光画像工学
Optical imaging and image processing
(VI)

Appendix
Geometrical optics, ray-tracing, lens aberration
幾何光学、光線追跡、レンズ系の収差

- Focal point and focal length of a lens
- Image formation, real image and virtual image
- Thick lens, principal plane
- Ray-tracing
- Paraxial approximation
- Lens aberration
  - Five Seidel aberrations
  - Chromatic aberration

Focal point and focal plane
焦点、焦点面

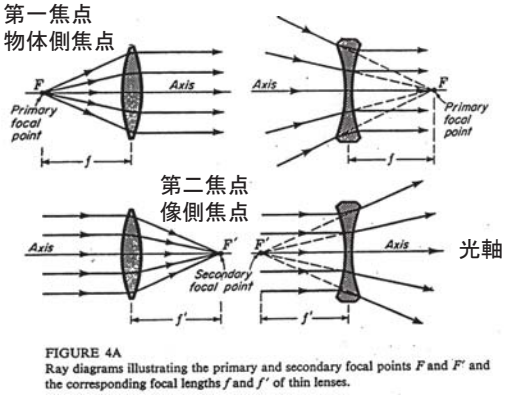


FIGURE 4A
Ray diagrams illustrating the primary and secondary focal points F and F' and the corresponding focal lengths f and f' of thin lenses.

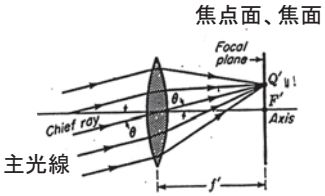


FIGURE 4B
How parallel incident rays are brought to a focus at the secondary focal plane of a thin lens.

Figures are from: Fundamentals of Optics, F. A. Jenkins and H. E. White, McGraw-Hill

Image formation
結像

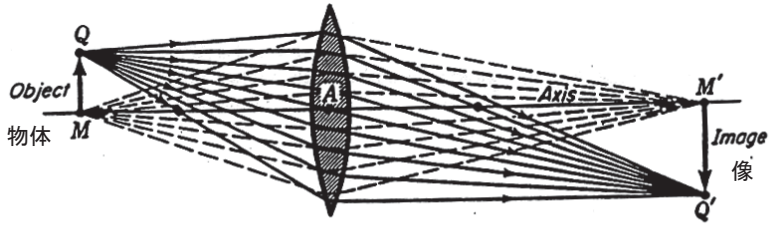


FIGURE 4C
Image formation by an ideal thin lens. All rays from an object point Q which pass through the lens are refracted to pass through the image point Q'.

“Real image” and “virtual image”
実像と虚像

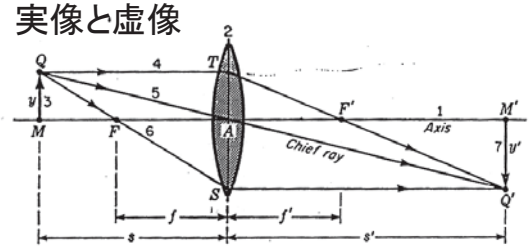


FIGURE 4D
The parallel-ray method for graphically locating the image formed by a thin lens.

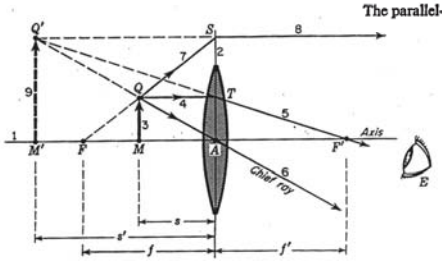


FIGURE 4F
The parallel-ray method for graphically locating the virtual image formed by a positive lens when the object is between the primary focal point and the lens.

Lens formula
(for thin lens)

$$\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$$

Ray-tracing
光線追跡

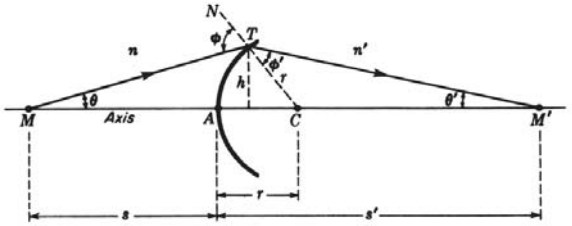


FIGURE 8D
Geometry used in deriving the ray-tracing formulas.

