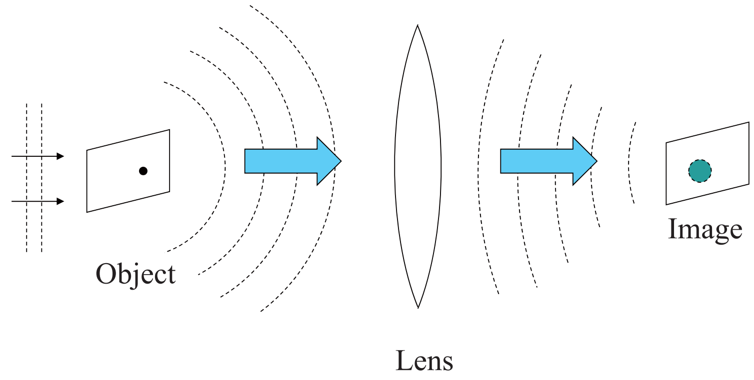
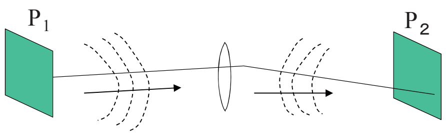
光画像工学
Optical imaging and image processing
(V)

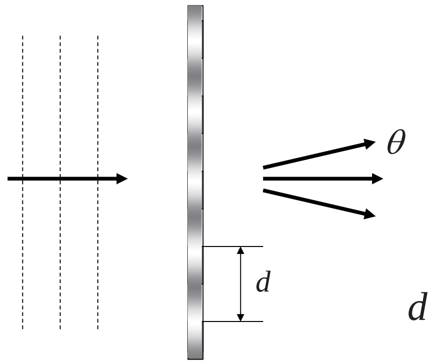
2.3 Diffraction and wave propagation
2.3 光の回折と伝搬



