光画像工学
Optical imaging and image processing
(IV)

2. Optical imaging systems
2. 光学的イメージングシステム

2. Optical imaging systems
- 2.1 Complex expression of waves
  - Complex amplitude, Wavefront
  - Plane wave, spherical wave
- 2.2 Interference
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- 2.3 Diffraction and wave propagation
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- 2.4 Imaging through a lens system
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- 2.5 Impulse response (PSF) and transfer function of a lens system
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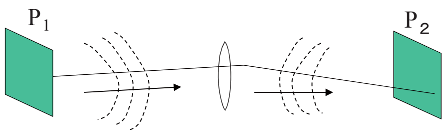
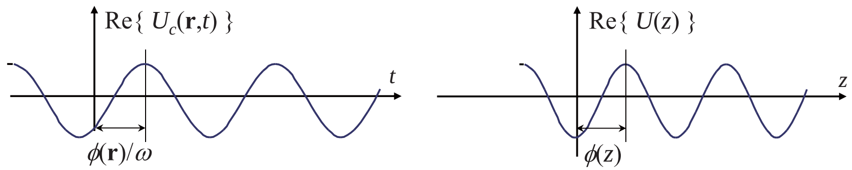
  - Diffraction limit, Rayleigh criterion, Numerical aperture

Appendix. Geometrical optics, ray-tracing, lens aberration

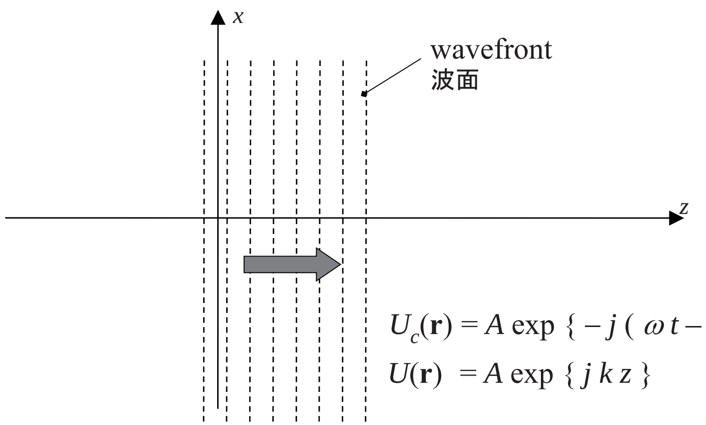


2.1 Complex expression of waves
2.1 波動の複素表示



- $U_c(\mathbf{r}, t) = A(\mathbf{r}) \exp\{-j(\omega t - \phi(\mathbf{r}))\}$
  $= U(\mathbf{r}) \exp(-j\omega t)$
- $U(\mathbf{r})$: Complex amplitude　　複素振幅
  $U(\mathbf{r}) = A(\mathbf{r}) \exp\{j\phi(\mathbf{r})\}$
- $A(\mathbf{r})$: Amplitude　　振幅
- $\phi(\mathbf{r})$: Phase　　位相
- $I(\mathbf{r})$: Intensity　　強度
  $I(\mathbf{r}) = \langle |U_c(\mathbf{r}, t)|^2 \rangle = |U(\mathbf{r})|^2 = A(\mathbf{r})^2$

Plane wave traveling in the *z*-direction
平面波



$$U_c(\mathbf{r}) = A \exp\{-j(\omega t - kz)\}$$
$$U(\mathbf{r}) = A \exp\{jkz\}$$

k: wave number　波数　$k = \dfrac{2\pi}{\lambda}$

Plane wave traveling along the wave vector **k** 波数ベクトル

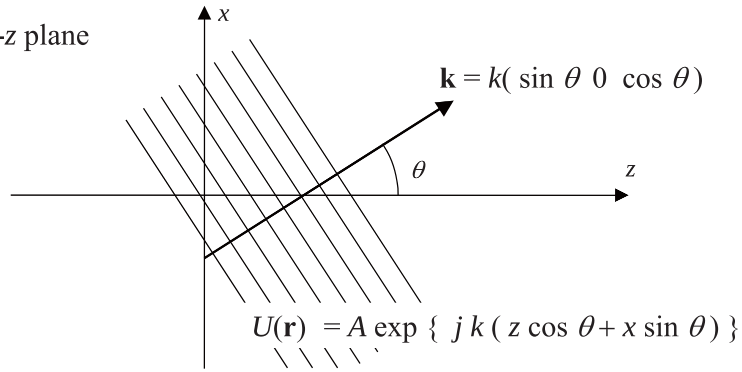
$$U_c(\mathbf{r}, t) = A(\mathbf{r}) \exp\{ -j (\omega t - \phi(\mathbf{r})) \}$$