Snell's law $n\sin\phi = n'\sin\phi'$

Image formation by a thick lens
厚いレンズによる結像

- H: Primary principal plane　　第一主面、物体側主面
- H": Secondary principal plane　　第二主面、像側主面

（第一主点、第二主点）

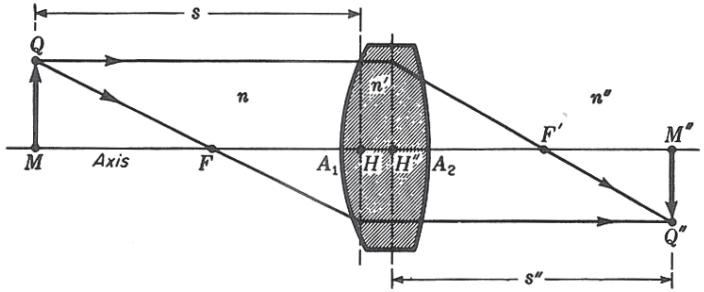


FIGURE 5E
The parallel-ray method of construction for graphically locating an image formed by a thick lens.

Example of ray-tracing

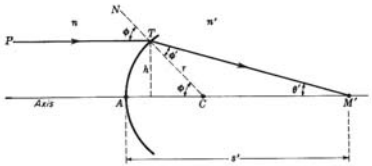


FIGURE 8E
Geometry for ray tracing with parallel incident light.

Table 8A RAY-TRACING CALCULATIONS FOR A SINGLE CONVEX SPHERICAL SURFACE*
$r = +5.0$ cm　　$n = 1.0$ cm　　$n' = 1.67200$

Eq.	Unknown	Relationship	$h = 3.0$	$h = 2.0$	$h = 1.0$	$h = 0$
(8e)	$\sin\phi$	$\frac{h}{r}$	+0.6000000	0.4000000	0.2000000	0.6000000
(8b)	$\sin\phi'$	$\frac{n}{n'}\sin\phi$	+0.3588517	0.2392344	0.1196172	0.3588517
		ϕ	+36.869898°	23.578178°	11.536959°	
		ϕ'	+21.029692°	13.841356°	6.8700110°	
(8f)	θ'	$\phi' - \phi$	−15.840206°	9.7368220°	4.6669480°	
		$\sin\theta'$	−0.2729554	0.1691228	0.0813636	0.2411483
(8d)	$r - s'$	$r\frac{\sin\phi'}{\sin\theta'}$	−6.5734494	7.0728015	7.3507809	7.4404775
		s'	+11.573449	12.072802	12.350781	12.440478

* Although the refractive index of air at normal temperature and pressure is 1.000292, it is customary in ray tracing to use the value 1.000000.

Paraxial Approximation
近軸近似

- $\sin\theta \approx \theta$

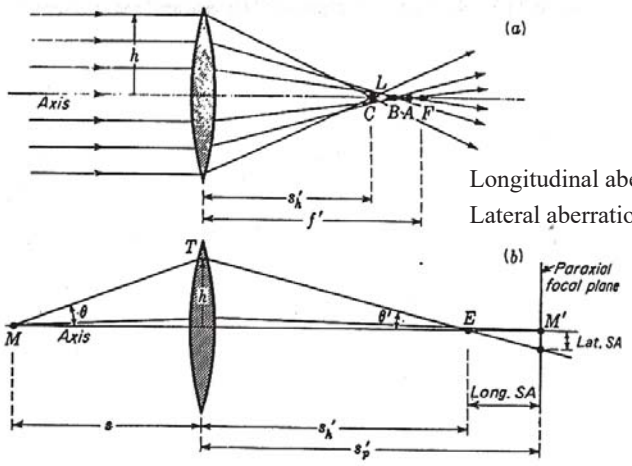
Table 9A VALUES OF sin θ AND ITS FIRST THREE EXPANSION TERMS

	sin θ	θ	$\frac{\theta^3}{3!}$	$\frac{\theta^5}{5!}$
10°	0.1736482	0.1745329	0.0008861	0.0000135
20°	0.3420201	0.3490658	0.0070888	0.0000432
30°	0.5000000	0.5235988	0.0239246	0.0003280
40°	0.6427876	0.6981316	0.0567088	0.0013829

$$\sqrt{\Delta x^2+\Delta y^2+\Delta z^2} = \Delta z\sqrt{1+(\frac{\Delta x}{\Delta z})^2+(\frac{\Delta y}{\Delta z})^2} \quad \text{Gaussian approximation}$$

$$\cong \Delta z\left[1+\frac{1}{2}(\frac{\Delta x}{\Delta z})^2+\frac{1}{2}(\frac{\Delta y}{\Delta z})^2\right]$$

Spherical aberration
球面収差



Longitudinal aberration (縦収差)
Lateral aberration (横収差)

FIGURE 9D
Illustrations of lateral and longitudinal spherical aberration of a lens.