2. Optical imaging systems
2. 光学的イメージングシステム



Diffraction Grating
回折格子



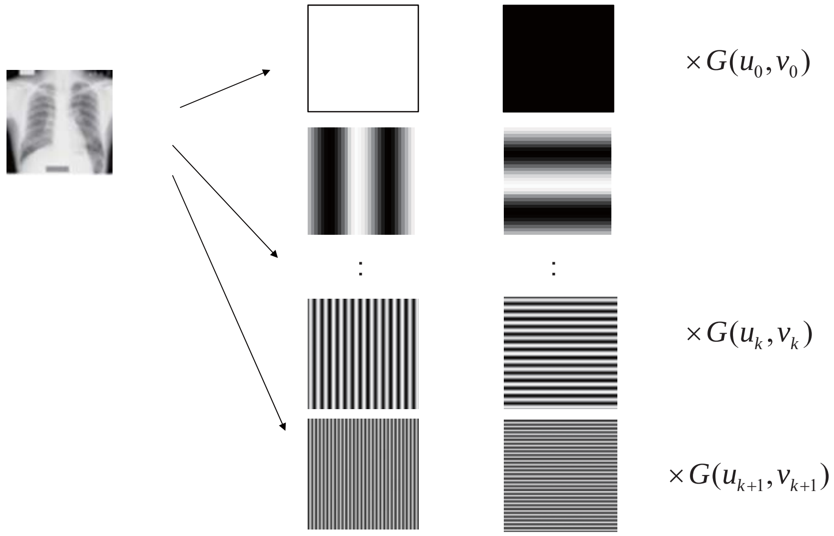
$$d \sin\theta = m\lambda$$

If sinusoidal grating,

$$d \sin\theta = 0、\pm\lambda$$

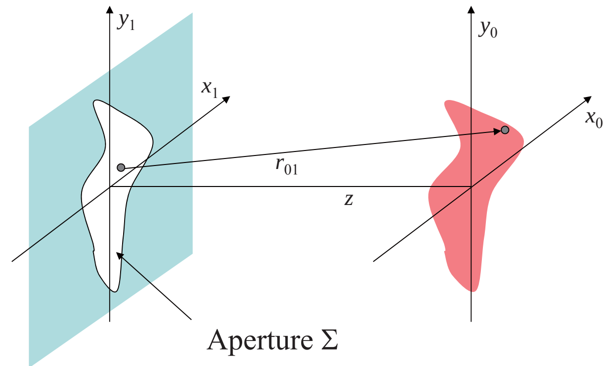
2-D Fourier transform

$$G(u,v)=\iint g(x,y)\exp\{-j2\pi(xu+yv)\}dxdy$$

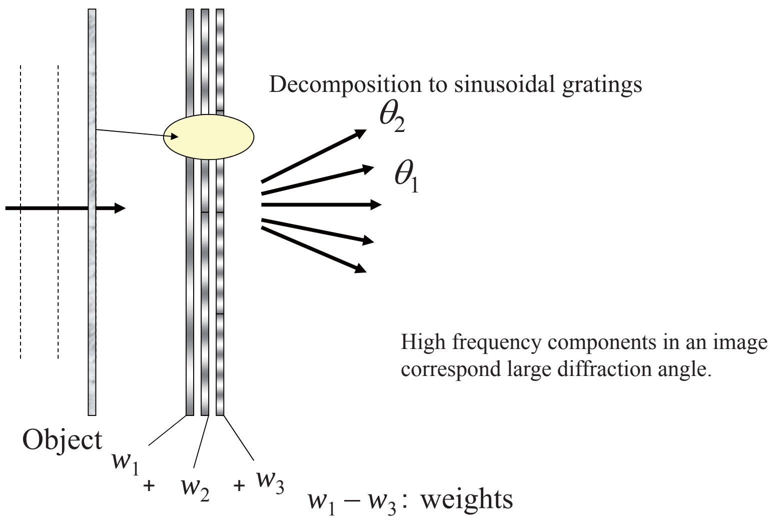


Scalar diffraction theory
スカラー回折理論

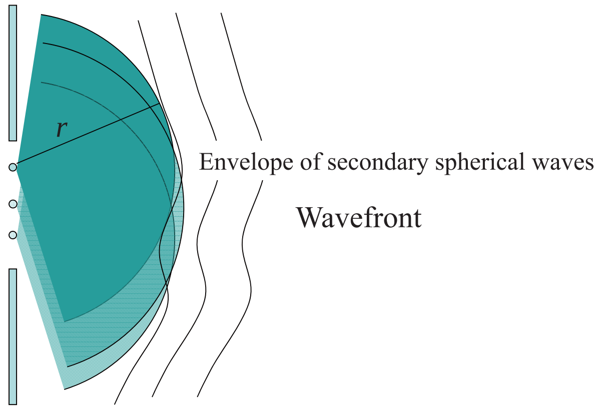
$$U(x_0,y_0)=\frac{1}{j\lambda z}\iint_{\Sigma}U(x_1,y_1)\exp(jkr_{01})dx_1dy_1$$



Superposition of sinusoidal gratings



Huygens-Fresnel Principle



Fresnel approximation,　フレネル近似

If $|x_0 - x_1| << z$ and $|y_0 - y_1| << z$

$$r_{01} = \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2 + (z_0 - z_1)^2}$$

$$= z\sqrt{1 + (\frac{x_0 - x_1}{z})^2 + (\frac{y_0 - y_1}{z})^2}$$

$$\cong z\left[1 + \frac{1}{2}(\frac{x_0 - x_1}{z})^2 + \frac{1}{2}(\frac{y_0 - y_1}{z})^2\right]$$

⇨ Paraxial approximation

Spherical wave is approximated by quadratic wave:

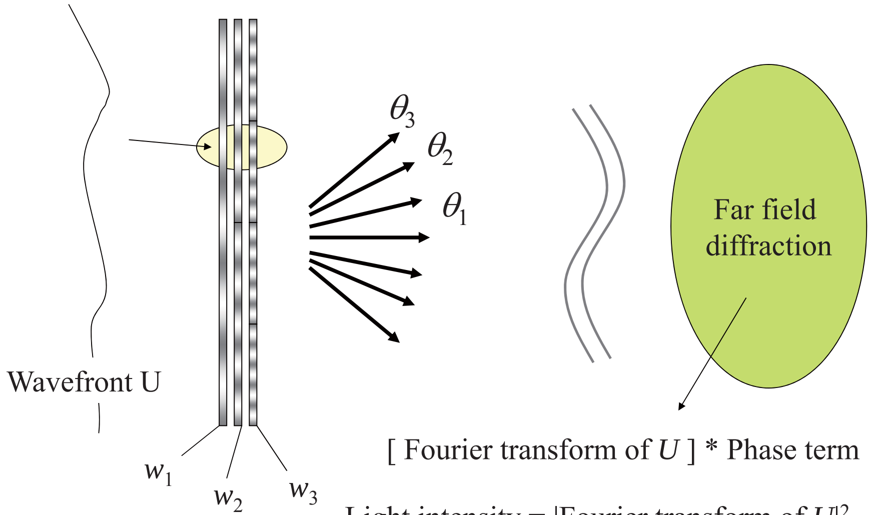
Spherical wave

$$U(x_0, y_0) = \frac{\exp(jkz)}{j\lambda z}\exp\{j\frac{k}{2z}[(x_0 - x_1)^2 + (y_0 - y_1)^2]\}$$

Fresnel diffraction

$$U(x_0, y_0) = \frac{\exp(jkz)}{j\lambda z}\iint_{\Sigma} U(x_1, y_1)\exp\{j\frac{k}{2z}[(x_0 - x_1)^2 + (y_0 - y_1)^2]\}dx_1 dy_1$$

Fraunhofer Diffraction
フラウンホファー回折



[Fourier transform of U] * Phase term

Light intensity = |Fourier transform of U|^2

Rewriting the Fresnel diffraction equation

$$g(x_0, y_0) = \iint h(x_0 - x_1, y_0 - y_1) f(x_1, y_1) dx_1 dy_1$$

$$= f(x_0, y_0) * h(x_0, y_0)$$ ⟶ Convolution

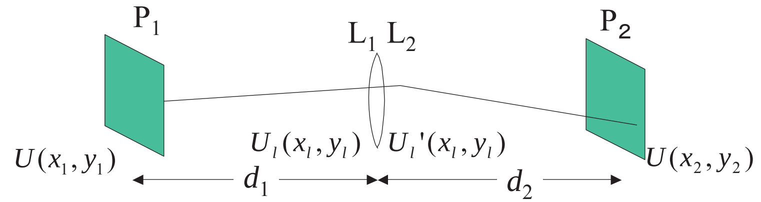
$$h(x_0, y_0; x_1, y_1) = \frac{\exp\{jkz\}}{j\lambda z}\exp\{j\frac{k}{2z}[(x_0 - x_1)^2 + (y_0 - y_1)^2]\}$$

$$U(x_0, y_0) = \frac{\exp\{jkz\}}{j\lambda z}\exp\{j\frac{k}{2z}(x_0^2 + y_0^2)\}\int\int_{-\infty}^{\infty} U(x_1, y_1)$$

$$\exp\{j\frac{k}{2z}(x_1^2 + y_1^2)\}\exp\{-j\frac{2\pi}{\lambda z}(x_0 x_1 + y_0 y_1)\}dx_1 dy_1$$

Fourier Transform of $U(x_1, y_1)\exp\{j\frac{k}{2z}(x_1^2 + y_1^2)\}$ * Phase Term

2.4 Imaging through a lens system
2.4 レンズ系による結像



$P_1 \to L_1$　Fresnel Diffraction

$$U_l(x_l, y_l) = \frac{\exp(jkd_1)}{j\lambda d_1}\int\int_{-\infty}^{\infty} U(x_1, y_1)\exp\{j\frac{k}{2d_1}[(x_l - x_1)^2 + (y_l - y_1)^2]\}dx_1 dy_1$$

$L_1 \to L_2$　Phase modulation by lens

Transformation of wavefront: Spherical wave → Plane wave; Spherical wave → Spherical wave

$$U_l'(x_l, y_l) = U_l(x_l, y_l) P(x_l, y_l)\exp\{-j\frac{k}{2f}(x_l^2 + y_l^2)\}$$

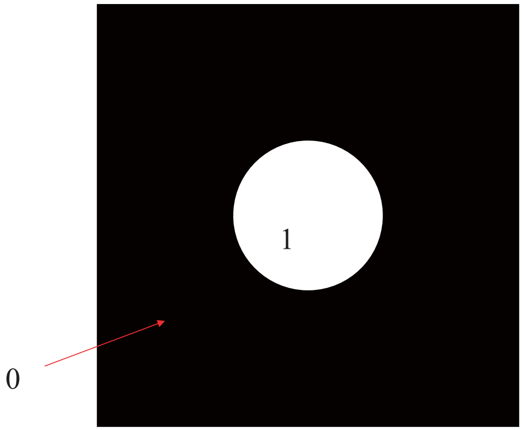
$L_2 \to P_2$　Fresnel Diffraction

$$U(x_2, y_2) = \frac{\exp(jkd_2)}{j\lambda d_2}\int\int_{-\infty}^{\infty} U_l'(x_l, y_l)\exp\{j\frac{k}{2d_2}[(x_2 - x_l)^2 + (y_2 - y_l)^2]\}dx_l dy_l$$