$$\phi(\mathbf{r}) = \mathbf{k} \cdot \mathbf{r} = k_x x + k_y y + k_z z$$

$\mathbf{k} = (k_x \; k_y \; k_z)$: wave vector, $\cdot$: inner product

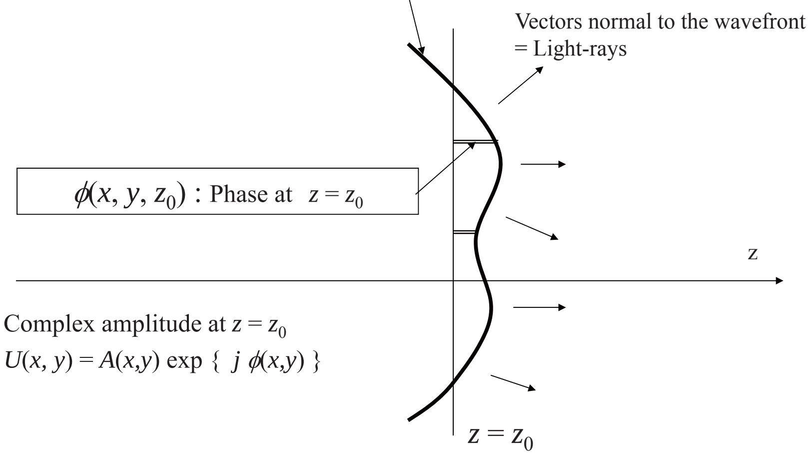
$$U(\mathbf{r}) = A(\mathbf{r}) \exp(j \, \mathbf{k} \cdot \mathbf{r}) = A(\mathbf{r}) \exp\{ j (k_x x + k_y y + k_z z) \}$$

x-z plane



Wavefront
波面

(Surface of equal phase)

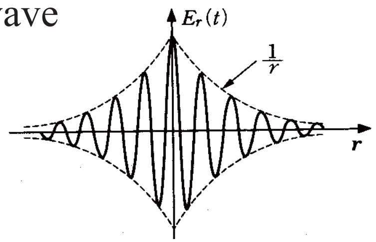


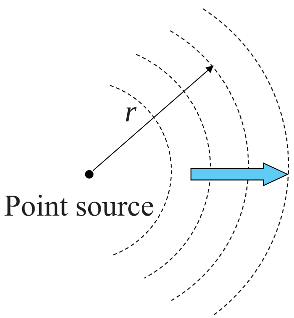
Complex amplitude at $z = z_0$

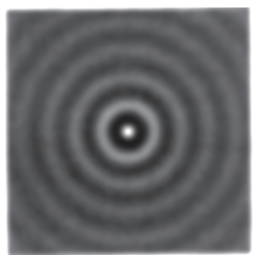
$$U(x, y) = A(x,y) \exp \{ j \, \phi(x,y) \}$$