Radius of curvature and the spherical aberration

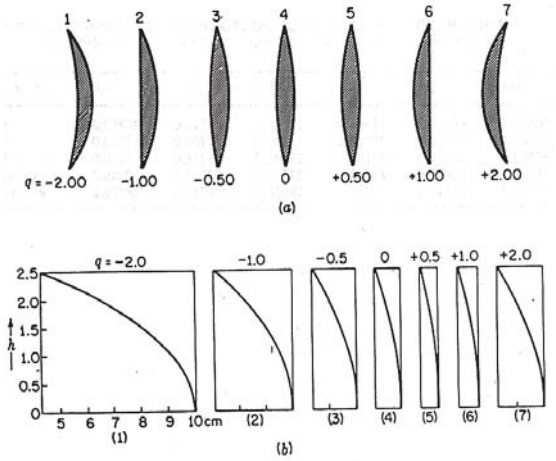


FIGURE 9F
(*a*) Lenses of different shapes but with the same power or focal length. The difference is one of bending. (*b*) Focal length versus ray height *h* for these lenses.

Coma
コマ収差

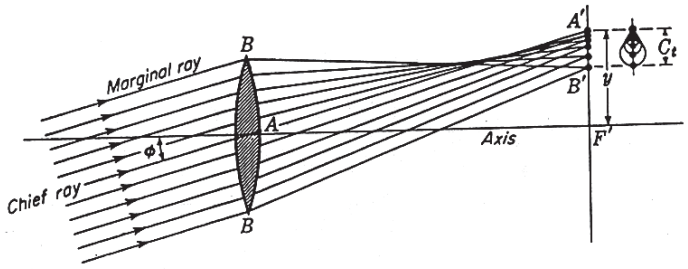


FIGURE 9I
Coma, the second of the five monochromatic aberrations of a lens. Only the tangential fan of rays is shown.

Spherical aberration and Coma

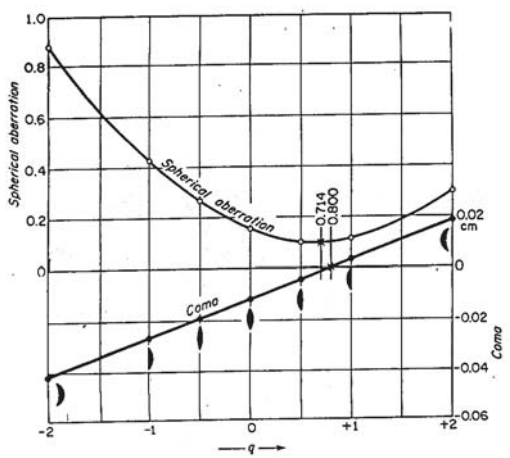
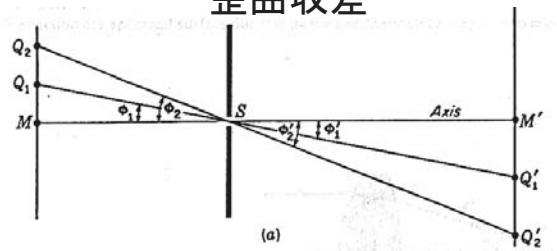


FIGURE 9L
Graphs comparing coma with longitudinal spherical aberration for a series of lenses having different shapes.

Distortion
歪曲収差



$MS:SM'=Q_1S:SQ_1'=Q_2S:SQ_2'$

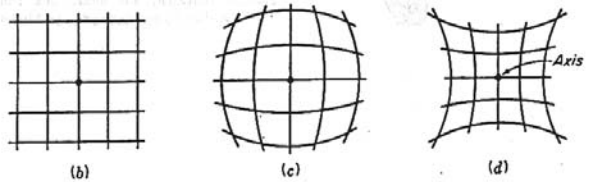
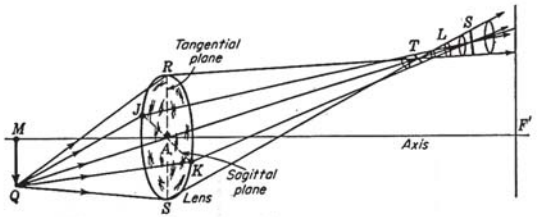


FIGURE 9T
(a) A pinhole camera shows no distortion. Images of a rectangular object screen shown with (b) no distortion, (c) barrel distortion, and (d) pincushion distortion.

Astigmatism and Curvature of field
非点収差と像面湾曲



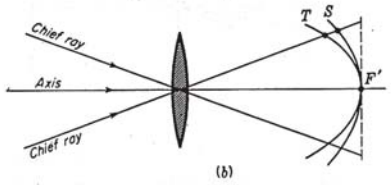


FIGURE 9P
(a) Perspective diagram showing the two focal lines which constitute the image of an off-axis object point Q. (b) Loci of the tangential and sagittal images. The two surfaces approximate paraboloids of revolution.