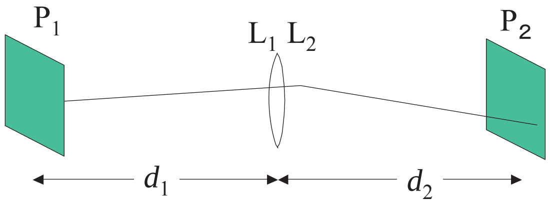
Lens aperture = Pupil function

瞳関数

$P(x_l, y_l)$



Wavefront at P_1 and P_2 planes



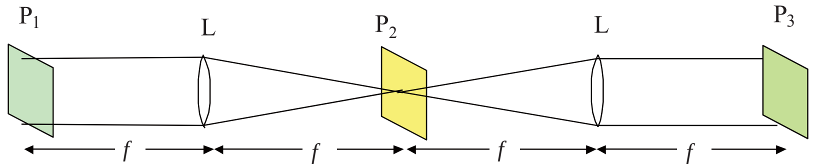
$$U(x_2, y_2) = A\int\int_{-\infty}^{\infty} h(x_2, y_2; x_1, y_1) U(x_1, y_1) dx_1 dy_1$$

$$h(x_2, y_2; x_1, y_1) = \frac{1}{\lambda^2 d_1 d_2} \exp\left[j\frac{k}{2d_2}(x_2^2 + y_2^2) \right] \exp\left[j\frac{k}{2d_1}(x_1^2 + y_1^2) \right]$$

$$\int\int_{-\infty}^{\infty} P(x, y) \exp\left[j\frac{k}{2}\left(\frac{1}{d_1} + \frac{1}{d_2} - \frac{1}{f}\right)(x^2 + y^2) \right]$$

$$\exp\left[-jk\left\{\left(\frac{x_1}{d_1} + \frac{x_2}{d_2}\right)x + \left(\frac{y_1}{d_1} + \frac{y_2}{d_2}\right)y\right\} \right] dxdy$$

Optical Fourier transform and Coherent optical filtering



P_1: Input , P_2: Filter (Fourier plane), P_3: Output

When $d_1 = f$ and $d_2 = f$,

$$U(x_2, y_2) = A\int\int_{-\infty}^{\infty} U(x_1, y_1) \exp\{j\frac{2\pi}{\lambda f}[x_1 x_2 + y_1 y_2]\} dx_1 dy_1$$

$u = x_2 / \lambda f , v = y_2 / \lambda f$

$$U(u, v) = C\ \boldsymbol{F}\{U(x_1, y_1)\}$$

Figure 7-11 (a) Image before filtering. (b) Image after use of high-pass spatial filter.

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Optical high-pass filtering
光学的ハイパスフィルターの例

W. T. Cathey, Optical Information Processing and Holography, John Wiley & Sons, New York, 1974

Phase contrast imaging
位相コントラスト法

- Phase shift of zero frequency component

$f(x,y) = \exp\{j\phi(x,y)\} \approx 1 + j\phi(x,y)$ ($\phi(x,y) << 1$)

$g(x,y) = \exp(j\frac{\pi}{2}) + j\phi(x,y) = j + j\phi(x,y)$

$I(x,y) = |g(x,y)|^2 \approx 1 + 2\phi(x,y)$

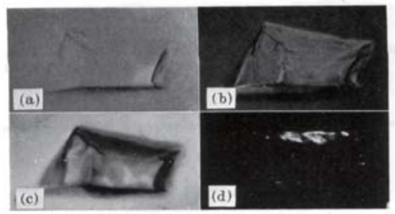


Fig. 8.34. Microscope images of glass fragments ($n = 1{\cdot}52$) mounted in clarite ($n = 1{\cdot}54$), 100 ×.
(a), Bright field image; (b) and (c), Phase contrast images; (d), Dark field image.
(After A. H. Bennett, H. Jupnik, H. Osterberg, and O. W. Richards, *Trans. Amer. Microscop. Soc.*, **65** (1946), 126.)

