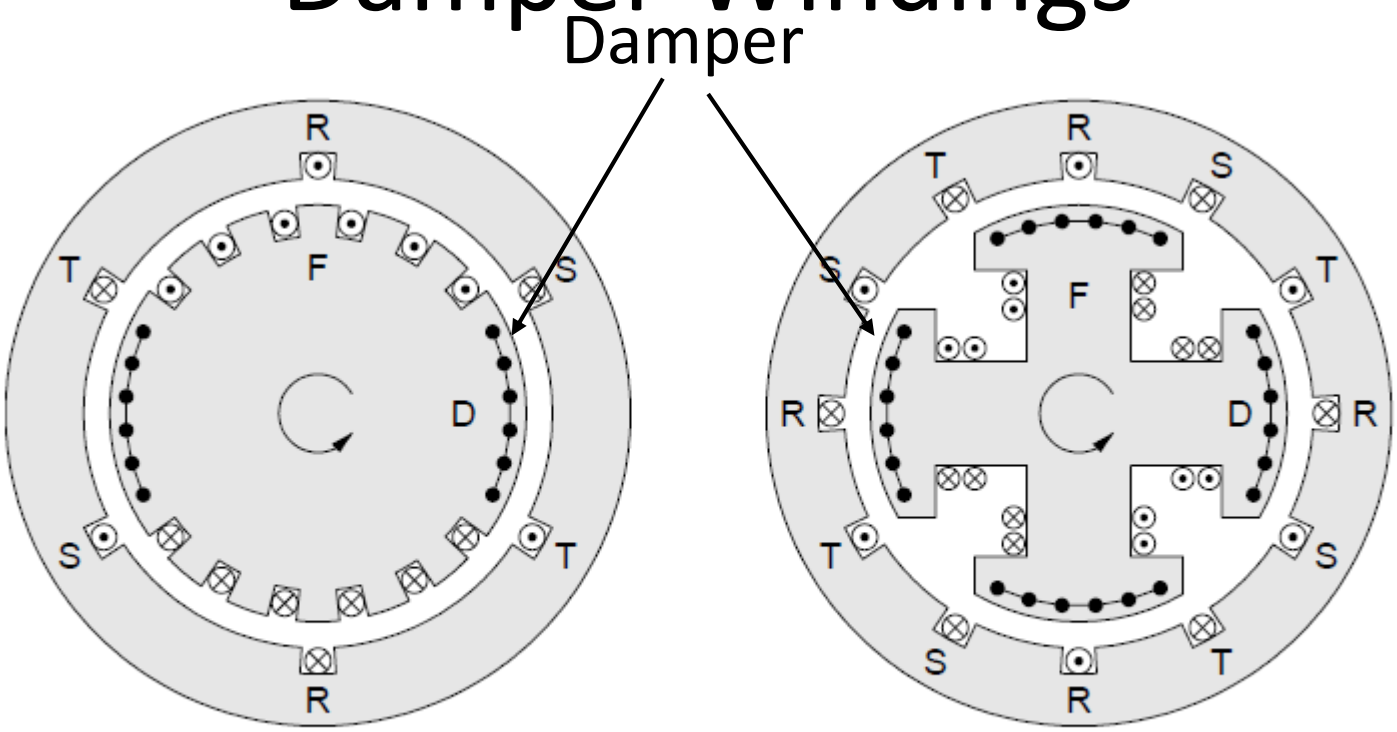
  $V_0 \Rightarrow V_1$

- Voltage Recover

  $V_1 \Rightarrow V_0$



Damper Windings



Round rotor
$p = 2,\ n = 3000\,\mathrm{min}^{-1}$ for $f = 50\,\mathrm{Hz}$

Salient pole rotor
$p = 4,\ n = 1500\,\mathrm{min}^{-1}$ for $f = 50\,\mathrm{Hz}$

- Damper Torque: $T_D \propto \omega_m$, and thus $T_D \propto \dfrac{d\delta}{dt}$

Engineering Articles

Mechanical Equations

Power Relationship Considering Dampers:

$$P_m - P = \omega_m J \frac{d\omega_m}{dt} + (R_\omega + R_D)\omega_m$$

$$= p\omega_m J \frac{d^2}{dt^2}\delta + (R_\omega + R_D)\frac{d}{dt}\delta$$

R_ω: Mechanical Friction

R_D: Equivalent Friction of Damper

$$\frac{d^2}{dt^2}\Delta\delta + \frac{R_\omega + R_D}{p\omega_m J}\frac{d}{dt}\Delta\delta + \frac{3VE_0\cos\delta_0}{p\omega_m J X_s}\Delta\delta = 0$$

Dynamic Equations

$$\frac{d^2}{dt^2}\Delta\delta + \frac{R_\omega + R_D}{p\omega_m J}\frac{d}{dt}\delta + \frac{3VE_0\cos\delta_0}{p\omega_m J X_s}\Delta\delta = 0$$

Solution becomes damped sinusoidal function.

Natural damped angular frequency:

$$\omega_n = \sqrt{\frac{3VE_0\cos\delta_0}{p\omega_m J X_s}}$$

Damping factor:

$$\zeta = \frac{R_\omega + R_D}{2\sqrt{p\omega_m J}}\sqrt{\frac{X_s}{3VE_0\cos\delta_0}}$$