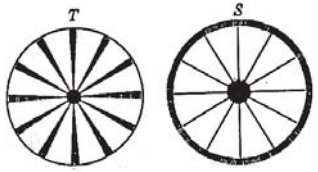


FIGURE 9Q
Astigmatic images of a spoked wheel.

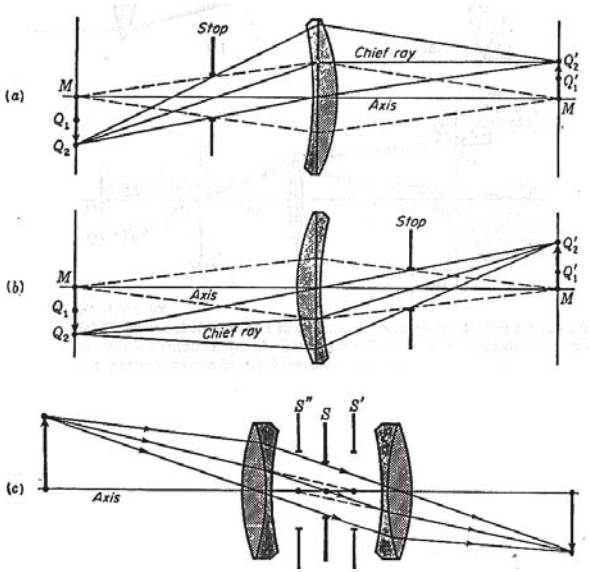


FIGURE 9U
(a) A stop in front of a lens giving rise to barrel distortion. (b) A stop behind a lens giving rise to pincushion distortion. (c) A symmetrical doublet with a stop between is relatively free of distortion.

Chromatic aberration
色収差

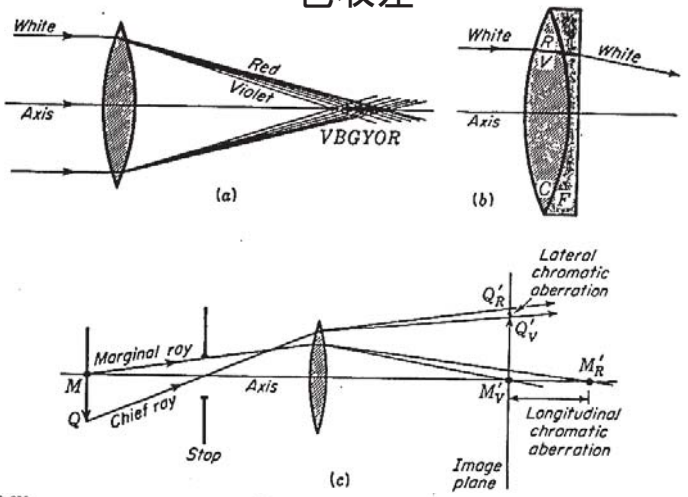


FIGURE 9X
(*a*) Chromatic aberration of a single lens. (*b*) A cemented doublet corrected for chromatic aberration. (*c*) Illustrating the difference between longitudinal chromatic aberration and lateral chromatic aberration.

Table 9E REFRACTIVE INDICES OF TYPICAL OPTICAL MEDIA FOR FOUR COLORS

Medium	Designation	ICT type	ν	n_C	n_D	n_F	n_G
Borosilicate crown	BSC	500/664	66.4	1.49776	1.50000	1.50529	1.50937
Borosilicate crown	BSC-2	517/645	64.5	1.51462	1.51700	1.52264	1.52708
Spectacle crown	SPC-1	523/587	58.7	1.52042	1.52300	1.52933	1.53435
Light barium crown	LBC-1	541/599	59.7	1.53828	1.54100	1.54735	1.55249
Telescope flint	TF	530/516	51.6	1.52762	1.53050	1.53790	1.54379
Dense barium flint	DBF	670/475	47.5	1.66650	1.67050	1.68059	1.68882
Light flint	LF	576/412	41.2	1.57208	1.57600	1.58606	1.59441
Dense flint	DF-2	617/366	36.6	1.61216	1.61700	1.62901	1.63923
Dense flint	DF-4	649/338	33.9	1.64357	1.64900	1.66270	1.67456
Extra dense flint	EDF-3	720/291	29.1	1.71303	1.72000	1.73780	1.75324
Fused quartz	SiO_2		67.9		1.4585		
Crystal quartz (*O* ray)	SiO_2		70.0		1.5443		
Fluorite	CaF_2		95.4		1.4338		

Summary of lens aberrations

- Seidel aberrations
  - Spherical aberration
  - Coma
  - Astigmatism
  - Curvature of field
  - Distortion
- Chromatic aberration

- ザイデルの５収差
  - 球面収差
  - コマ収差
  - 非点収差
  - 像面湾曲
  - 歪曲収差
- 色収差