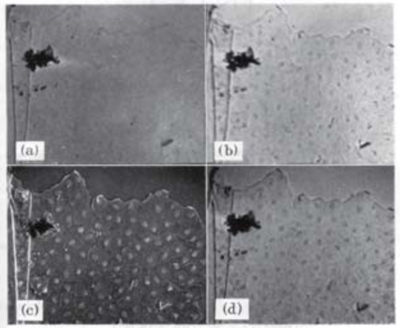
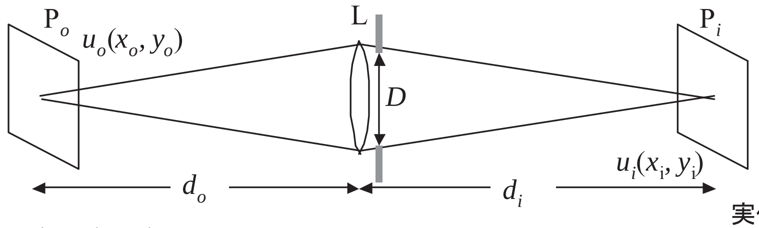


Fig. 8.35. Microscope images of epithelium from frog nictitating membrane, 100 ×.
(a), Bright field image at full aperture, N.A. = 0·25; (b), Bright field image with aperture half filled; (c), Phase-contrast image (bright contrast) at full aperture; (d), Phase-contrast image (dark contrast) at full aperture.
(After A. H. Bennett, H. Jupnik, H. Osterberg, and O. W. Richards, *Trans. Amer. Microscop. Soc.*, **65** (1946), 119.)

M. Born and E. Wolf, Principle of Optics, 6th Edition, Pergamon Press, 1980

Impulse response of an lens system



実像
Real Image

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$ Lens formula

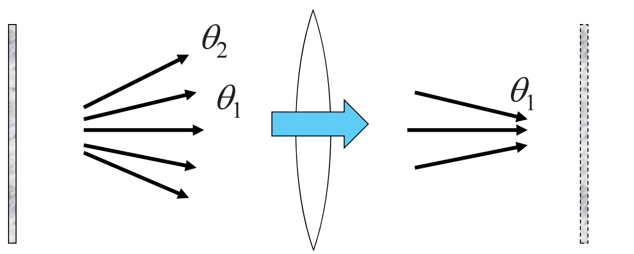
$$M = \frac{d_i}{d_o}$$ Magnification

$$h(x_i, y_i; x_o, y_o) = \frac{1}{\lambda^2 d_i d_o} \int\int_{-\infty}^{\infty} P(x,y) \exp\left[-j\frac{2\pi}{\lambda d_i}\{(x_i + Mx_o)x + (y_i + My_o)y\}\right] dxdy$$

⇨ Impulse response = Fourier transform of the pupil function

2.5 Impulse response (PSF) and transfer function of a lens system
2.5 レンズ系の点像分布関数と伝達関数

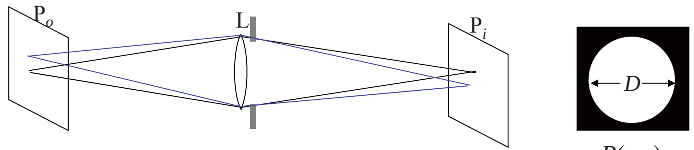
Imaging by a lens system



Object

Image

Transfer function of lens system

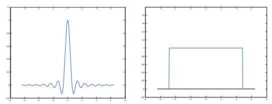


$P(x,y)$

- Coherent transfer function $H_c(u,v)$

PSF $u_i(x_i, y_i) = P_f(\frac{x_i}{\lambda d_i}, \frac{y_i}{\lambda d_i}) * u_o(-\frac{x_i}{M}, -\frac{y_i}{M}) = h_c(x_i, y_i) * u_o(-\frac{x_i}{M}, -\frac{y_i}{M})$

$U_i(u,v) = H_c(u,v) U_o(u,v)$

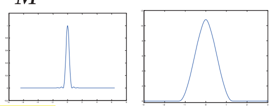


- Optical transfer function (Incoherent)

PSF $|u_i(x_i, y_i)|^2 = |P_f(\frac{x_i}{\lambda d_i}, \frac{y_i}{\lambda d_i})|^2 * |u_o(-\frac{x_i}{M}, -\frac{y_i}{M})|^2$