Spherical wave
球面波

$$U(r) = \frac{U_0 \exp\{ j k r \}}{r}$$

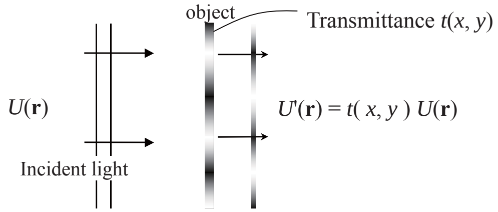


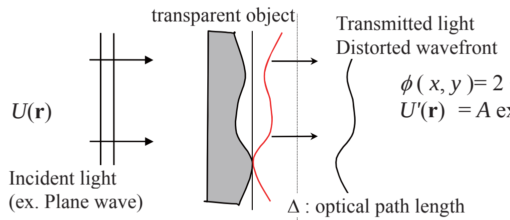




$$r = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$$

Complex amplitude modulation 複素振幅変調





$$\phi(x, y) = 2 \pi \, \Delta(x, y) / \lambda$$

$$U'(\mathbf{r}) = A \exp\{ j (k z_0 + \phi(x, y)) \}$$

Phase modulation

Amplitude transmittance $t(x,y)$ →→ $U'(x, y) = t(x, y) \, U(x, y)$

2.2 Interference
2.2 光の干渉

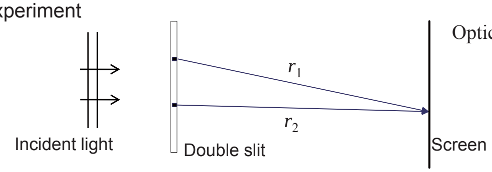
- Interference of two wavefronts

$U_{1c}(\mathbf{r}, t) = A_1(\mathbf{r}) \exp \{ -j (\omega_1 t - \phi_1(\mathbf{r})) \}$

$U_{2c}(\mathbf{r}, t) = A_2(\mathbf{r}) \exp \{ -j (\omega_2 t - \phi_2(\mathbf{r})) \}$

$$I(\mathbf{r}) = \langle | U_{1c}(\mathbf{r}, t) + U_{2c}(\mathbf{r}, t) |^2 \rangle$$
$$= A_1(\mathbf{r})^2 + A_2(\mathbf{r})^2$$
$$+ 2 A_1(\mathbf{r}) A_2(\mathbf{r}) \langle \cos \{ (\omega_1 - \omega_2) t - (\phi_1(\mathbf{r}) - \phi_2(\mathbf{r})) \} \rangle$$

Young's experiment



Optical path difference (OPD) = $|r_1 - r_2|$

$$\phi_1(\mathbf{r}) = r_1 \cdot \frac{2\pi}{\lambda}$$

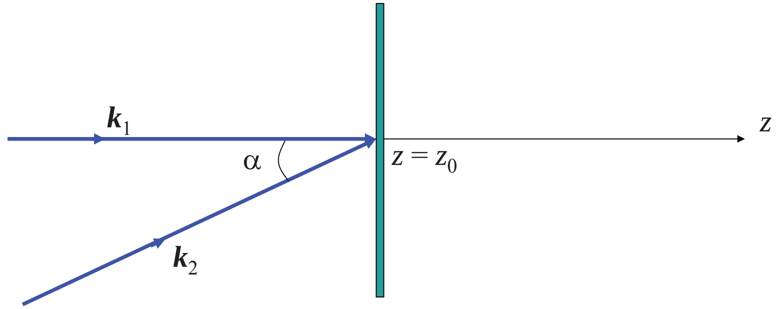
$$|\phi_1(\mathbf{r}) - \phi_2(\mathbf{r})| = \frac{2\pi}{\lambda} \cdot \text{OPD}$$

Interference of two plane waves

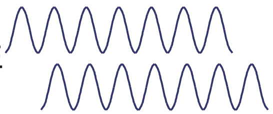
$$I = A_1^2 + A_2^2 + 2A_1A_2 \cos\{(\boldsymbol{k}_2 - \boldsymbol{k}_1)\boldsymbol{r} - (\phi_2 - \phi_1)\}$$

$$\boldsymbol{k}_1\boldsymbol{r} = kz_0$$
$$\boldsymbol{k}_2\boldsymbol{r} = kx\sin\alpha + kz_0\cos\alpha$$
$$(\boldsymbol{k}_2 - \boldsymbol{k}_1)\boldsymbol{r} = kx\sin\alpha + kz_0(1-\cos\alpha)$$

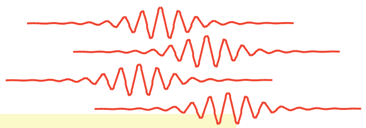
Constant



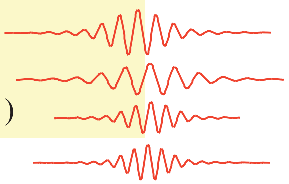
Coherence
コヒーレンス、干渉性



- if $\omega_1 \neq \omega_2$, $\langle \cos \{ (\omega_1 - \omega_2) t + \phi \} \rangle = 0$
  for the observation time >> 2π / ω
  $I(\mathbf{r}) = A_1(\mathbf{r})^2 + A_2(\mathbf{r})^2$
  → incoherent (temporal)



