1 Lecture - Introduction of the course, Finite and Infinite Sets

1.1 Introduction of the course

- name of the course: Calculus II
- instructor name: Jan Brezina
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- room: main building H219
- teaching assistant: Murakami (murakami.h.ah@m.titech.ac.jp, H316)

About lecture

- purpose of the course: strengthen logical thinking and get deeper understanding of what we learned in spring semester
- advantage over other courses - math and English
- why English: thinking in another language gives you new ways of looking at things
- can take lecture and exercise separately, but better together
- class is in English, tests too
- content is similar to Japanese classes

Class materials

- personal notes
- handouts
- any book in any language dealing with the topics of this course

Grading

- score 60 - 100 points is a pass
- depending on a person - just pass or care about good score - your choice
- grades from lecture and exercise are NOT related

Lecture grading

Midterm exam Final exam	80 points
Homework	20 points

- about homework
  - 8 homework
  - if you cant do your homework it indicates that you don ' t understand the lecture
  - the assignment of homework will be given with the content of Wednesday lecture
  - homework submission is before/after the following week's Tuesday lecture
  - late submissions are NOT accepted,
  - each properly submitted homework is worth 2 points, if you submit 4 or more, you get extra 2 points, if you submit 8 you get extra 4 points
  - homework that written last minute, empty, difficult to read, dirty, etc. is worth 0 points

Publishing of materials

- weekly on OCW-i,
- homework, solutions of homework, solution of tests, etc.

Consultation

- always welcome - me or TA
- setup an appointment (personally, by email)
- come in group preferably
- consult with your classmates regularly

1.2 Finite and Infinite Sets

- cardinality,
- finite and infinite sets,
- countably infinite set,
- countably infinite set can be arranged into a sequence,
- countable and uncountable sets,
- examples of countable sets $\mathbb{N}$, $2\mathbb{N}$, $\mathbb{N} \times \mathbb{N}$, $\mathbb{Q}^+$, $\mathbb{Q}$,
- $(0, 1)$ and $\mathbb{R}$ are uncountable

1.3 Homework

1) Prove the statement in Remark 4.1.

2) Prove Lemma 4.1.

3) Prove Theorem 4.4. (If you dare, prove it for any finite number of countable sets!)

4) Prove that if $|A| = |B|$ and $|B| = |C|$ then $|A| = |C|$. (transitive law)

5) Show that $\mathbb{Z}$ and $S = \{\dots, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, \dots\}$ have the same cardinality.

6) Prove that the set A is countably infinite.

   - $A = \{\ln(n) : n \in \mathbb{Z}^+\} \subseteq \mathbb{R}$,
   - $A = \{(m, n) \in \mathbb{Z}^+ \times \mathbb{Z}^+ |\ m \leq n\}$,

7) Prove that the set of all irrational numbers is uncountable.

8) If S is an infinite set, prove that S contains a countably infinite subset.