$g(x_i, y_i) = h'(x_i, y_i) * f(x_i, y_i)$

$G(u,v) = H'(u,v) F(u,v)$



$H(u,v) = H'(u,v) / H'(0,0)$: Optical transfer function (OTF)

Modulation Transfer Function

$$\mathrm{MTF} = \frac{\text{Contrast of output image } (u,v)}{\text{Contrast of input image}(u,v)}$$

Ideal incoherent imaging system

$$\mathrm{MTF} = |\,\mathrm{OTF}\,|$$

$$\mathrm{OTF}(u,v) = \mathrm{MTF}(u,v)\ \exp\{j\phi(u,v)\}$$

$\phi(u,v)$: Phase transfer function (PTF)

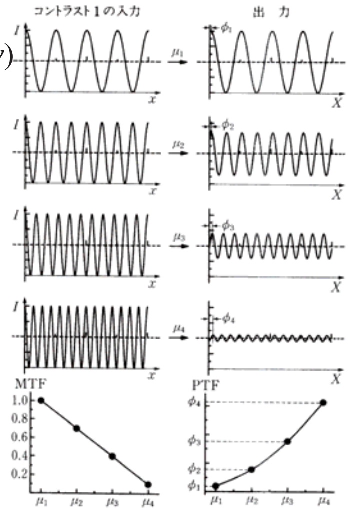
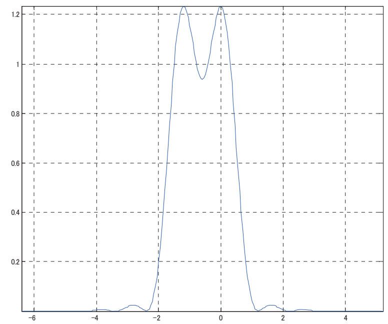


図 5.11 光強度の入出力特性

吉村、「光情報工学の基礎」

2.6 Resolution of a lens system
2.6 レンズ系の分解能

- Rayleigh criterion -



Rayleigh limit $L = 1.22\dfrac{\lambda d_i}{D}$

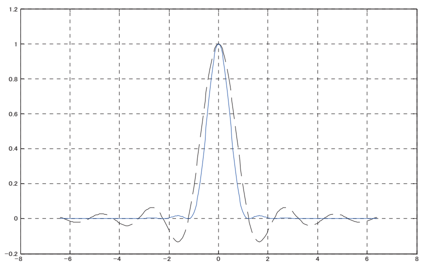
回折限界・解像限界 (Diffraction limit)

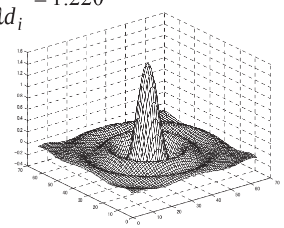
Impulse response of a circular aperture

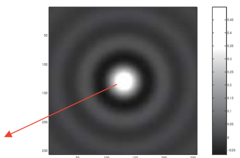
$$h'(x_i, y_i) = |P_f(\frac{x_i}{\lambda d_i}, \frac{y_i}{\lambda d_i})|^2$$

$$= |\frac{\pi D^2}{2} \cdot \frac{J_1(\frac{\pi D r_i}{\lambda d_i})}{\frac{\pi D r_i}{\lambda d_i}}|^2$$

$h'(r_i) = 0$ for $\dfrac{Dr_i}{\lambda d_i} = 1.220$





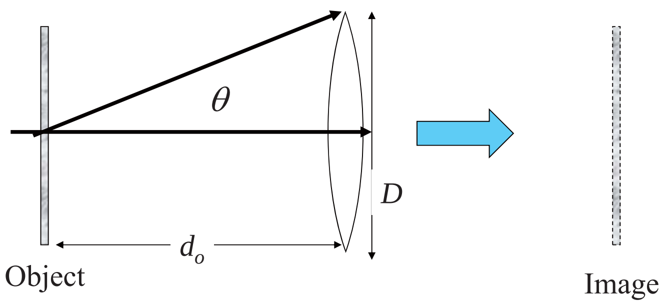


$2J_1(\pi z)/\ \pi z,\ |\,2J_1(\pi z)/\pi z|^2$

Airy Disk

Estimating the resolution of a lens system

$p \sin\theta = \lambda$



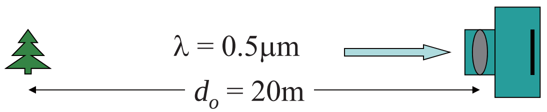
$$\sin\theta \approx \theta = D/(2\,d_o)$$