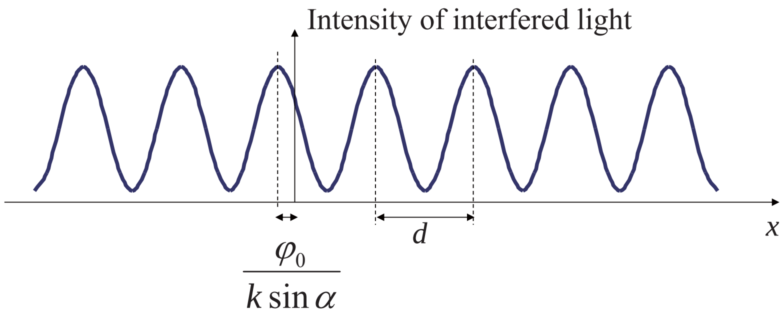
- if $\omega_1 = \omega_2$, → coherent
  $I(\mathbf{r}) = \langle | U_{1c}(\mathbf{r}, t) + U_{2c}(\mathbf{r}, t) |^2 \rangle$
  $= | U_1(\mathbf{r}) + U_2(\mathbf{r}) |^2$
  $= A_1(\mathbf{r})^2 + A_2(\mathbf{r})^2 +$
  $2 A_1(\mathbf{r}) A_2(\mathbf{r}) \cos (\phi_1(\mathbf{r}) - \phi_2(\mathbf{r}))$



- (Partially coherent)

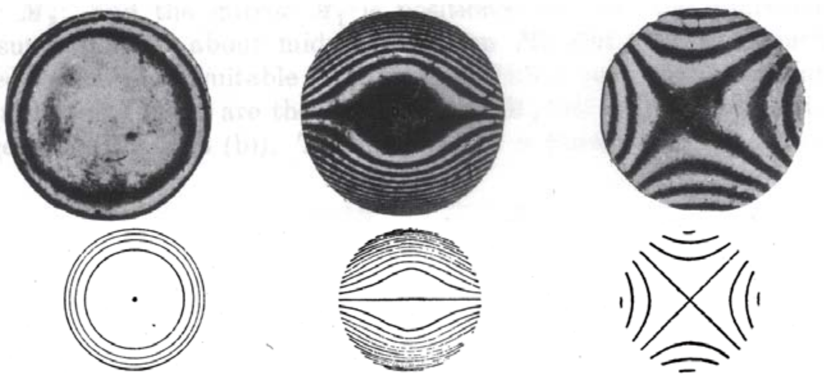
Interference fringe
干渉縞

$$I = A_1^2 + A_2^2 + 2A_1A_2 \cos\{kx\sin\alpha - \varphi_0\}$$
$$\varphi_0 = kz_0(1-\cos\alpha) - (\phi_2 - \phi_1)$$



$$kd\sin\alpha = \frac{2\pi d\sin\alpha}{\lambda} = 2\pi \;\rightarrow\; d\sin\alpha = \lambda$$

Examples of interference fringes
(Measurement of lens aberration)



(*a*) *Spherical aberration*
$\Phi_{max} = 0{\cdot}4\lambda_0$
Gaussian focal plane

(*b*) *Coma*
$\Phi_{max} = 2\lambda_0$
Gaussian focal plane

(*c*) *Astigmatism*
$\Phi_{max} = 2{\cdot}1\lambda_0$
Central plane

M. Born & E. Wolf, Principles of Optics

Fig. 7.42. Twyman–Green interference patterns from lenses showing primary aberrations. Φ_{max} is the maximum wave aberration in the exit pupil. The patterns above are observed, those below are calculated.
(After R. Kingslake, *Trans. Opt. Soc., London*, **27** (1927), 94.)