$$U_{\max} = 1/p_{\min} = \sin\theta/\lambda \approx D/(2\lambda d_o)$$

Rayleigh limit (image plane) $L = 1.22\dfrac{\lambda d_i}{D}$ ⟶ Object plane $\dfrac{L}{M} = 1.22\dfrac{\lambda d_i}{DM} = 1.22\dfrac{\lambda d_o}{D}$

Example

$$u_{o\,\max} = \frac{D}{\lambda d_o}$$



$\lambda = 0.5\mu m$

$d_o = 20m$

$f = 20mm$
$F = 2$

- Lens diameter $D = f / F = 10mm$
- $u_{o\,\max} = 10 / (0.5 \times 10^{-3} \times 20 \times 10^{3}) = 1.0\ (mm^{-1})$
- Object larger than ≅ 1mm can be resolved.

60x Plan Apochromat Objective

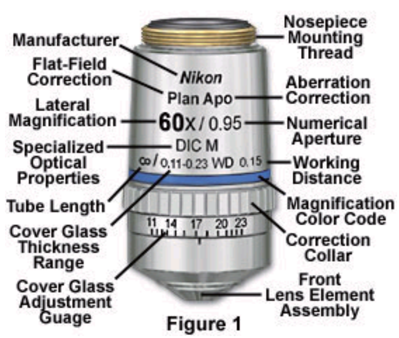


Figure 1

Objective Numerical Aperture and Working Distance

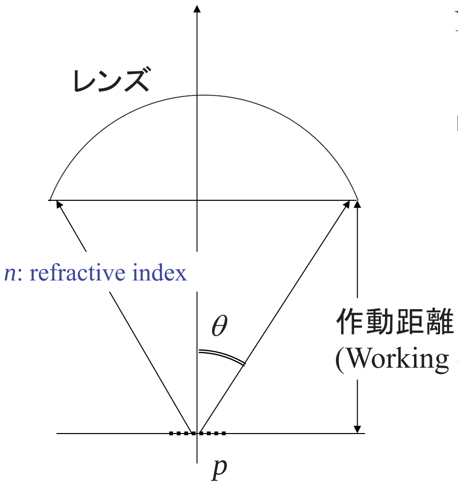
Optical Correction* and Magnification	Numerical Aperture	Working Distance (Millimeters)
ACH 10x	0.25	6.10
ACH 20x	0.40	3.00
ACH 40x	0.65	0.45
ACH 60x	0.80	0.23
ACH 100x (Oil)	1.25	0.13
PL 4x	0.10	22.0
PL 10x	0.25	10.5
PL 20x	0.40	1.20
PL 40x	0.65	0.56
PL 100x (Oil)	1.25	0.15
PL FL 4x	0.13	17.0
PL FL 10x	0.30	10.00
PL FL 20x	0.50	1.60
PL FL 40x	0.75	0.51
PL FL 100x (Oil)	1.30	0.10
PL APO 1.25x	0.04	5.1
PL APO 2x	0.06	6.20
PL APO 4x	0.16	13.00
PL APO 10x	0.40	3.10
PL APO 20x	0.70	0.65
PL APO 40x	0.85	0.20
PL APO 60x (Oil)	1.40	1.10
PL APO 100x (Oil)	1.40	0.10

*Abbreviations:
ACH, Achromat
PL, Plan Achromat
PL FL, Plan Fluorite
PL APO, Plan Apochromat

Table 2

http://www.microscopyu.com/articles/optics/objectivespecs.html

Numerical Aperture (開口数)



作動距離
(Working distance)

$$\mathrm{NA} = n \sin\theta \longrightarrow p \geq a\frac{\lambda}{NA}$$

$$p \sin\theta = \lambda$$

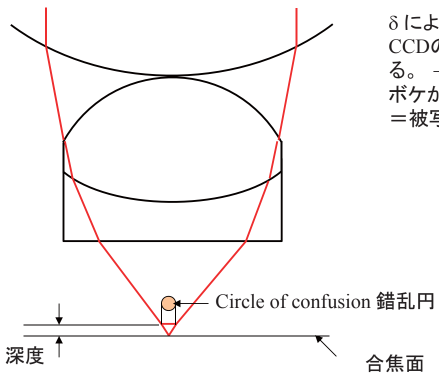
レイリー限界

$$1.22\frac{\lambda d_o}{D} \cong 1.22\frac{\lambda}{2\ NA} = 0.61\frac{\lambda}{NA}$$

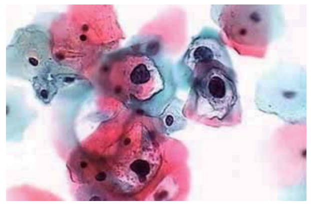
NA = 0.25
λ = 500nm
$a = 0.61$

$p \geq 1.22\mu m$

Depth of field 被写界深度



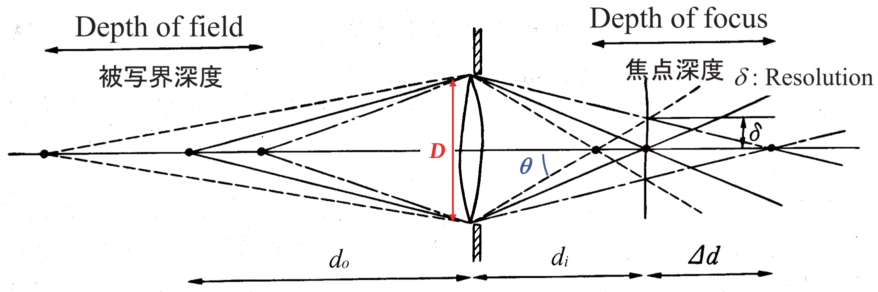
δによるボケが眼の分解能、レンズ系の分解能、CCDの画素間隔より小さければ、ボケは許容できる。 → 許容錯乱円
ボケが許容錯乱円となる深度の範囲
＝被写界深度



http://www.medlab.com.au/cytology/

■ レンズのNAが大きければ被写界深度は浅くなる
→ 分解能と被写界深度は一般的にトレードオフ

Defocus / Depth of focus 焦点はずれ／焦点深度



Depth of focus $\Delta d = \frac{d_i + \Delta d}{D}(2\delta) \approx \frac{d_i}{D}(2\delta) = \frac{2\delta}{2\tan\theta} \approx \frac{\delta}{\sin\theta} = \frac{\delta}{NA}$

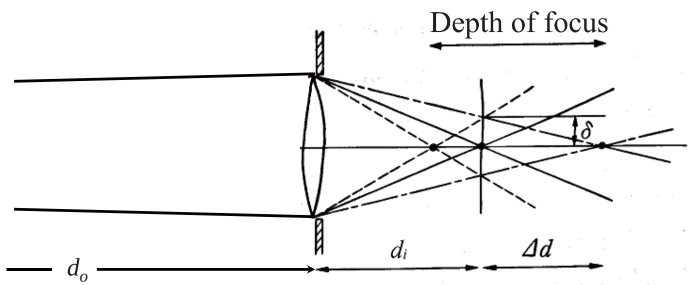
Depth of field $\Delta d_o \approx \frac{d_o}{D}(2\delta\frac{d_o}{d_i}) = \frac{d_o^2}{d_i D}(2\delta) = \frac{d_o^2}{d_i^2}\frac{\delta}{\tan\theta} \approx \frac{M^2\delta}{NA}$

δ: Permissible circle of confusion
determined by the diffraction limit, sensor resolution, etc.
回折限界、センサー分解能などにより決まる

For $\delta <$ Diffraction limit, $\delta = 0.61\frac{\lambda}{NA} = c\frac{\lambda}{NA}$ ⇨ $\Delta d = c\frac{\lambda}{(NA)^2}$

Demo: http://www.matter.org.uk/tem/depth_of_field.htm

Depth of focus
(When d_o is large, i.e., $d_i \approx f$)



$\Delta d = \frac{d_i}{D}(2\delta) \approx \frac{f}{D}(2\delta) = 2\delta F$

F-number $F = \frac{f}{D}$

For $\delta <$ Diffraction limit, $(d_i \approx f)$ $\delta = 1.22\frac{\lambda f}{D} = 1.22\lambda F$

Depth of focus $\Delta d = 2.44\lambda F^2$