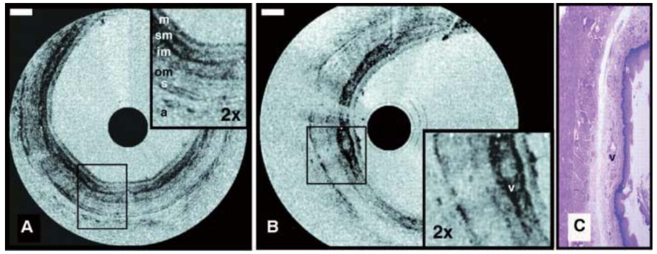


Figure 2. OCT imaging of the rabbit esophagus in vivo (22). (A) This image allows visualization of the esophageal layers of the rabbit including the mucosa (m), the submucosa (sm), the inner muscular layer (im), the outer muscular layer (om), the serosa (s), and the adipose and vascular supportive tissues (a). (B) A blood vessel (v) is apparent within the submucosa of the esophagus. (C) Corresponding histology for (B) (H&E stain). Bars, 500 µm.

G. J. Tearney, M. E. Brezinski, B. E. Bouma, S. A. Boppart, C. Pitvis, J. F. Southern, and J. G. Fujimoto, "In vivo endoscopic optical biopsy with optical coherence tomography," Science 276, 2037-2039 (1997).

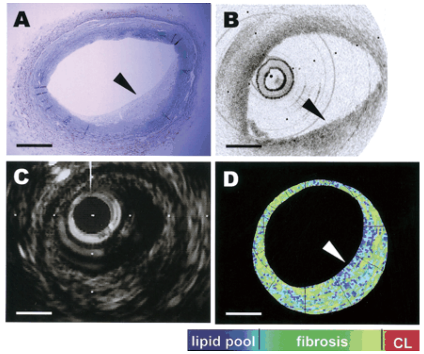
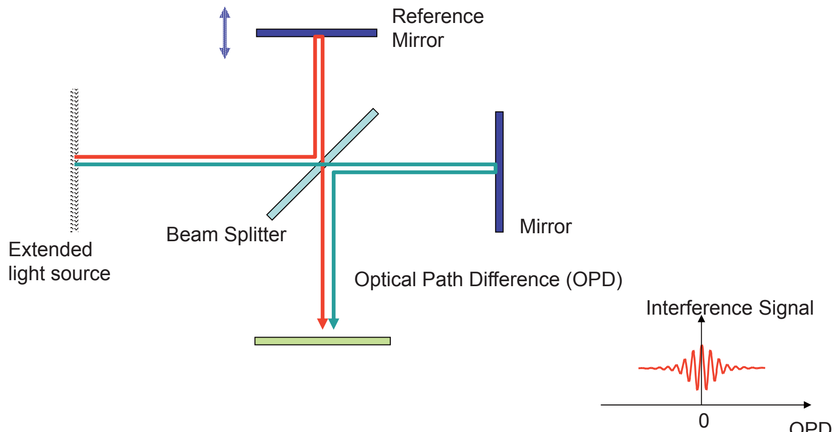


Figure 2 Example of a discrepancy between the optical coherence tomography and integrated backscatter intravascular ultrasound diagnosis. (A) Histological image (Masson's trichrome staining). (B) Optical coherence tomography image. (C) Conventional intravascular ultrasound image. (D) Integrated backscatter intravascular ultrasound image. A fibrous lesion was detected by optical coherence tomography as a homogeneous, highly backscattering (signal-rich) region (arrowhead in B). However, the fibrous lesion was misclassified as intimal hyperplasia by the integrated backscatter intravascular ultrasound image (arrowhead in D) because the fibrous lesion consisted of fibrosis with minimal collagen fiber. CL = calcification. Bar = 1 mm.

M. Kawasaki, B. E. Bouma, J. Bressner, S. L. Houser, S. K. Nadkarni, B. D. MacNeill, I.-K. Jang, H. Fujiwara, and G. J. Tearney, "Diagnostic Accuracy of Optical Coherence Tomography and Integrated Backscatter Intravascular Ultrasound Images for Tissue Characterization of Human Coronary Plaques," J. Am. Coll. Cardiol. 48, 81-88 (2006)

Michelson Interferometer



When white light source is used, interference signal can be observed only when OPD ≈ 0